Microeconomic Theory I - Midterm Fall 2014

Exam Time: 11/12 2:20pm-5:20pm. You have 3 hours; allocate your time wisely.

Part A (42 + bonus 8%): Prediction Markets for Daibuck Mayoral Election

Zi Lu and Gee Lan both care only about their own consumption under different states of the world. Gee Lan thinks GP will win the Daibuck mayoral election this year with probability 0.9. Zi Lu thinks there is an equal chance for GP to win or lose. The preference scaling function of each person is logarithmic: ($u(x) = \ln(x)$).

1. (2%) What is the expected utility function of Zi Lu and Gee Lan over the states $s = 1$ (GP wins) and $s = 2$ (GP loses)?
2. (4%) What are their degree relative risk aversion, $R(x)$? Are they risk averse, risk neutral or risk loving?
3. (3%) Can we use a representative agent to replace a group of people who have the same expected utility function as Zi Lu (or Gee Lan)? Why or why not?
4. (2%) Consider zFuture in Daibuck that offers event futures for each state. Assuming Zi Lu and Gee Lan both have the same total wealth (W, W), write down their consumer problems when facing state claim market price ratio p_1/p_2.
5. (6%) Write down the Lagrangians and derive the Kuhn-Tucker conditions of these consumer problems. (Hint: Define notation for subjective proabilities and present your answer using such notation.)
6. (6%) What is the individual demand for each state claim? What is the market demand for each state claim?
7. (5%) What is the Walrasian equilibrium state claim price ratio?
8. (8%) What is the equilibrium consumption of Zi Lu and Gee Lan (c_1, c_2)? How much can they lose under the worse case scenario compared to auturky (consuming their initial wealth)? Is your answer consistent with their risk preferences as found in question 2? Explain.
9. (6%) How would your equilibrium price ratio change if the beliefs of Zi Lu and Gee Lan change? (Hint: How do your Kuhn-Tucker conditions depend on the beliefs of Zi Lu and Gee Lan?)
10. (bonus 8%) Consider figures on the last page.[1] What is required to apply the above analysis to obtain "market beliefs" on candidates' winning probabilities? Are inferred market beliefs consistent with other poll results? Why or why not?

---

[1] These are screen shots of http://xfuture.org/contract/68ee9fea-15fc-4380-9f95-963eebfbac3e and http://xfuture.org/contract/0429bc5b-3898-42c5-88e4-d5a0275302d6.

Part B (30%): Riz and Lucy Avery's Preferences

Consider Lucy and Riz Avery who both obey expected utility theory. Lucy Avery has expected utility function $u(x) = \min\{x - a, l \cdot (x - a)\}$ where $l \geq 0$ and $a =$ constant, while Riz Avery has expected utility function satisfying $v'(x) = x^{-r}$ with $r > 0$. Both face the ten lottery choices of Holt and Laury (2002) listed below:

You will roll a ten-sided die and get paid according to your decision (choice A or B):

Decision	Lottery A	Lottery B	Your choice (A or B)
Question 1	1：Gain NT\$200 2～10：Gain NT\$160	1：Gain NT\$385 2～10：Gain NT\$10	
Question 2	1～2 ：Gain NT\$200 3～10：Gain NT\$160	1～2 ：Gain NT\$385 3～10：Gain NT\$10	
Question 3	1～3 ：Gain NT\$200 4～10：Gain NT\$160	1～3 ：Gain NT\$385 4～10：Gain NT\$10	
Question 4	1～4 ：Gain NT\$200 5～10：Gain NT\$160	1～4 ：Gain NT\$385 5～10：Gain NT\$10	
Question 5	1～5 ：Gain NT\$200 6～10：Gain NT\$160	1～5 ：Gain NT\$385 6～10：Gain NT\$10	
Question 6	1～6 ：Gain NT\$200 7～10：Gain NT\$160	1～6 ：Gain NT\$385 7～10：Gain NT\$10	
Question 7	1～7 ：Gain NT\$200 8～10：Gain NT\$160	1～7 ：Gain NT\$385 8～10：Gain NT\$10	
Question 8	1～8 ：Gain NT\$200 9～10：Gain NT\$160	1～8 ：Gain NT\$385 9～10：Gain NT\$10	
Question 9	1～9 ：Gain NT\$200 10：Gain NT\$160	1～9 ：Gain NT\$385 10：Gain NT\$ 10	
Question 10	1～10：Gain NT\$200	1～10：Gain NT\$385	

