Theory of Choice

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(Lecture 4, Micro Theory I)

Preferences, Utility and Choice

- Empirically, we see people make choices
- Can we come up with a theory about why people made these choices?
- Preferences: People choose certain things instead of others because they prefer them
  - As an individual, preferences are primitive; my choices are made based on my preferences
- Can we do some reverse engineering?

Preferences, Utility and Choice

- Revealed Preferences: Inferring someone's preferences by his/her choices
  - As an econometrician, choices are primitive; preferences are revealed by observing them
- Not formally discussed in Riley's book, but the idea of revealed preferences is everywhere…

- Can we do further reverse engineering?

Preferences, Utility and Choice

Choices ←→ Preferences ←→ Utility

- Can we describe preferences with a function?
- Utility: A function that describes preferences
  - Someone's true utility may not be the same as what economists assume, but they behave as if
  - Reverse engineering: Program a robot that makes the same choice as you do…
- What are the axioms needed for a preference to be described by a utility function?

Why do we care about this?

- Need objective function to constrain-maximize
- Cannot observe one's real utility (objective)
  - Neuroeconomics is trying this, but not there yet (Except places that ignore human rights...)
- Can we find an as if utility function (economic model) to describe one's preferences?
  - Can elicit preferences by asking people to make a lot of choices (= revealed preference!)
- If yes, we can use it as our objective function

Preferences: How alternatives are ordered?

- A binary relation for household h: $\succsim_h$
  $\vec{x}^1 \succsim_h \vec{x}^2$ ($\vec{x}^1$ is ordered as least as high as $\vec{x}^2$)
  - But order may not be defined for all bundles...
- Weak inequality order:
  $$\vec{x}^1 \succsim_h \vec{x}^2 \text{ if and only if } \vec{x}^1 \geq \vec{x}^2$$
  - Cannot define order between (1,2) and (2,1)...

Preferences: Completeness and Transitivity

- To represent preferences with utility function, consumers have to be able to compare all bundles

- Complete Axiom: (Total Order)

  For any consumption bundle $\vec{x}^1, \vec{x}^2 \in X$,

  either $\vec{x}^1 \succsim_h \vec{x}^2$ or $\vec{x}^2 \succsim_h \vec{x}^1$.

  - Also need consistency across pair-wise rankings…

- Transitive Axiom:

  For any consumption bundle $\vec{x}^1, \vec{x}^2, \vec{x}^3 \in X$,

  if $\vec{x}^1 \succsim_h \vec{x}^2$ and $\vec{x}^2 \succsim_h \vec{x}^3$, then $\vec{x}^1 \succsim_h \vec{x}^3$.

