General Equilibrium and Efficiency with Production

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2008/12/12

(Lecture 13, Micro Theory I)

What We Learned from Exchange Economy and Robinson Crusoe?

- Pareto Efficient Allocation (PEA) – Optimum
  - Cannot make one better off without hurting others
- Walrasian Equilibrium (WE) – Price Vector
  - When Supply Meets Demand
  - Even if there is only one person Robinson Crusoe
- 1st Welfare Theorem: WE are PEA
  - Schizophrenic Robinson Crusoe achieves optimum
- 2nd Welfare Theorem: PEA supported as WE
- These also apply to the general case as well!

The General Model: Firms

- *F* Firms: *f*=*1, 2, …, F*
- Production Plan $y^f = (y_1^f, \cdots, y_n^f) \in \gamma^f$
- Production Set $\gamma^f \subset \mathbb{R}^n$
- Production Plan for the Economy $\{y^f\}_{f=1}^F$
- Aggregate Production Plan

$$y = \sum_{f=1}^{F} y^f$$

- Aggregate Production Set γ

The General Model: Consumers

- n Commodities (are private): *1, 2, …, n*
- H Consumers: $h = 1, 2, \cdots, H$
  - Consumption Set: $X^h \subset \mathbb{R}^n$
  - Endowment: $\omega^h = (\omega_1^h, \cdots, \omega_n^h) \in X^h$
  - Consumption Vector: $x^h = (x_1^h, \cdots, x_n^h) \in X^h$
  - Utility Function: $U^h(x^h) = U^h(x_1^h, \cdots, x_n^h)$
  - Consumption Allocation: $\{x^h\}_{h=1}^H$
  - Aggregate Consumption and Endowment:
    $x = \sum_{h=1}^H x^h$ and $\omega = \sum_{h=1}^H \omega^h$

The General Model: Shareholdings

- All firms are owned by consumers.
- Consumer h has θ^{hf} ownership shares in firm f
- Ownership share must sum up to one:

$$\sum_{h=1}^{H} \theta^{hf} = 1, f = 1, \cdots, F$$

The General Model: Feasible Allocation

- A allocation is feasible if
- The sum of the net demands doesn't exceed aggregate production plan: $x - \omega \leq y$
- A feasible plan for the economy $\left\{\hat{x}^h\right\}_{h=1}^{H}, \left\{\hat{y}^f\right\}_{f=1}^{F}$ is Pareto efficient if
- there is no other feasible plan $\left\{x^h\right\}_{h=1}^{H}, \left\{y^f\right\}_{f=1}^{F}$
- that is
- strictly preferred by at least one: $U^i(x^i) > U^i(\hat{x}^i)$
- and weakly preferred by all: $U^h(x^h) \geq U^h(\hat{x}^h)$

Walrasian Equilibrium

- Price-taking firms and consumers: $\text{Prices } p > 0$
- Consumers: *h=1, 2, …, H*
  - Firm Profit: $p \cdot y^f$
  - Total Dividend Payments: $\sum_f \theta^{hf} p \cdot y^f$
  - Wealth: $W^h = p \cdot \omega^h + \sum_f \theta^{hf} p \cdot y^f$
  - Budget Set: $\{x^h \in X^h | p \cdot x^h \leq W^h\}$
- Consumer interest best served by profit maximzing firm managers

Walrasian Equilibrium

- Price-taking: Prices $p > 0$
- Firm f chooses production plan $\overline{y}^f$ to max. profit
  $$p \cdot \overline{y}^f \geq p \cdot y^f, \text{ for all } y^f \in \gamma^f, f = 1, \cdots, F$$
- Consumer h chooses most preferred consumption in her budget set:
  $$\overline{x}^h \succ_h x^h \text{ for all } x^h \text{ such that } p \cdot x^h \leq W^h$$
- Vector of Excess Demand: $\overline{e} = \overline{x} - \omega - \overline{y}$
  - Total consumption $\overline{x} = \sum \overline{x}^h$
  - Total production $\overline{y} = \sum \overline{y}^f$

Definition: Walrasian Equilibrium Prices

- The price vector $p \geq 0$ is a Walrasian Equilibrium price vector if
- there is no market in excess demand ($\bar{e} \leq 0$),
- and $p_j = 0$ for any market that is in excess supply ($\bar{e}_j < 0$).

- We are now ready to state and prove the "Adam Smith Theorem" (WE ➔ PEA)…

Proposition 5.2-1: First Welfare Theorem

- If preferences of each consumer satisfies LNS, then the Walrasian Equilibrium allocation is Pareto efficient.
- Proof:

Proposition 5.2-3: Second Welfare Theorem

- Let $\{\hat{x}^h\}_{h=1}^{H}, \{\hat{y}^f\}_{f=1}^{F}$ be a PEA. Suppose:

1. Consumption vectors are private
2. Consumption sets X^h are convex
3. Utility functions $U^h(\cdot)$ are continuous, quasi-concave, and satisfies LNS
4. There is some $\underline{x}^h \in X^h$ such that $\underline{x}^h < \hat{x}^h$
5. Production sets γ^f convex and of free disposal

- Then there exist a price vector $p \geq 0$ such that

$$x^h \succ_h \hat{x}^h \Rightarrow p \cdot x^h > p \cdot \hat{x}^h$$

$$y^f \in \gamma^f \Rightarrow p \cdot y^f \leq p \cdot \hat{y}^f$$

Summary of 5.2

- Pareto Efficiency:
  - Cannot make one better off without hurting others
- Walrasian Equilibrium: market clearing prices
- Welfare Theorems:
  - First: Walrasian Equilibrium is Pareto Efficient
  - Second: Pareto Efficient allocations can be supported as Walrasian Equilibria (with transfer)
- Homework: Exercise 5.2-1~3