Micro Theory I: Final

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Total Score: 40%, plus bonus 10%. You should start with the question you think is the easiest.

1 The Taxi Driver Problem [10%]

Consider the decision-making problem of taxi drivers. The taxi fare the driver can collect consists of a fixed surcharge p_0 per ride, and a per milleage fee p_1. The cost the driver incurs equals to a gasoline cost of r_1 per mile and the search cost $C(n)$. Assume that $C(n) = C(n_1, n_2)$ where n_1 is the number of customers found on the street, and n_2 is the number of customers arranged by the company. In order to find n_1 customers on the road, the driver has to drive around randomly. Hence, we assume it would take $L(n_1)$ miles to find n_1 customers. On the other hand, having the company arrange customers for you requires a commission fee r_2 per customer, but does not require additional driving to search. Hence, the total search cost is $C(n_1, n_2) = L(n_1) \cdot r_1 + n_2 \cdot r_2$. Assume there are n customers, each taking a taxi ride for a distance q_i, $i = 1, \cdots, n$. The distance of the ride for each of the n_1 consumers found on the street is an i.i.d. random variable: $q_i \sim$ Poisson (λ_1) (miles), while that for the n_2 consumers arranged by the company is i.i.d. $q_j \sim$ Poisson (λ_2) (miles).

1. (2%) Suppose $\lambda_1 = \lambda_2$. State the cost-minimization problem for the taxi driver given a certain number of customers $n = \overline{n}$. What is the first order condition to this problem?
2. (1%) State the expected profit for the taxi driver (to be used below).
3. (2%) Assume the taxi driver only has time to take $n = \overline{n}$ drivers. Solve the taxi driver's profit maximizing problem, given r_1, r_2, p_0, p_1.
4. (3%) Verify the law of imput demand: $\Delta n_1 \cdot \Delta r_1 \leq 0$, $\Delta n_2 \cdot \Delta r_2 \leq 0$. Can you obtain the same relationship via revealed preferences? Why or why not?
5. (2%) How would your answers to Question 3 change if $\lambda_1 \neq \lambda_2$?

2 An Academia Economy [14% + 6%]

Suppose that Joseph Wang can produce journal articles (y_2) using labor ($-y_1$). The production set $\mathcal{Y} = \hat{\mathcal{Y}} \cup \{0\}$ where

$$\hat{\mathcal{Y}} = \left\{ (y_1, y_2) \middle| y_1 \leq -\gamma, y_1 + \gamma + \frac{1}{64} y_2^2 \leq 0 \right\}$$

He enjoys both leisure and reading journal articles, so his utility function is $U(x) = 2x_1 + \ln x_2$. He has an endowment of 2γ units of time.

1. (1%) Depict the production set in a neat figure.
2. (2%) Show that the optimal output of journal articles is 4.
3. (2%) Joseph the manager is in charge of the journal production "firm." Solve for the firm's supply curve for journal articles as function of the "real wage" p_1/p_2.
4. (2%) Wang the Consumer is in charge of the article consumption "household." Solve for the consumer's demand curve for journal articles.
5. (2%) Hence or otherwise show that if there is a Walrasian Equilibrium (WE), the real wage must be 8.
6. (2%) Compute the implied equilibrium profit as a function of γ.
7. (1%) Hence obtain the values of γ for which there exists a WE.
8. (2%) Explain why this is an example where the First Welfare Theorem holds, but the Second Welfare Theorem fails. In particular, you should state the critical assumption that is violated in the Second Welfare Theorem, and show how all assumptions in the First Welfare Theorem holds.
9. (bonus 2%) Suppose instead the production set is $\hat{\mathcal{Y}} = \left\{ (y_1, y_2) \middle| y_1 \leq 0, y_1 + 2y_2 \leq 0 \right\}$. Solve for the optimal output of journal articles.
10. (bonus 3%) Can the optimal output be implemented as a Walrasian Equilibrium? If yes, solve for the Walrasian equilibrium; if not, explain why it cannot be implemented.
11. (bonus 1%) Explain why one could view Joseph the manager as an aggregate of numerous academic researchers all producing journal articles, and Wang the Consumer as the combination of many academic readers all consuming journal articles. In other words, this describes a market for journal articles in an "academia economy."

3 Consumption Vouchers [10% + 4%]

All individuals have Cobb-Douglas preferences $U^h(x) = u(x_1^h) + \delta u(x_2^h)$. where $u(x_t) = \ln x_{t1} + 2\ln x_{t2}$. The aggregate endowment is $\omega = (\omega_{11}, \omega_{12}, \omega_{21}, \omega_{22}) = (2, 2, 1, 1)$.

1. (1%) Why can you view this economy as one representative agent?
2. (2%) If commodities are non-storable, what is the Walrasian equilibrium of this economy?
3. (1%) Show that there is an equilibrium in which the future spot prices and spot prices are identical. What is the equilibrium interest rate in this economy?
4. (2%) Next suppose that there are firms that can store at no cost. That is each firm has a production set $Y^f = \left\{ y^f \middle| y_{2j}^f \leq -y_{1j}^f, j = 1, 2 \right\}$. Assume that storage takes place, what are the Walrasian equilibrium spot and future prices. (Hint: Consider the behavior of the firms.)
5. (2%) Under what conditions will there be storage?
6. (2%) Suppose due to an economic downturn due to a financial crisis, aggregate endowment is instead $\omega = (\omega_{11}, \omega_{12}, \omega_{21}, \omega_{22}) = (1, 1, 2, 2)$. What is the optimal consumption (when storage and lending is allowed)?
7. (bonus 2%) Since all banks are hit by the financial crisis, no lending can take place, (though storage is allowed). What is the Walrasian equilibrium of this economy? Would there be storage in equilibrium?
8. (bonus 2%) Next suppose the government decides to issue consumption vouchers in the first period, funded by future taxes (of the same amount) in the second period. If the government wants to induce optimal consumption, what is the amount of vouchers it should issue? What is the Walrasian equilibrium in this case?

4 Prediction Markets [6%]

Hsiao-Ying thinks that the economy will recover this year with probability 0.1. Mark thinks that the economy has an equal chance to recover or not recover this year. The preference sacling function of each person is logarithmic ($v(c) = \ln c$).

1. (4%) If the total wealth of Mark and Shiao-Ying is the same, show that the equilibrium state claims price ratio will be 3/7. (I.e. Equilibrium market belief is 30%-70%.)
2. (2%) How much would a person with a wealth W actually bet?