Edgeworth Box Experiment

Joseph Tao-yi Wang
2019/10/2
(Lecture 8b, Micro Theory I)

Road Map for Chapter 3

- Pareto Efficiency Allocation (PEA)
 - Cannot make one better off without hurting others
- Walrasian (Price-taking) Equilibrium (WE)
 - When Supply Meets Demand
 - Focus on Exchange Economy First
- 1st Welfare Theorem:
 - Any WE is PEA (Adam Smith Theorem)
- 2nd Welfare Theorem:
 - Any PEA can be supported as a WE with transfers

2x2 Exchange Economy

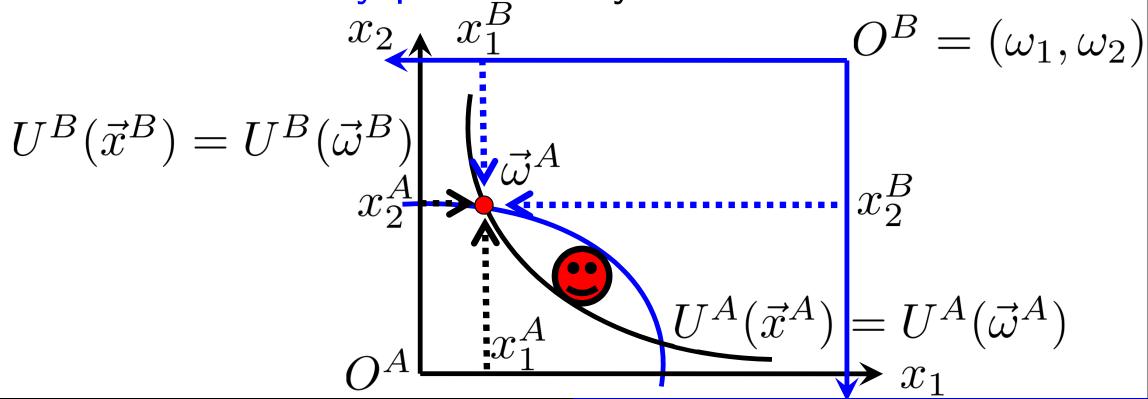
- 2 Commodities: Good 1 and 2
- 2 Consumers: Alex and Bev h = A, B
 - Endowment: $\vec{\omega}^h = (\omega_1^h, \omega_2^h), \, \omega_i = \omega_i^A + \omega_i^B$
 - Consumption Set: $\vec{x}^h = (x_1^h, x_2^h) \in \mathbb{R}^2_+$
 - Strictly Monotonic Utility:

$$U^{h}(\vec{x}^{h}) = U^{h}(x_{1}^{h}, x_{2}^{h}), \quad \frac{\partial U^{h}}{\partial x_{i}^{h}}(\vec{x}^{h}) > 0$$

- Edgeworth Box
 - These consumers could be representative agents, or literally TWO people (bargaining)

Pareto Efficiency

- A feasible allocation is Pareto efficient if
- there is no other feasible allocation that is
- strictly preferred by at least one consumer
- and is weakly preferred by all consumers.

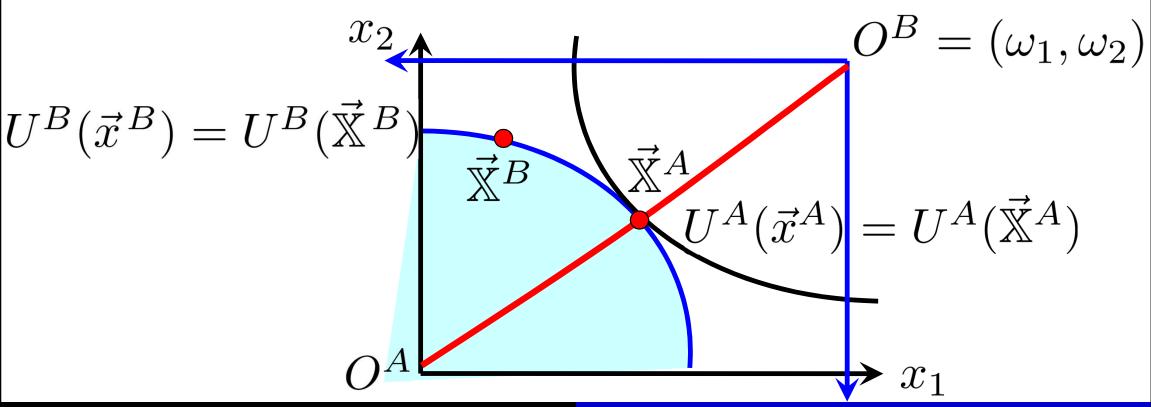


Pareto Efficient Allocations

For
$$\vec{\omega} = (\omega_1, \omega_2)$$
, consider

$$\max_{\vec{x}^A, \vec{x}^B} \left\{ U^A(\vec{x}^A) | U^B(\vec{x}^B) \ge U^B(\vec{X}^B), \vec{x}^A + \vec{x}^B \le \vec{\omega} \right\}$$

Need
$$MRS^A(\vec{X}^A) = MRS^B(\vec{X}^A)$$
 (interior solution)



PEA with Cobb-Douglas Utility

$$\max_{x,y} U^{A}(x,y) = x^{\alpha}y^{1-\alpha}$$
s.t. $U^{B} = (\overline{x} - x)^{\beta}(\overline{y} - y)^{1-\beta} \ge U^{B}$

$$\mathcal{L} = x^{\alpha}y^{1-\alpha} - \lambda \cdot \left[U^{B} - (\overline{x} - x)^{\beta}(\overline{y} - y)^{1-\beta}\right]$$
FOC: (for interior solutions)
$$\frac{\partial \mathcal{L}}{\partial x} = \alpha \cdot \frac{y^{1-\alpha}}{x^{1-\alpha}} - \beta\lambda \cdot \frac{(\overline{y} - y)^{1-\beta}}{(\overline{x} - x)^{1-\beta}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = (1 - \alpha) \cdot \frac{x^{\alpha}}{y^{\alpha}} - (1 - \beta)\lambda \cdot \frac{(\overline{x} - x)^{\beta}}{(\overline{y} - y)^{\beta}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = U^{B} - (\overline{x} - x)^{\beta}(\overline{y} - y)^{1-\beta} = 0$$

PEA with Cobb-Douglas Utility

Meaning of FOC: $MRS^A = MRS^B$

$$\lambda = \frac{\alpha \cdot \frac{y^{1-\alpha}}{x^{1-\alpha}}}{\beta \cdot \frac{(\overline{y} - y)^{1-\beta}}{(\overline{x} - x)^{1-\beta}}} = \frac{(1-\alpha) \cdot \frac{x^{\alpha}}{y^{\alpha}}}{(1-\beta) \cdot \frac{(\overline{x} - x)^{\beta}}{(\overline{y} - y)^{\beta}}}$$

$$\Rightarrow \alpha \cdot y \cdot (1-\beta) \cdot (\overline{x} - x) = \beta \cdot (\overline{y} - y) \cdot (1-\alpha) \cdot x$$

$$= \frac{\beta(1-\alpha) \cdot \overline{y} \cdot x}{\alpha(1-\beta)(\overline{x} - x) + \beta(1-\alpha) \cdot x} = \frac{\gamma \overline{y} \cdot x}{(\gamma - 1)x + \overline{x}}$$

$$= \frac{\frac{8}{3} \cdot 50 \cdot x}{\frac{5}{3}x + 50} = \frac{400x}{5x + 150} \quad \alpha = 0.6, \quad \gamma = \frac{\beta(1-\alpha)}{\alpha(1-\beta)}$$

Joseph Tao-yi Wang 2x2 Exchange Economy

with Cobb-Douglas Utility 48 47 46 45 44 43 42 41 40 39 38 37 36 35 34 33 32 31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 Contract Curve (all PEA) 400x466 473 479 453 459 466 473 480 175 456 464 472 479 5x + 1501&A7&B7 B8 A5 A4 A9 A3 A2 B10 200 190 179 168

9 | 11 | 13 | 15 | 17 | 18 | 20 | 21 | 23 | 24 | 25 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 36 | 37 | 38 | 39 | 39 | 40 | 41 | 41 | 42 | 43 | 43 | 44 | 45 | 45 | 46 | 46 | 47 | 47 | 48 | 48 | 49 | 49 | 50 | 50

Walrasian Equilibrium - 2x2 Exchange Economy

- All Price-takers: Price vector $\vec{p} \ge 0$
- 2 Consumers: Alex and Bev $h \in \mathcal{H} = \{A, B\}$
 - Endowment: $\vec{\omega}^h = (\omega_1^h, \omega_2^h), \, \omega_i = \omega_i^A + \omega_i^B$
 - Consumption Set: $\vec{x}^h = (x_1^h, x_2^h) \in \mathbb{R}^2_+$
 - Wealth: $W^h = \vec{p} \cdot \vec{\omega}^h$
- Market Demand: $\vec{x}(\vec{p}) = \sum_h \vec{x}^h(\vec{p}, \vec{p} \cdot \vec{\omega}^h)$ (Solution to consumer problem)
- Vector of Excess Demand: $\vec{z}(\vec{p}) = \vec{x}(\vec{p}) \vec{\omega}$
 - Vector of total Endowment: $\vec{\omega} = \sum_h \vec{\omega}^h$

Definition: Market Clearing Prices

- Let Excess Demand for Commodity j be $z_j(\vec{p})$
- The Market for Commodity j Clears if
 - Excess Demand = 0 or Price = 0 (and ED < 0)
 - Excess demand = shortage; negative ED means surplus

$$z_j(\vec{p}) \leq 0$$
 and $p_j \cdot z_j(\vec{p}) = 0$

- Why is this important?
- 1. Walras Law
 - The last market clears if all other markets clear
- 2. Market clearing defines Walrasian Equilibrium

Local Non-Satiation Axiom (LNS)

- For any consumption bundle $\vec{x} \in C \subset \mathbb{R}^n$ and any δ -neighborhood $N(\vec{x}, \delta)$ of \vec{x} , there is some bundle $\vec{y} \in N(\vec{x}, \delta)$ s.t. $\vec{y} \succ_h \vec{x}$
- LNS implies consumer must spend all income
- If not, we have $\vec{p} \cdot \vec{x}^h < \vec{p} \cdot \vec{\omega}^h$ for optimal \vec{x}^h
- But then there exist δ -neighborhood $N(\vec{x}^h, \delta)$
- In the budget set for sufficiently small $\delta > 0$
- LNS $\Rightarrow \vec{y} \in N(\vec{x}^h, \delta), \vec{y} \succ_h \vec{x}^h, \vec{x}^h \text{ is not optimal!}$

Local Non-Satiation Axiom (LNS)

For any consumption bundle $\vec{x} \in C \subset \mathbb{R}^n$ and any δ -neighborhood $N(\vec{x}, \delta)$ of \vec{x} , there is some bundle $\vec{y} \in N(\vec{x}, \delta)$ s.t. $\vec{y} \succ_h \vec{x}$

- LNS implies consumer must spend all income
- If not, we have $\vec{p} \cdot \vec{x}^h < \vec{p} \cdot \vec{\omega}^h$ for optimal \vec{x}^h
- But then there exist δ -neighborhood $N(\vec{x}^h, \delta)$
- In the budget set for sufficiently small $\delta > 0$
- LNS $\Rightarrow \vec{y} \in N(\vec{x}^h, \delta), \vec{y} \succ_h \vec{x}^h, \vec{x}^h \text{ is not optimal!}$

Walras Law

• For any price vector \vec{p} , the market value of excess demands must be zero, because:

$$\vec{p} \cdot \vec{z}(\vec{p}) = \vec{p} \cdot (\vec{x} - \vec{\omega}) = \vec{p} \cdot \left(\sum_{h} (\vec{x}^h - \vec{\omega}^h)\right)$$

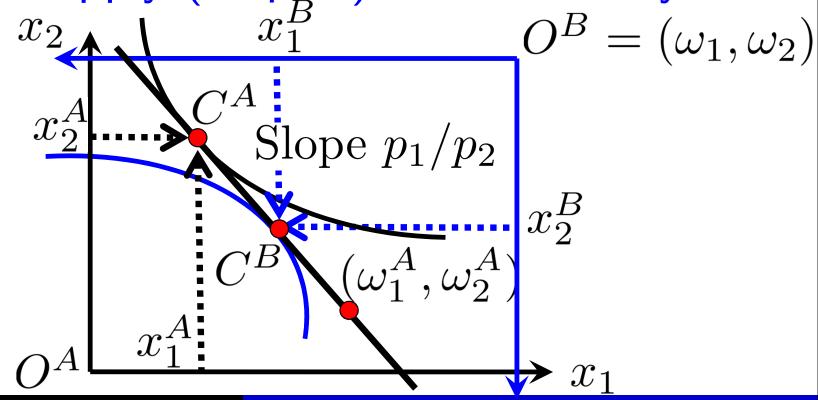
$$= \sum_{h} (\vec{p} \cdot \vec{x}^h - \vec{p} \cdot \vec{\omega}^h) = 0 \text{ by LNS}$$

$$p_1 z_1(\vec{p}) + p_2 z_2(\vec{p}) = 0$$

If one market clears, so must the other.

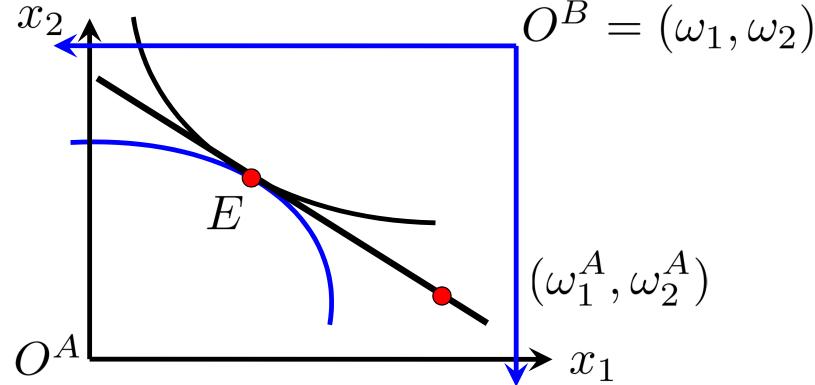
Definition: Walrasian Equilibrium

- The price vector $\vec{p} \ge \vec{0}$ is a Walrasian Equilibrium price vector if all markets clear.
 - WE = price vector!!!
- EX: Excess supply (surplus) of commodity 1...



Definition: Walrasian Equilibrium

- Lower price for commodity 1 if excess supply
 - Until Markets Clear



- Cannot raise Alex's utility without hurting Bev
 - Hence, we have FWT...

First Welfare Theorem: WE -> PEA

- If preferences satisfy LNS, then a Walrasian Equilibrium allocation (in an exchange economy) is Pareto efficient.
- Sketch of Proof:
- 1. Any weakly (strictly) preferred bundle must cost at least as much (strictly more) as WE
- 2. Markets clear
 - > Pareto preferred allocation not feasible

Walrasian Equilibrium: Consumer A Problem

$$\max_{x,y} U^{A}(x,y) = x^{\alpha}y^{1-\alpha}$$
s.t. $P_{x} \cdot x + P_{y} \cdot y \leq I^{A} = P_{x} \cdot \omega_{x}^{A} + P_{y} \cdot \omega_{y}^{A}$

$$\mathcal{L} = x^{\alpha}y^{1-\alpha} - \lambda \cdot \left[P_{x} \cdot x + P_{y} \cdot y - I^{A}\right]$$
FOC: (for interior solutions)
$$\frac{\partial \mathcal{L}}{\partial x} = \alpha \cdot \frac{y^{1-\alpha}}{x^{1-\alpha}} - \lambda \cdot P_{x} = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = (1-\alpha) \cdot \frac{x^{\alpha}}{y^{\alpha}} - \lambda \cdot P_{y} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = P_{x} \cdot x + P_{y} \cdot y - I^{A} = 0$$

Walrasian Equil.: Consumer Optimal Choice

Meaning of FOC:
$$MRS^A = \frac{P_x}{P_y}$$

$$\frac{P_x}{P_y} = \frac{\alpha}{1 - \alpha} \cdot \frac{y}{x} \quad \Rightarrow x = \frac{\alpha}{1 - \alpha} \cdot \frac{P_y}{P_x} \cdot y$$

$$\Rightarrow I^A = P_x \cdot x + P_y \cdot y = \frac{P_y}{1 - \alpha} \cdot y$$

$$\Rightarrow y_A^* = (1 - \alpha) \cdot \frac{I^A}{P_y}, \quad x_A^* = \alpha \cdot \frac{I^A}{P_x}$$

$$x_A^* = \alpha \cdot \frac{I^A}{P_x}$$

Similarly,
$$y_B^* = (1 - \beta) \cdot \frac{I^B}{P_y}$$
, $x_B^* = \beta \cdot \frac{I^B}{P_x}$

The Walrasian Equilibrium: Markets Clear

$$x_A^* = \alpha \cdot \frac{P_x \omega_x^A + P_y \omega_y^A}{P_x} = \alpha \omega_x^A + \alpha \cdot \frac{P_y}{P_x} \cdot \omega_y^A$$
$$x_B^* = \beta \cdot \frac{P_x \omega_x^B + P_y \omega_y^B}{P_x} = \beta \omega_x^B + \beta \cdot \frac{P_y}{P_x} \cdot \omega_y^B$$

Markets Clear:
$$x_A^* + x_B^* = \omega_x^A + \omega_x^B$$

$$\Rightarrow \left(\alpha \cdot \omega_y^A + \beta \cdot \omega_y^B\right) \cdot \frac{P_y}{P_x} = (1 - \alpha)\omega_x^A + (1 - \beta)\omega_x^B$$

$$\frac{P_y}{P_x} = \frac{(1 - \alpha)\omega_x^A + (1 - \beta)\omega_x^B}{\alpha \cdot \omega_y^A + \beta \cdot \omega_y^B}$$

Walras. Equil. in Edgeworth Box Experiment

$$\alpha = 0.6, \beta = 0.8$$

$$(\omega_x^A, \omega_y^A) = (44, 9), \quad (\omega_x^B, \omega_y^B) = (6, 41),$$

$$\Rightarrow \frac{P_y}{P_x} = \frac{(1 - \alpha)\omega_x^A + (1 - \beta)\omega_x^B}{\alpha \cdot \omega_y^A + \beta \cdot \omega_y^B}$$

$$= \frac{(0.4)44 + (0.2)6}{0.6 \cdot 9 + 0.8 \cdot 41} = \frac{17.6 + 1.2}{5.4 + 32.8} = \frac{94}{191}$$

$$\Rightarrow \frac{P_x}{P_y} = \frac{191}{94} \approx 2.032$$

Iras. Equil. in Edgeworth Box Experiment Contract Curve (all PEA) 400x66 473 5x + 15034 1&A7&B7 A9 A3 ≈ 2.032

Experimental Results: 江淳芳個經: Contract Curve (all PEA) 400x5x + 15034 ≈ 2.032

Experimental Results: 汀淳芳個經: 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 50 49 48 47 46 45 44 43 42 41 453 459 466 473 🕰 40 456 463 470 477 39 456 464 472 479 38 12 37 13 36 35 455 464 473 34 16 33 17 32 18 31 19 30 20 29 21 28 22 27 23 460 472 26 24 Α1 25 25 24 26 23 27 22 28 В8 176 21 29 A10 20 30 В9 B5 19 31 18 32 A4 A2 17 33 Α7 Bl 16 34 В7 15 35 A3 B10 14 36 Α6 13 37 В4 12 38 11 39 10 40 9 41 A8 ≈ 2.032 8 42 7 43 6 44 5 45 4 46 3 47 2 48 200 190 179 168 1 175 165 156 49

36 36 37 38 39 39 40 41 41 42 43 43 44 44 45 45 46 46 47 47 48 48 48 49 49

13 15 17 18 20 21 23 24 25 27 28 29 30 31 32 33 34

0 50

Experimental Results: 黃貞穎個經:第一回合 Contract Curve (all PEA) 400x5x + 150

35 36 36 37 38 39 39 40 41 41 42 43 43 44 44 45 45 46 46 47 47 48 48 48 49 49 50 50

Experimental Results: 黃貞穎個經: 第二回合 ≈ 2.032

Experimental Results: 袁國芝個經: Contract Curve (all PEA) 400x5x + 150Average(without A1&B5) 182 170 200 190 179 168

3 5 7 9 11 13 15 17 18 20 21 23 24 25 27 28 29 30 31 32 33 34 35

Experimental Results: 袁國芝個 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 50 49 48 47 A1&B5 46 45 44 43 42 41 40 453 459 466 473 🔏 456 463 470 477 39 456 464 472 479 38 12 37 13 36 35 15 34 16 33 17 32 18 31 19 30 20 29 21 28 22 27 23 26 24 25 25 A2 24 26 23 27 22 28 176 21 29 20 30 19 31 B1 A3 Average(without A1&B5) 18 32 17 33 В7 16 34 A8 15 35 В9 14 36 13 37 12 38 A4 В3 11 39 A5 В8 10 40 9 41 8 42 ≈ 2.032 7 43 6 44 5 45 3 47 2 48 200 190 179 168 1 49 175 165 156 0 50

Core

7 9 11 13 15 17 18 20 21 23 24 25 27 28 29 30 31 32 33 34

36 36 37 38 39 39 40 41 41 42 43 43 44 44 45 45 46 46 47 47 48 48 48 49 49 50

Fxp. Results: 王道一(碩一)個論一: 第一回合 Contract Curve (all PEA) 400x5x + 150

Exp. Results: 王道一(大一)經原一: 第一回合 Contract Curve (all PEA) 400x5x + 150 ≈ 2.032

What Have We Learned?

- Bilateral trade happens in the Eye
- Prices converge toward WE prices
- Final positions converge toward core and WE
 - Average closer in 2nd round; variance decreases
- Still a lot of noise (but does not effect results)
- Markets work without full information (Hayek)
- What provided the force of competition?
 - Existence of perfect substitute (other A and Bs)
- How can we get further converge?
 - Experience? Larger space? Other trading rules?