Aversion to Risk

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(Lecture 16, Micro Theory I)

Dealing with Uncertainty

- Preferences over risky choices (Section 7.1)
- One simple model: Expected Utility

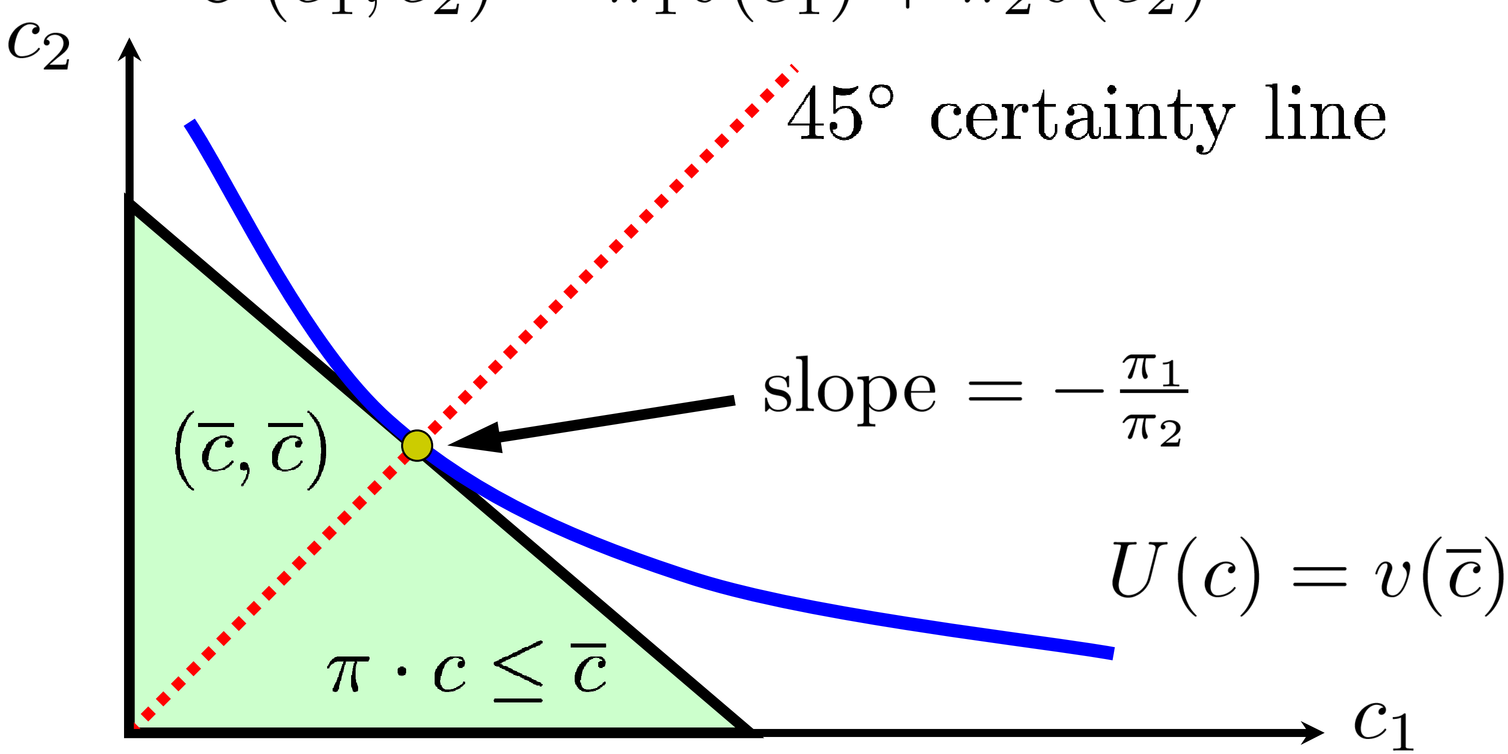
$$U(c_1, c_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

- How can old tools be applied to analyze this?
- How is “risk aversion” measured?
- What about differences in risk aversion?
- How does a risk averse person trade state claims? (Wealth effects? Individual diff.?)

Dealing with Uncertainty

- Two states: s=1: KMT wins; s=2: DPP wins
- π_s: Prob. of state s c_s: consumption in state s

$$U(c_1, c_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

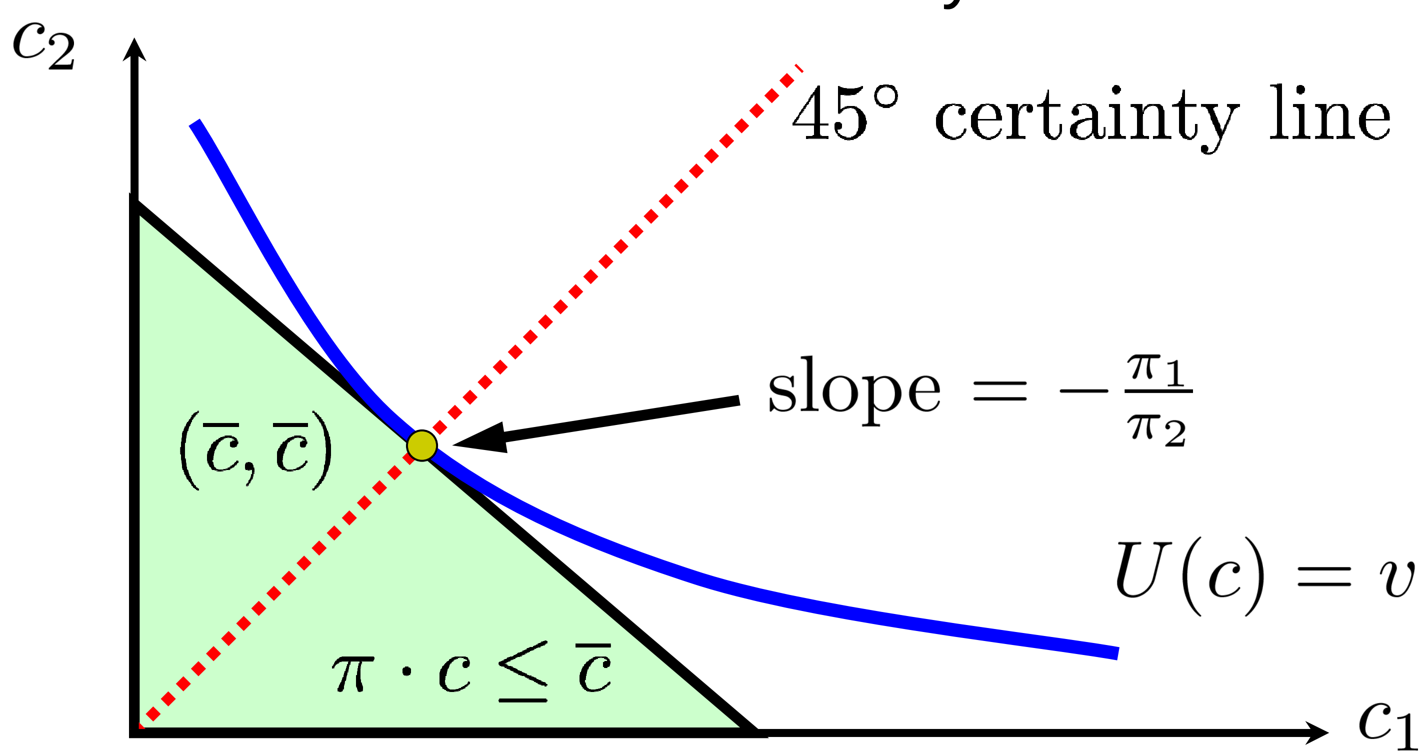


Risk Aversion: Concave *v(c)*

- Upper contour sets of *U(.)* is convex

$$U(c_1, c_2) = \pi_1 v(c_1) + (1 - \pi_1) v(c_2) \leq v(\overline{c})$$

  - Prefers certain bundle to risky ones with same EV

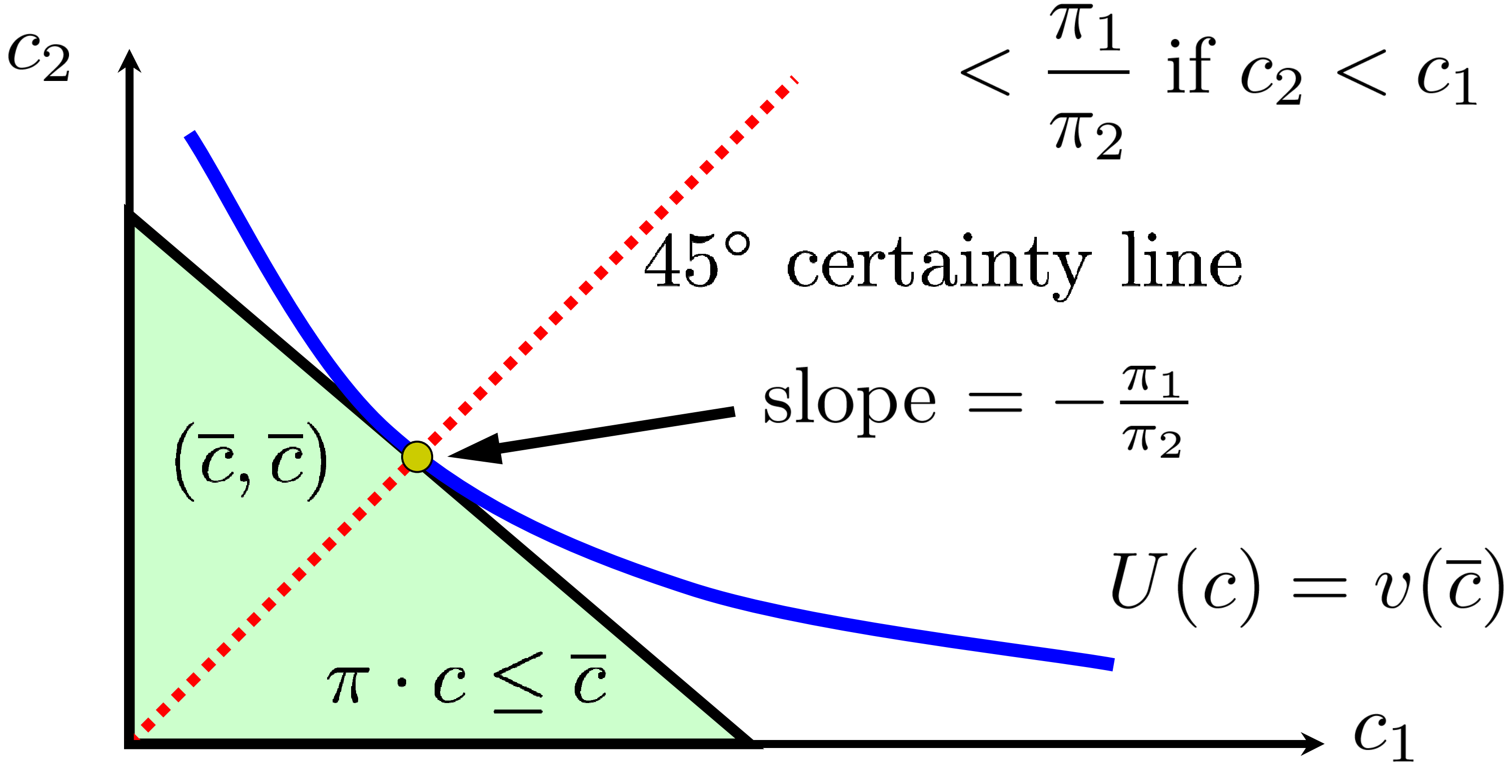


Risk Aversion: Concave *v(c)*

$$c_2 > c_1 \Rightarrow v'(c_1) > v'(c_2)$$

$$MRS(c_1, c_2) = \left.\frac{dc_2}{dc_1}\right|_{U=\overline{U}} = \frac{\frac{\partial U}{\partial c_1}}{\frac{\partial U}{\partial c_2}} = \frac{\pi_1 v'(c_1)}{\pi_2 v'(c_2)}$$

$$< \frac{\pi_1}{\pi_2} \text{ if } c_2 < c_1$$

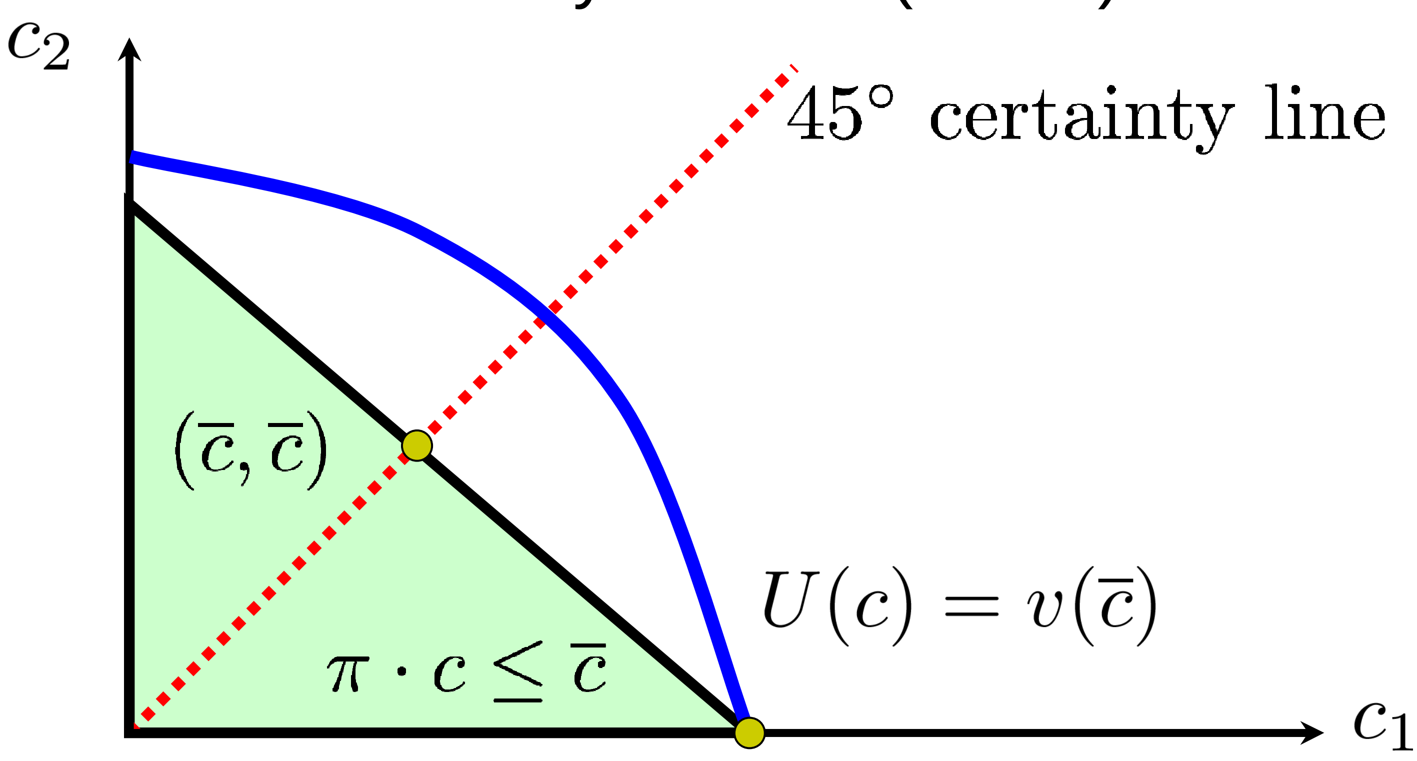


Extremely Risk Loving: Convex *v(c)*

- Upper contour sets of *U(.)* is convex

$$U(c_1, c_2) = \pi_1 v(c_1) + (1 - \pi_1) v(c_2) \geq v(\overline{c})$$

  - Prefers most risky bundles (weird!)



Jensen's Inequality

- For any probability vector π and consumption vector c, if $v(c)$ is concave, then

$$\sum_{s=1}^{S} \pi_s v(c_s) \leq v(\overline{c}) \text{ where } \overline{c} = \sum_{s=1}^{S} \pi_s c_s$$

- Proof:
- Easy if $v(c)$ is continuously differentiable, since

$$\text{Concavity implies } v(c_s) \leq v(\overline{c}) + v'(\overline{c})(c_s - \overline{c})$$

- Weighted average yields the inequality. QED.

Measure Risk Aversion

- Let: $M = MRS(c_1, c_2) = \left.\frac{dc_2}{dc_1}\right|_{U=\overline{U}} = \frac{\pi_1 v'(c_1)}{\pi_2 v'(c_2)}$
- Then:

$$\ln M = \ln v'(c_1) - \ln v'(c_2) + \ln\left(\frac{\pi_1}{\pi_2}\right)$$

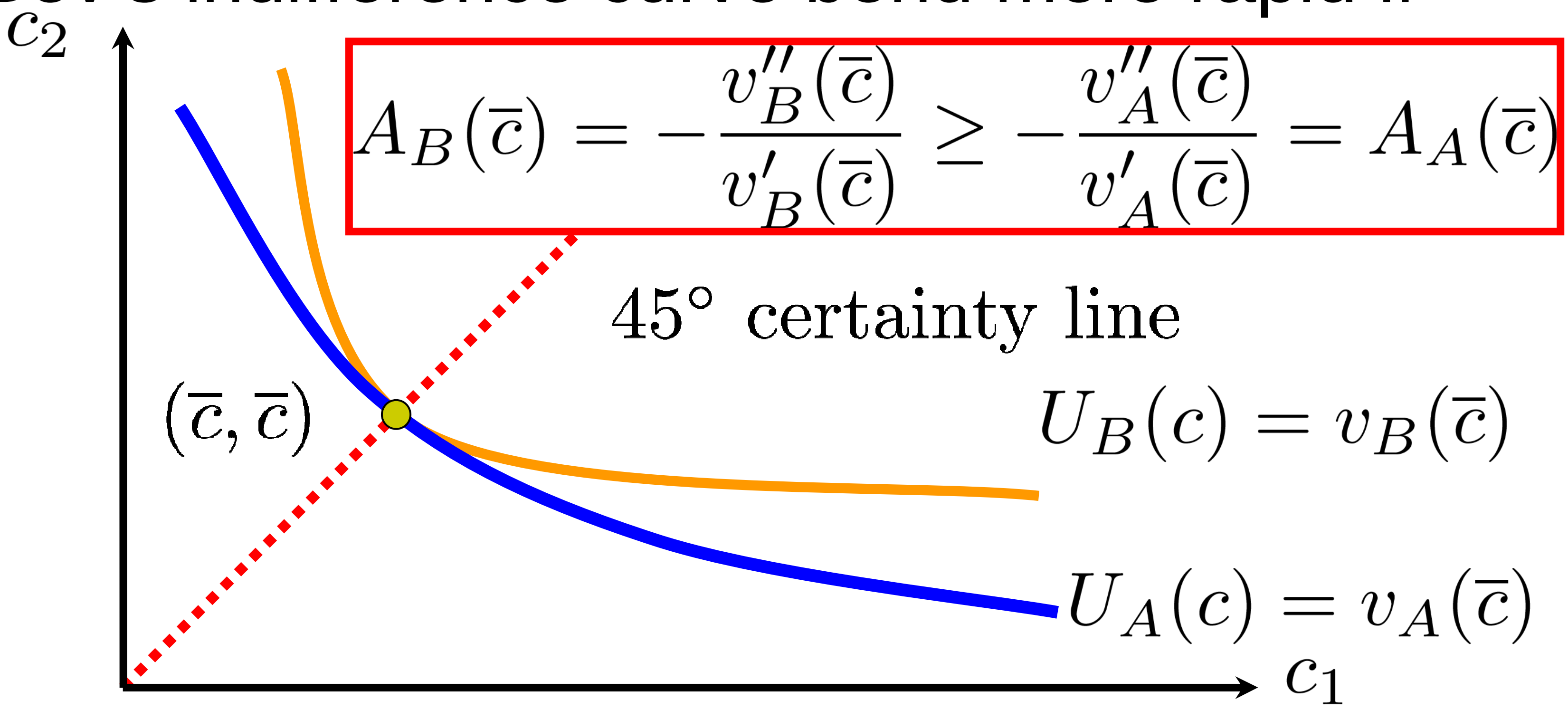
- So, $\frac{\partial}{\partial c_1}\ln M = \frac{v''(c_1)}{v'(c_1)}, \qquad \frac{\partial}{\partial c_2}\ln M = -\frac{v''(c_2)}{v'(c_2)}$
- Thus, $\frac{1}{M}\frac{dM}{dc_1} = \frac{\partial}{\partial c_1}\ln M + \frac{\partial}{\partial c_2}\ln M \left.\frac{dc_2}{dc_1}\right|_{U=\overline{U}}$

$$= \frac{v''(c_1)}{v'(c_1)} - \frac{v''(c_2)}{v'(c_2)} \cdot \left(-\frac{\pi_1}{\pi_2}\right)$$

Measuring Risk Aversion

- At $(\bar{c}, \bar{c})$, $\dfrac{1}{M}\dfrac{dM}{dc_1} = \dfrac{v''(\bar{c})}{v'(\bar{c})} \cdot \left(1 + \dfrac{\pi_1}{\pi_2}\right)$
- Bev's indifference curve bend more rapid if

$$A_B(\bar{c}) = -\frac{v_B''(\bar{c})}{v_B'(\bar{c})} \geq -\frac{v_A''(\bar{c})}{v_A'(\bar{c})} = A_A(\bar{c})$$



Measuring Risk Aversion

- Absolute Risk Aversion $A(c) = -\frac{v''(\overline{c})}{v'(\overline{c})}$
- Relative Risk Aversion $R(c) = -\frac{cv''(\overline{c})}{v'(\overline{c})}$
- Indifference curve bend more rapid if *A(c)* high
- Can also obtain:
- *A(c)* higher ➔ acceptable gambles set smaller
  - But need to first establish the relationship between two people's (risk averse) utility functions…

Proposition 7.2-1: Differences in Risk Aversion

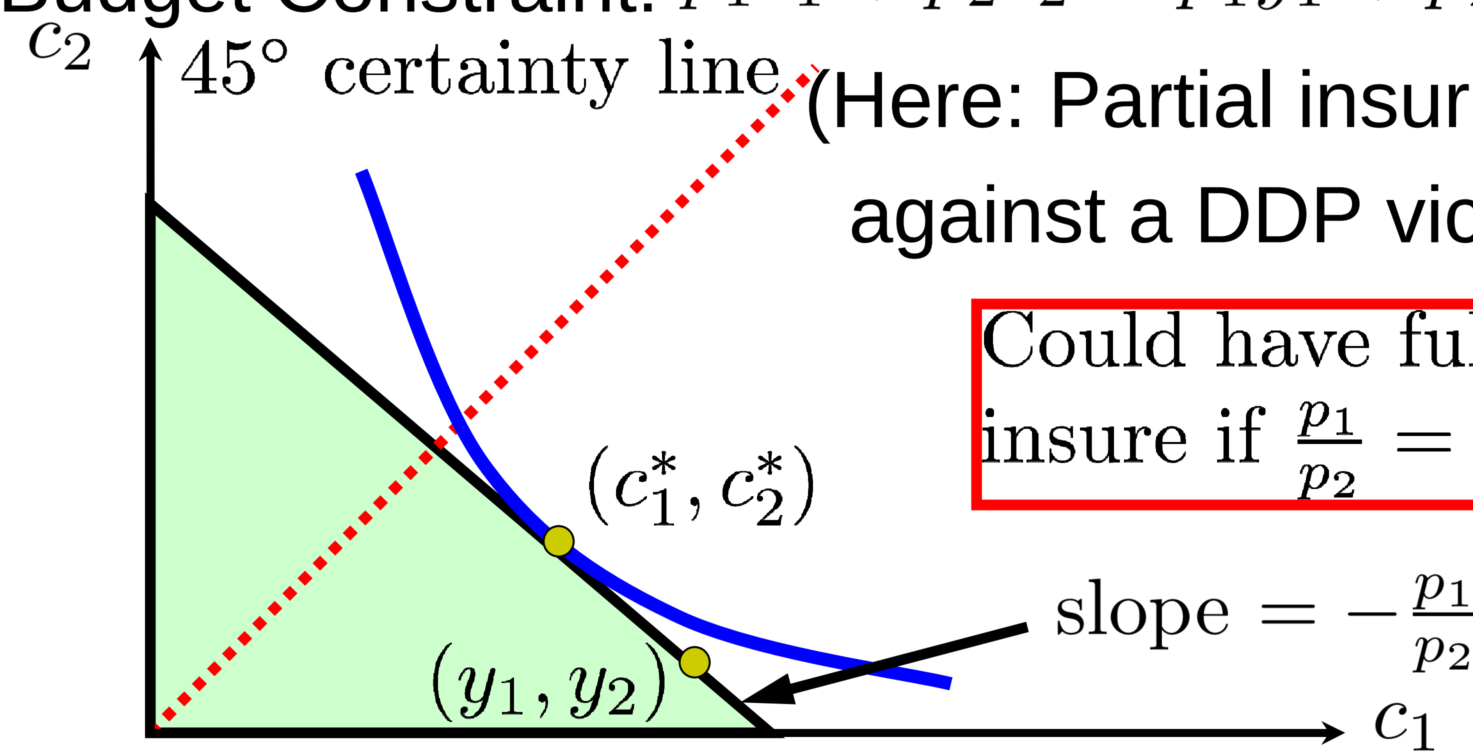
- Two (von Neumann-Morgenstern) expected utility functions: v_A, v_B
- Then $$A_B(c) = -\frac{v_B''(c)}{v_B'(c)} \geq -\frac{v_A''(c)}{v_A'(c)} = A_A(c)$$
- iff the mapping $f(\cdot) : v_A \rightarrow v_B$ is concave.
- Proof:

Proposition 7.2-2:Risk Aversion & the Set of Acceptable Gambles

- If $$A_B(c) = -\frac{v_B''(c)}{v_B'(c)} \geq -\frac{v_A''(c)}{v_A'(c)} = A_A(c)$$
- and both start with the same wealth $\bar{c}$. Then,
- The set of acceptable gambles to B is a subset of the set of gambles acceptable to A.
- Proof:

Trading in State Claim Markets

- y_s: Endowment in state s, $y_1 > y_2$
- p_s: current price of unit consumption in state s
- Budget Constraint: $p_1 c_1 + p_2 c_2 = p_1 y_1 + p_2 y_2$



(Here: Partial insurance against a DDP victory)

Could have fully insure if $\frac{p_1}{p_2} = \frac{\pi_1}{\pi_2}$

Wealth↑, how would riskiness of optimal choice change?

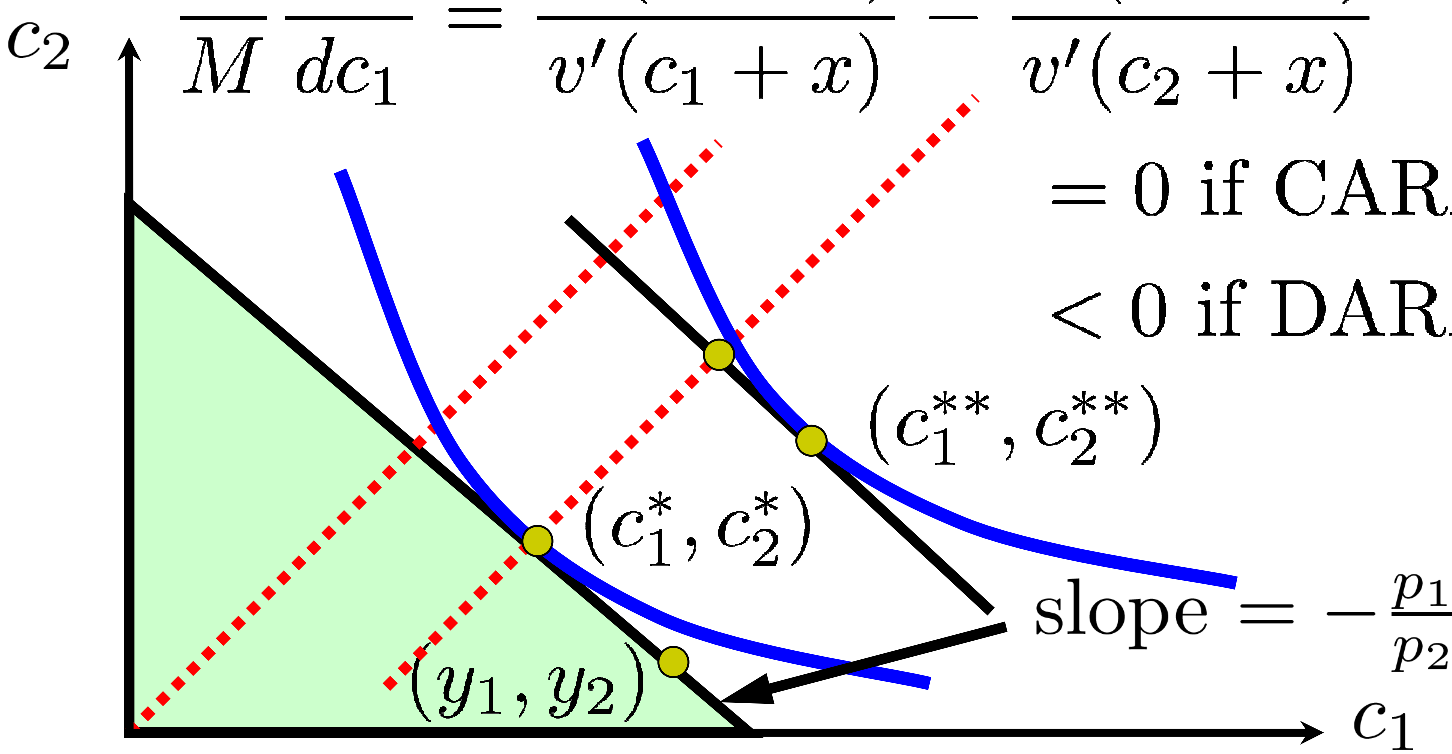
- Move from (c_1, c_2) to $(c_1 + x, c_2 + x)$

$$\ln M = \ln v'(c_1 + x) - \ln v'(c_2 + x) + \ln\left(\frac{\pi_1}{\pi_2}\right)$$

$$\frac{1}{M}\frac{dM}{dc_1} = \frac{v''(c_1 + x)}{v'(c_1 + x)} - \frac{v''(c_2 + x)}{v'(c_2 + x)}$$

$$= 0 \text{ if CARA}$$

$$< 0 \text{ if DARA}$$



Would a more risk averse person invest less risky?

- Yes…

Summary of 7.2

- Homework: Exercise 7.2-1~8