1. (3%) Find the Von Neumann-Morgenstern utility function $v(.)$ of Riz Avery and its corresponding degree of relative risk aversion $R(x)$.
2. (2%) Show that for $x > a$, Lucy Avery exhibits both constant relative risk aversion and constant absolute risk aversion.
3. (8%) Show that Lucy Avery's utility function is concave: If $x^k = kx^0 + (1-k)x^1$, then $u(x^k) \geq ku(x^0) + (1-k)u(x^1)$. Show that the inequality is strict unless $x^0$and x^1 are both greater/small than a.

4. (4%) Show that if Lucy chooses lottery B in Question k, she would also choose lottery B in Question (k+1).
5. (6%) Show that if a person follows expected utility theory and chooses lottery B in Question k, he would also choose lottery B in Question (k+1). What is the critical assumption required for the above statement to be true?
6. (3%) If $a < 10$, show that Lucy would choose lottery A for Questions 1~4 and lottery B otherwise.
7. (4%) Now suppose $a = 200$, $l = 2$ and $r = 0$. Would Lucy choose more or less lottery A's than Riz? Why?

Part C (28 + bonus 12%): The People-Salmon Problem

Consider the relationship between the People of Daiwan (P) and returning salmon A-Shin (A). The people of Daiwan have utility function $u(x) = \min\{x - a, l \cdot (x - a)\}$ with $l \geq 0$ and a = constant. A-Shin decides whether it wants use genuine ($e = 1$) or suspicious ($e = 0$) material to produce food oil, and this affects whether the national health insurance surplus is $y_1 > 0$ ($s = 1$) or $y_0 < 0$ ($s = 0$). $\pi_1(e)$, the probability of $s = 1$, is $\pi_1(1) = 0.9$ and $\pi_1(0) = 0.5$. The people of Daiwan care about the national health insurance surplus and design a punishment/reward scheme to induce A-Shin to stop using suspicious material. Assume the two parties have expected utility:

$$U_P = \sum_{s=0}^{1} \pi_s(e) u(y_s - w_s) \qquad U_A = \sum_{s=0}^{1} \pi_s(e) v(w_s) - C(e)$$

where $v'(x) = x^{-r}$, $r \geq 0$, $C(1) = c_1 > C(0) = c_0$, and A-Shin's outside option (not being a salmon) yields reservation utility $\overline{U}_A = 0$. First assume the people of Daiwan can observe whether A-Shin uses suspicious material because the Daiwan government has access to espionage technology and abuses it.

1. (2%) Show that the increasing likelihood ratio assumption holds. In other words, show that for $\tilde{e} > e$, $\dfrac{\pi_1(\tilde{e})}{\pi_1(e)} > \dfrac{\pi_0(\tilde{e})}{\pi_0(e)}$
2. (16%) Write down the optimization problem of the people of Daiwan and derive the Kuhn-Tucker conditions for its solution. (Hint: You can write the minimum as a constraint to avoid having a discontinuous point in the utility function, such as $\max_{x,y} \{\min(x, y)\} = \max_{x,y} \{x | y \geq x\}$, and discuss different cases depending on whether $y_s - w_s > a$.)

3. (4%) Suppose $l = 1$ and $r > 0$. Show that the optimal reward/punishment scheme involves the A-Shin receiving the same reward/payment regardless of the national health insurance is running a surplus or deficit (i.e. $w_1 = w_0$).
4. (4%) Suppose $r = 0$ and $l > 1$. What is the optimal reward/punishment scheme for the people of Daiwan?

Now suppose more realistically that the people of Daiwan cannot observe whether salmon A-Shin uses suspicious material.

5. (2%) If $r = 0$ and and $l > 1$, what is the optimal reward/punishment scheme for the people of Daiwan if we assume $0.4 \cdot (y_1 - y_0) \geq c_1 - c_0$?
6. (bonus 10%) If $l = 1$ and $r = 1$, when y_1 is sufficiently large, design an optimal reward/punishment scheme for the people of Daiwan to induce $e = 1$.
7. (bonus 2%) Did the people of Daiwan adopt such an optimal reward/punishment scheme? Why or why not? (What assumptions could fail in real life?)

Figures for bonus question in Part A: