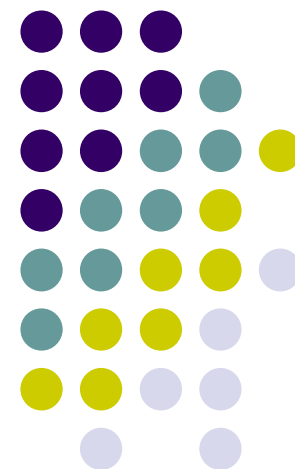


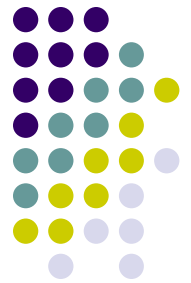
General Equilibrium and Efficiency with Production

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(Lecture 13, Micro Theory I)



What We Learned from Exchange Economy and Robinson Crusoe?



- Pareto Efficient Allocation (PEA) – Optimum
 - Cannot make one better off without hurting others
- Walrasian Equilibrium (WE) – Price Vector
 - When Supply Meets Demand
 - Even if there is only one person Robinson Crusoe
- 1st Welfare Theorem: WE are PEA
 - Schizophrenic Robinson Crusoe achieves optimum
- 2nd Welfare Theorem: PEA supported as WE
- These also apply to the general case as well!

The General Model: Firms



- F Firms: $f=1, 2, \dots, F$
- Production Plan $y^f = (y_1^f, \dots, y_n^f) \in \gamma^f$
- Production Set $\gamma^f \subset \mathbb{R}^n$
- Production Plan for the Economy $\{y^f\}_{f=1}^F$
- Aggregate Production Plan

$$y = \sum_{f=1}^F y^f$$

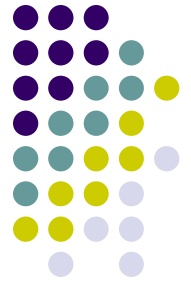
- Aggregate Production Set γ

The General Model: Consumers



- n Commodities (are private): $1, 2, \dots, n$
- H Consumers: $h = 1, 2, \dots, H$
 - Consumption Set: $X^h \subset \mathbb{R}^n$
 - Endowment: $\omega^h = (\omega_1^h, \dots, \omega_n^h) \in X^h$
 - Consumption Vector: $x^h = (x_1^h, \dots, x_n^h) \in X^h$
 - Utility Function: $U^h(x^h) = U^h(x_1^h, \dots, x_n^h)$
 - Consumption Allocation: $\{x^h\}_{h=1}^H$
 - Aggregate Consumption and Endowment:
$$x = \sum_{h=1}^H x^h \text{ and } \omega = \sum_{h=1}^H \omega^h$$

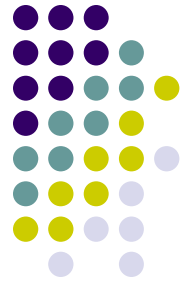
The General Model: Shareholdings



- All firms are owned by consumers.
- Consumer h has θ^{hf} ownership shares in firm f
- Ownership share must sum up to one:

$$\sum_{h=1}^H \theta^{hf} = 1, f = 1, \dots, F$$

The General Model: Feasible Allocation



- A allocation is **feasible** if
- The sum of the net demands **doesn't exceed** aggregate production plan:

$$x - \omega \leq y$$
- A feasible plan for the economy $\{\hat{x}^h\}_{h=1}^H, \{\hat{y}^f\}_{f=1}^F$ is **Pareto efficient** if
- there is no other feasible plan $\{x^h\}_{h=1}^H, \{y^f\}_{f=1}^F$
- that is
- **strictly preferred** by at least one:
- and **weakly preferred** by all:

$$U^i(x^i) > U^i(\hat{x}^i)$$

$$U^h(x^h) \geq U^h(\hat{x}_6^h)$$



Walrasian Equilibrium

- Price-taking firms and consumers: Prices $p > 0$
- Consumers: $h=1, 2, \dots, H$
 - Firm Profit: $p \cdot y^f$
 - Total Dividend Payments: $\sum_f \theta^{hf} p \cdot y^f$
 - Wealth: $W^h = p \cdot \omega^h + \sum_f \theta^{hf} p \cdot y^f$
 - Budget Set: $\{x^h \in X^h \mid p \cdot x^h \leq W^h\}$
- Consumer interest best served by profit maximizing firm managers



Walrasian Equilibrium

- Price-taking: Prices $p > 0$
- Firm f chooses production plan \bar{y}^f to max. profit
 $p \cdot \bar{y}^f \geq p \cdot y^f$, for all $y^f \in \gamma^f$, $f = 1, \dots, F$
- Consumer h chooses most preferred consumption in her budget set:
 $\bar{x}^h \succ_h x^h$ for all x^h such that $p \cdot x^h \leq W^h$
- Vector of Excess Demand: $\bar{e} = \bar{x} - \omega - \bar{y}$
 - Total consumption $\bar{x} = \sum \bar{x}^h$
 - Total production $\bar{y} = \sum \bar{y}^f$

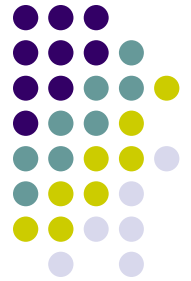
Definition: Walrasian Equilibrium Prices



- The price vector $p \geq 0$ is a **Walrasian Equilibrium price vector** if
- there is no market in excess demand ($\bar{e} \leq 0$),
- and $p_j = 0$ for any market that is in excess supply ($\bar{e}_j < 0$).

- We are now ready to state and prove the “Adam Smith Theorem” (WE \rightarrow PEA)...

Proposition 5.2-1: First Welfare Theorem



- If preferences of each consumer satisfies LNS, then the Walrasian Equilibrium allocation is Pareto efficient.
- Proof:

Proposition 5.2-3: Second Welfare Theorem



- Let $\{\hat{x}^h\}_{h=1}^H, \{\hat{y}^f\}_{f=1}^F$ be a PEA. Suppose:
 1. Consumption vectors are private
 2. Consumption sets X^h are convex
 3. Utility functions $U^h(\cdot)$ are continuous, quasi-concave, and satisfies LNS
 4. There is some $\underline{x}^h \in X^h$ such that $\underline{x}^h < \hat{x}^h$
 5. Production sets γ^f convex and of free disposal
- Then there exist a price vector $p \geq 0$ such that
$$x^h \succ_h \hat{x}^h \Rightarrow p \cdot x^h > p \cdot \hat{x}^h$$
$$y^f \in \gamma^f \Rightarrow p \cdot y^f \leq p \cdot \hat{y}^f$$



Summary of 5.2

- Pareto Efficiency:
 - Cannot make one better off without hurting others
- Walrasian Equilibrium: market clearing prices
- Welfare Theorems:
 - First: Walrasian Equilibrium is Pareto Efficient
 - Second: Pareto Efficient allocations can be supported as Walrasian Equilibria (with transfer)
- Homework: Exercise 5.2-1~3