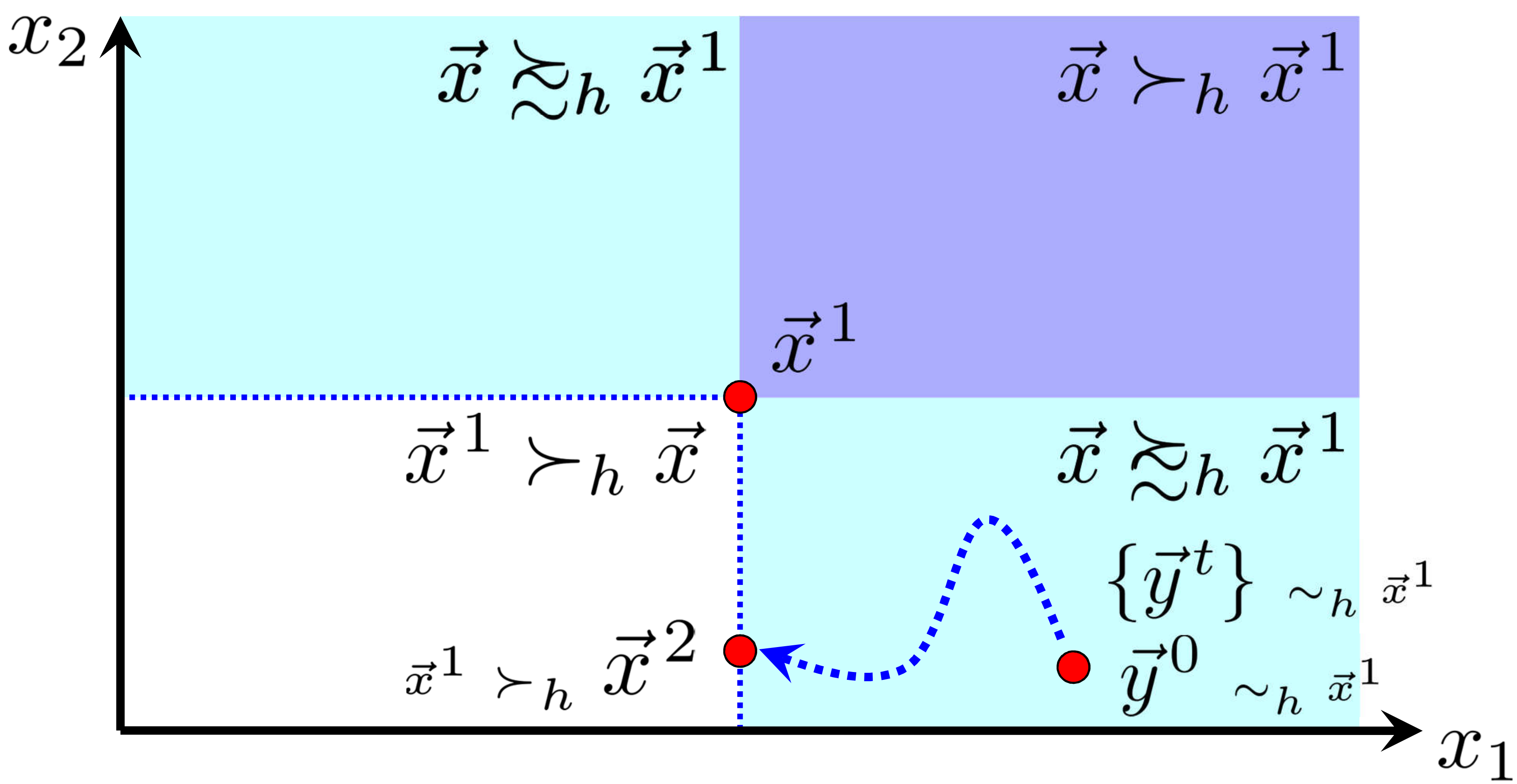
Preferences: Indifference; Strictly Preferred

- Indifference:
  $\vec{x}^1 \sim_h \vec{x}^2$ if and only if $\vec{x}^1 \succsim_h \vec{x}^2$ and $\vec{x}^2 \succsim_h \vec{x}^1$
- Strictly Preferred:
  $\vec{x}^1 \succ_h \vec{x}^2$ if and only if $\vec{x}^1 \succsim_h \vec{x}^2$, but $\vec{x}^2 \not\succsim_h \vec{x}^1$
  $\vec{x}^2 \succ_h \vec{x}^1$ if and only if $\vec{x}^2 \succsim_h \vec{x}^1$, but $\vec{x}^1 \not\succsim_h \vec{x}^2$
- Indifference order and strict preference order are both transitive, but not complete (total)
- The two axioms above are not enough...

Example: "Not-Less-Than" Order

- "Not-less-than" order: (Complete)

  $\vec{x}^1 \succsim_h \vec{x}^2$ if and only if $\vec{x}^1 \nless \vec{x}^2$



Continuous Preferences

- Why is non-continuous order a problem?

$$\vec{y}^t (\sim_h \vec{x}^1) \to \vec{x}^2, \text{ but } \vec{x}^1 \succ_h \vec{x}^2$$

- Corresponding utility also not continuous!

$$U(\vec{y}^t) = U(\vec{x}^1) \to U(\vec{x}^2) < U(\vec{x}^1)$$

- Continuous Order:

Suppose $\{\vec{x}^t\}_{t=1,2,\cdots} \to \vec{x}^0$. For any bundle $\vec{y}$,

If for all t, $\vec{x}^t \succsim_i \vec{y}$ then $\vec{x}^0 \succsim_i \vec{y}$.

If for all t, $\vec{y} \succsim_i \vec{x}^t$ then $\vec{y} \succsim_i \vec{x}^0$.

Where Do These Postulates Apply?

- More applicable to daily shopping (familiar…)
  - Can you rank things at open-air markets in Turkey?
- What if today's choice depends on past history or future plans? Consider: $\vec{x}_t = (\vec{x}_{1t}, \vec{x}_{2t}, \cdots, \vec{x}_{nt})$

  Then use $\vec{x} = (\vec{x}_1, \vec{x}_2, \cdots, \vec{x}_t, \cdots, \vec{x}_T)$
- What if there is uncertainty about the complete bundle? Consider: $(\vec{x}_1, \vec{x}_2^g, \vec{x}_2^b; \pi^g, \pi^b)$
- Would adding time and uncertainty make the commodities less "familiar"?

LNS (rules out "total indifference")

- Back to full information, static 1-period case
- An "everything-is-as-good-as-everything" order satisfies all other postulates so far
  - But isn't really useful for explaining choices...
- Local non-satiation (LNS):

For any consumption bundle $\vec{x} \in C \subset \mathbb{R}^n$

and any $\vec{\delta}$-neighborhood $N(\vec{x}, \vec{\delta})$ of $\vec{x}$,

there is some bundle $\vec{y} \in N(\vec{x}, \vec{\delta})$ s.t. $\vec{y} \succ_h \vec{x}$

Preferences: Strict Monotonicity

- Another similar strong assumption is
- "More is always strictly preferred."
  - Natural for analyzing consumption of commodity groups (food, clothing, housing...)
- Strict Monotonicity:

$$\text{If } \vec{y} > \vec{x}, \text{ then } \vec{y} \succ_h \vec{x}.$$

Preferences: Convexity

- Final postulate: "Individuals prefer variety."
- Convexity:

  Let C be a convex subset of $\mathbb{R}^n$

  For any $\vec{x}^0, \vec{x}^1 \in C$, if $\vec{x}^0 \succsim_h \vec{y}$ and $\vec{x}^1 \succsim_h \vec{y}$, then $(1-\lambda)\vec{x}^0 + \lambda\vec{x}^1 = \vec{x}^\lambda \succsim_h \vec{y}$, $0 < \lambda < 1.$
- Strict Convexity:

  For any $\vec{x}^0, \vec{x}^1, \vec{y} \in C$, if $\vec{x}^0 \succsim_h \vec{y}$ and $\vec{x}^1 \succsim_h \vec{y}$, then $\vec{x}^\lambda \succ_h \vec{y}$, $0 < \lambda < 1.$

Prop. 2.1-1: When's Utility Function Continuous?

Utility Function Representation of Preferences

If preferences are complete, reflective $(\vec{x} \succsim_h \vec{x})$, transitive and continuous on $X \subset \mathbb{R}^n$, they can be represented by a function $U(\vec{x})$ which is continuous over X.

→ Can use utility function to represent preferences

→ Use it as objective in constrained maximization

- Special Case: Strict Monotonicity

Special Case: Strict Monotonicity

Consider $\vec{x}^0, \vec{x}^1 \in X$, $\vec{x}^1 > \vec{x}^0 \Rightarrow \vec{x}^1 \succ_h \vec{x}^0$

For $T = \{\vec{x} \in X | \vec{x}^1 \succsim_h \vec{x} \succsim_h \vec{x}^0\}$,

Claim:

For any $\vec{y} \in T$, there exists some weight $\lambda \in [0, 1]$ such that $\vec{y} \sim_h \vec{x}^\lambda$ where $\vec{x}^\lambda = (1 - \lambda)\vec{x}^0 + \lambda \vec{x}^1$

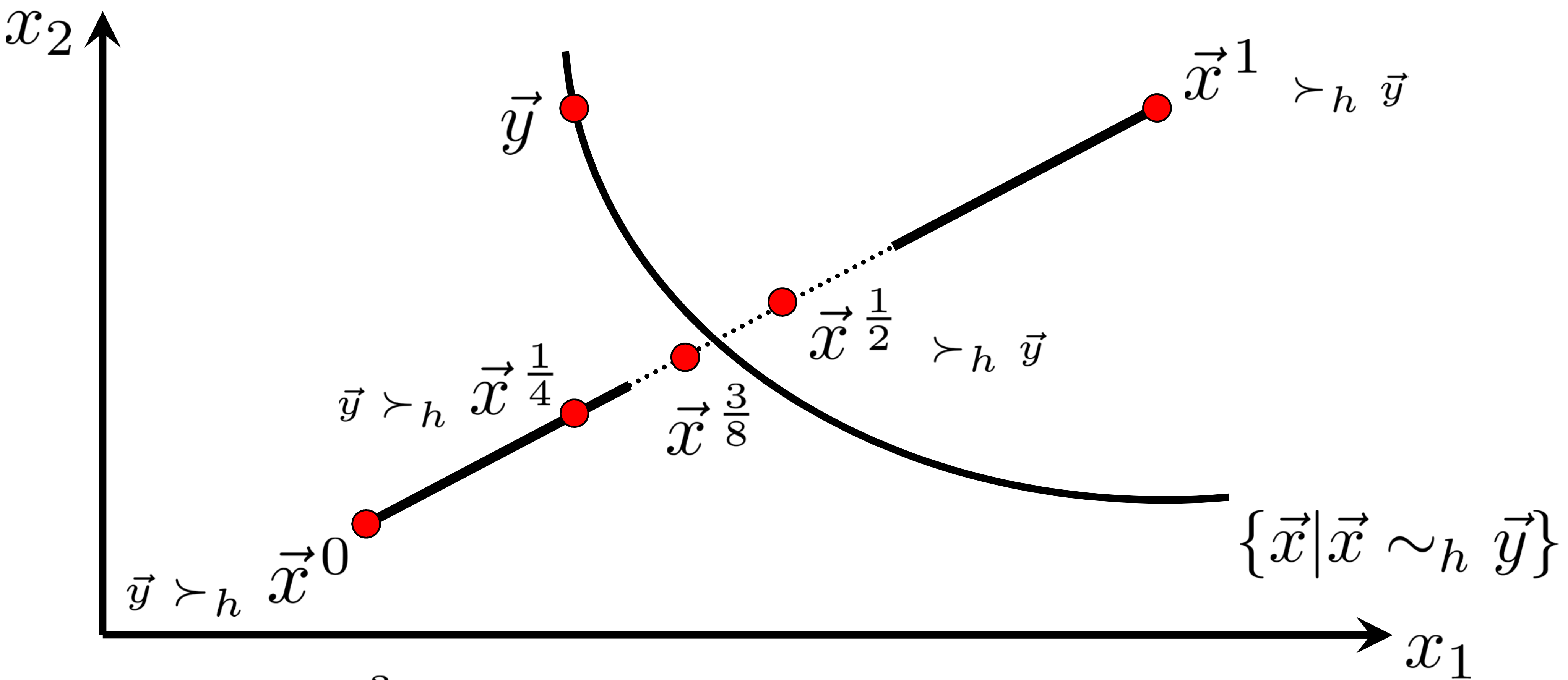
Moreover, $\lambda(\vec{y}) : T \to [0, 1]$ is continuous.

Proof:

Consider the sequence of intervals $\{\vec{x}^{\nu_t}, \vec{x}^{\mu_t}\}$,

Appeal to the completeness of real numbers...

Special Case: Strict Monotonicity



Either $\vec{x}^{\frac{3}{8}} \sim_h \vec{y}$ (done),
$\vec{x}^{\frac{3}{8}} \succ_h \vec{y}$(consider $\vec{x}^{\frac{3}{16}}$), or $\vec{y} \succ_h \vec{x}^{\frac{3}{8}}$(consider $\vec{x}^{\frac{7}{16}}$).

Special Case: Strict Monotonicity

Goal: Find $\vec{x}^{\hat{\lambda}} \sim_h \vec{y}$ as the limiting point of

Sequences $\vec{x}^{\nu_t} (\succsim_h \vec{y})$ and $(\vec{y} \succsim_h) \vec{x}^{\mu_t}$

Start with $\nu_0 = 1$, $\mu_0 = 0$. Let $\lambda_{t+1} = \frac{1}{2}(\nu_t + \mu_t)$

If $\vec{y} \sim_h \vec{x}^{\lambda_{t+1}}$, we are done.

If $\vec{y} \succ_h \vec{x}^{\lambda_{t+1}}$, $\nu_{t+1} = \nu_t$, $\mu_{t+1} = \lambda_{t+1}$

If $\vec{x}^{\lambda_{t+1}} \succ_h \vec{y}$, $\nu_{t+1} = \lambda_{t+1}$, $\mu_{t+1} = \mu_t$

$\vec{x}^1 = \vec{x}^{\nu_0} \succsim_h \cdots \succsim_h \vec{x}^{\nu_n} \succ_h \vec{y}$ $\Rightarrow \vec{x}^{\nu_n} \to \vec{x}^{\hat{\lambda}} \succsim_h \vec{y}$

$\vec{y} \succsim_h \vec{x}^{\hat{\lambda}} \leftarrow \vec{x}^{\mu_n} \Leftarrow$ $\vec{y} \succ_h \vec{x}^{\mu_n} \succsim_h \cdots \succsim_h \vec{x}^{\mu_0} = \vec{x}^0$

Completeness of $\mathbb{R} \Rightarrow \hat{\lambda}(\vec{y})$ exists, and $\vec{x}^{\hat{\lambda}} \sim_h \vec{y}$

Convex Preferences = Quasi-Concave Utility

- Quasi-Concave Utility Function:
- U is quasi-concave on X if for any $\vec{x}^0, \vec{x}^1 \in X$
- and convex combination $\vec{x}^\lambda = (1-\lambda)\vec{x}^0 + \lambda\vec{x}^1$

$$U(\vec{x}^\lambda) \geq \min\left\{U(\vec{x}^0), U(\vec{x}^1)\right\} \qquad \lambda \in [0,1]$$

- Convex Preferences:

Let X be a convex subset of $\mathbb{R}^n$

For any $\vec{x}^0, \vec{x}^1 \in X$, if $\vec{x}^0 \succsim_h \vec{y}$ and $\vec{x}^1 \succsim_h \vec{y}$,

then $(1-\lambda)\vec{x}^0 + \lambda\vec{x}^1 = \vec{x}^\lambda \succsim_h \vec{y}$, $0 < \lambda < 1$.

Convex Preferences to Quasi-Concave Utility

- For any $\vec{x}^0, \vec{x}^1 \in X$ and convex combination
$$\vec{x}^\lambda = (1-\lambda)\vec{x}^0 + \lambda\vec{x}^1, \lambda \in [0,1]$$
- Preferences are convex, represented by U
- Without loss of generality, assume $\vec{x}^0 \succsim_h \vec{x}^1$
- Then,
$$(1-\lambda)\vec{x}^0 + \lambda\vec{x}^1 = \vec{x}^\lambda \succsim_h \vec{x}^1.$$
- Hence,
$$U(\vec{x}^\lambda) \geq U(\vec{x}^1) = \min\left\{U(\vec{x}^0), U(\vec{x}^1)\right\}$$

Quasi-Concave Utility to Convex Preferences

- For any $\vec{x}^0, \vec{x}^1 \in X$ and convex combination
$$\vec{x}^\lambda = (1-\lambda)\vec{x}^0 + \lambda\vec{x}^1, \lambda \in [0,1]$$
- Preferences are represented by U
- If $\vec{x}^0 \succsim_h \vec{y}$ and $\vec{x}^1 \succsim_h \vec{y}$, we have
$$U(\vec{x}^1) \geq U(\vec{y}), U(\vec{x}^0) \geq U(\vec{y})$$
- Since U is quasi-concave,
$$U(\vec{x}^\lambda) \geq \min\left\{U(\vec{x}^0), U(\vec{x}^1)\right\} \geq U(\vec{y})$$
- Hence, $\vec{x}^\lambda \succsim_h \vec{y}$.

Summary of 2.1

- Preference Axioms
  - Complete
  - Transitive
  - Continuous
    - Monotonic
    - Convex / Strictly Convex
- Utility Function Representation
- Homework: Exercise 2.1-4 (Opt. 2.1-2)

In-Class Homework

- Exercise 2.1-1: Transitivity

a) Show that the transitive axiom implies if $\vec{x} \succ_h \vec{y}$ and $\vec{y} \succ_h \vec{z}$, then $\vec{x} \succ_h \vec{z}$.

b) Is it also the case that if $\vec{x} \succ_h \vec{y}$ and $\vec{y} \succsim_h \vec{z}$, then $\vec{x} \succ_h \vec{z}$?

In-Class Homework

- Complete, Transitive and Continuous?!
  - Determine whether the following preference are complete, transitive or continuous.
  - Would they have corresponding utility function $U(x)$? Why or why not?

a) "Not-less-than" order: $\vec{x}^1 \succsim_h \vec{x}^2$ iff $\vec{x}^1 \nless \vec{x}^2$

b) "Not-better-than" order: $\vec{x}^1 \succsim_h \vec{x}^2$ iff $\vec{x}^1 \ngtr \vec{x}^2$

c) Lexical Graphic order: $\vec{x}^1 \succsim_h \vec{x}^2$ iff

$\exists\, i$ such that $x_1^1 = x_1^2, \cdots, x_{i-1}^1 = x_{i-1}^2, x_i^1 \geq x_i^2$

In-Class Homework

- Exercise 2.1-3: Suf. C. for Convex Preferences
- Let $U(x)$ be a utility function and $f(.)$ be an increasing function.

a) If $u(x) = f(U(x))$ is concave, show that preferences are convex.

b) If u is strictly concave show that preferences are strictly convex.

c) If f is strictly concave are preferences strictly convex?