Edgeworth Box Experiment

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(Lecture 8a, Micro Theory I)

PEA with Cobb-Douglas Utility

$$\max_{x,y} U^A(x,y) = x^\alpha y^{1-\alpha}$$

$$\text{s.t. } U^B = (\overline{x}-x)^\beta (\overline{y}-y)^{1-\beta} \geq U^B$$

$$\mathcal{L} = x^\alpha y^{1-\alpha} + \lambda \cdot \left[(\overline{x}-x)^\beta (\overline{y}-y)^{1-\beta} - U^B\right]$$

FOC: (for interior solutions)

$$\frac{\partial \mathcal{L}}{\partial x} = \alpha \cdot \frac{y^{1-\alpha}}{x^{1-\alpha}} - \beta\lambda \cdot \frac{(\overline{y}-y)^{1-\beta}}{(\overline{x}-x)^{1-\beta}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = (1-\alpha) \cdot \frac{x^\alpha}{y^\alpha} - (1-\beta)\lambda \cdot \frac{(\overline{x}-x)^\beta}{(\overline{y}-y)^\beta} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = (\overline{x}-x)^\beta (\overline{y}-y)^{1-\beta} - U^B = 0$$

PEA with Cobb-Douglas Utility

Meaning of FOC: $MRS^A = MRS^B$

$$\lambda = \frac{\alpha \cdot \dfrac{y^{1-\alpha}}{x^{1-\alpha}}}{\beta \cdot \dfrac{(\overline{y}-y)^{1-\beta}}{(\overline{x}-x)^{1-\beta}}} = \frac{(1-\alpha) \cdot \dfrac{x^{\alpha}}{y^{\alpha}}}{(1-\beta) \cdot \dfrac{(\overline{x}-x)^{\beta}}{(\overline{y}-y)^{\beta}}}$$

$$\Rightarrow \alpha \cdot y \cdot (1-\beta) \cdot (\overline{x}-x) = \beta \cdot (\overline{y}-y) \cdot (1-\alpha) \cdot x$$

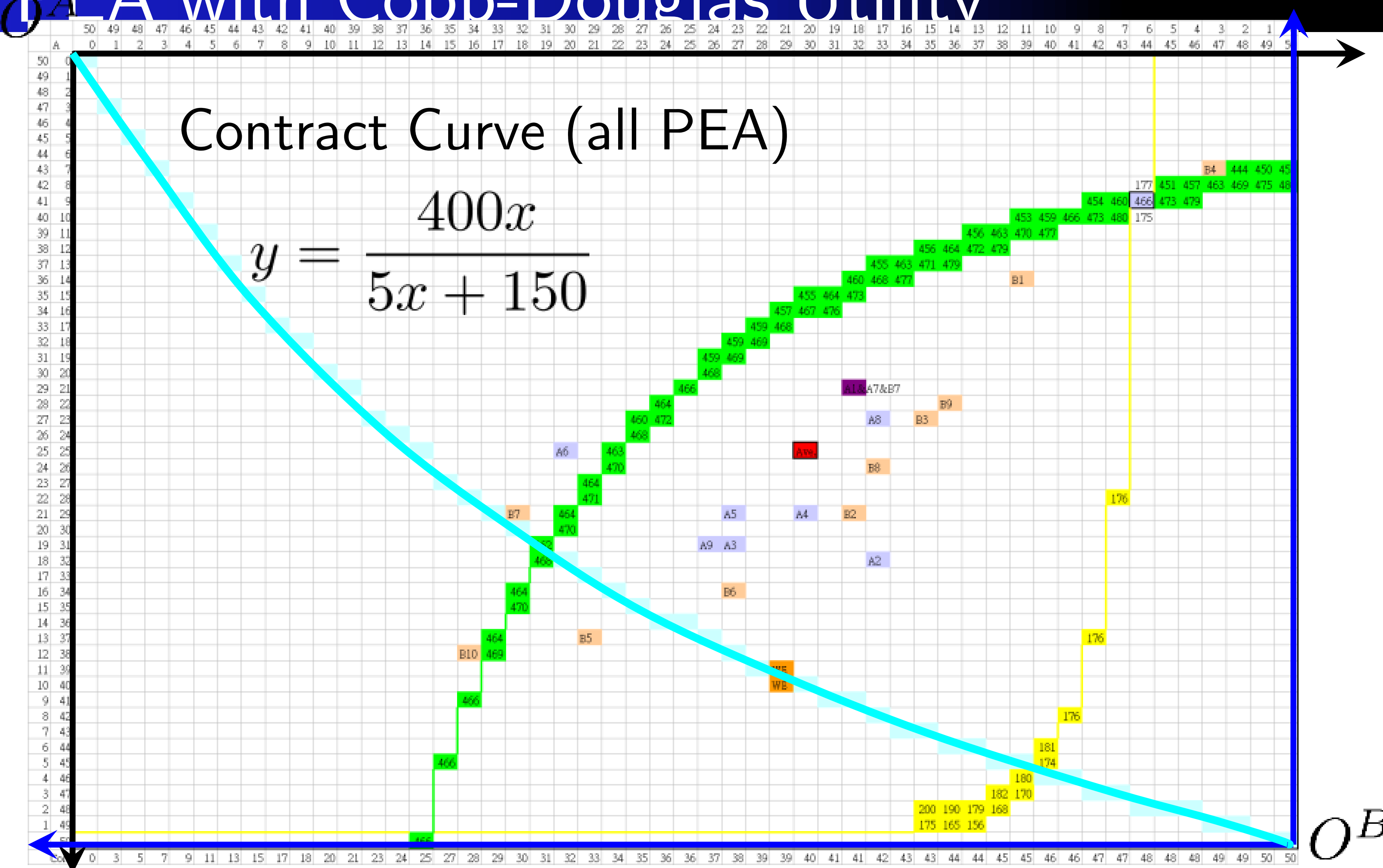
$$y = \frac{\beta(1-\alpha) \cdot \overline{y} \cdot x}{\alpha(1-\beta)(\overline{x}-x) + \beta(1-\alpha) \cdot x} = \frac{\gamma \overline{y} \cdot x}{(\gamma-1)x + \overline{x}}$$

$$= \frac{\frac{8}{3} \cdot 50 \cdot x}{\frac{5}{3}x + 50} = \frac{400x}{5x+150}$$

$\alpha = 0.6,$ $\beta = 0.8$

$$\gamma = \frac{\beta(1-\alpha)}{\alpha(1-\beta)}$$

PEA with Cobb-Douglas Utility



Walrasian Equilibrium: Consumer A Problem

$$\max_{x,y} U^A(x,y) = x^\alpha y^{1-\alpha}$$

$$\text{s.t. } P_x \cdot x + P_y \cdot y \leq I^A = P_x \cdot \omega_x^A + P_y \cdot \omega_y^A$$

$$\mathcal{L} = x^\alpha y^{1-\alpha} + \lambda \cdot \left[I^A - P_x \cdot x - P_y \cdot y\right]$$

FOC: (for interior solutions)

$$\frac{\partial \mathcal{L}}{\partial x} = \alpha \cdot \frac{y^{1-\alpha}}{x^{1-\alpha}} - \lambda \cdot P_x = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = (1-\alpha) \cdot \frac{x^\alpha}{y^\alpha} - \lambda \cdot P_y = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I^A - P_x \cdot x - P_y \cdot y = 0$$

Walrasian Equil.: Consumer Optimal Choice

Meaning of FOC: $MRS^A = \frac{P_x}{P_y}$

$$\frac{P_x}{P_y} = \frac{\alpha}{1-\alpha} \cdot \frac{y}{x} \quad \Rightarrow x = \frac{\alpha}{1-\alpha} \cdot \frac{P_y}{P_x} \cdot y$$

$$\Rightarrow I^A = P_x \cdot x + P_y \cdot y = \frac{P_y}{1-\alpha} \cdot y$$

$$\Rightarrow y_A^* = (1-\alpha) \cdot \frac{I^A}{P_y}, \quad x_A^* = \alpha \cdot \frac{I^A}{P_x}$$

Similarly, $y_B^* = (1-\beta) \cdot \frac{I^B}{P_y}$, $x_B^* = \beta \cdot \frac{I^B}{P_x}$

The Walrasian Equilibrium: Markets Clear

$$x_A^* = \alpha \cdot \frac{P_x\omega_x^A + P_y\omega_y^A}{P_x} = \alpha\omega_x^A + \alpha \cdot \frac{P_y}{P_x} \cdot \omega_y^A$$

$$x_B^* = \beta \cdot \frac{P_x\omega_x^B + P_y\omega_y^B}{P_x} = \beta\omega_x^B + \beta \cdot \frac{P_y}{P_x} \cdot \omega_y^B$$

Markets Clear: $x_A^* + x_B^* = \omega_x^A + \omega_x^B$

$$\Rightarrow \left(\alpha \cdot \omega_y^A + \beta \cdot \omega_y^B\right) \cdot \frac{P_y}{P_x} = (1-\alpha)\omega_x^A + (1-\beta)\omega_x^B$$

$$\frac{P_y}{P_x} = \frac{(1-\alpha)\omega_x^A + (1-\beta)\omega_x^B}{\alpha \cdot \omega_y^A + \beta \cdot \omega_y^B}$$

Walras. Equil. in Edgeworth Box Experiment

$$\alpha = 0.6, \beta = 0.8$$

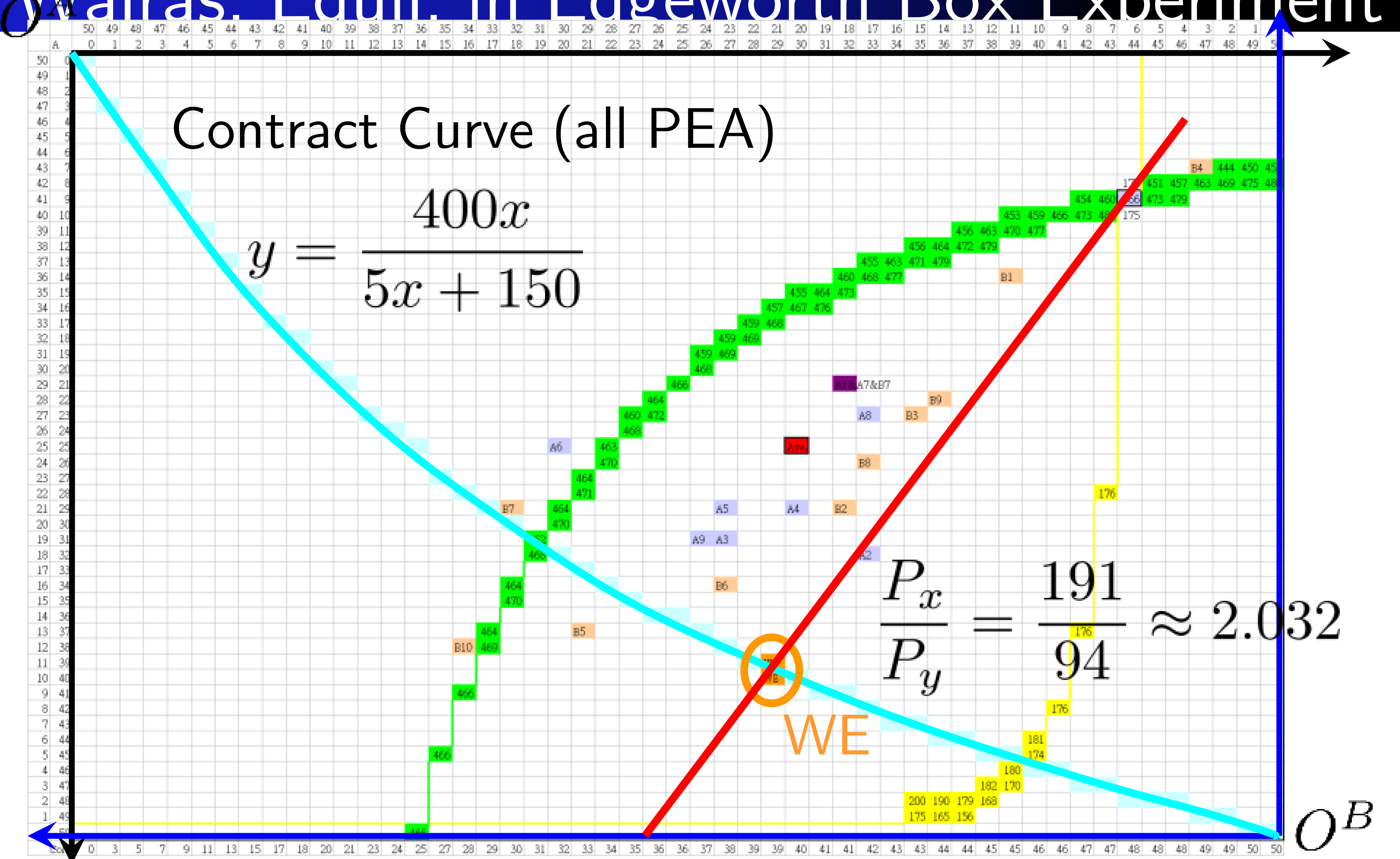
$$(\omega_x^A, \omega_y^A) = (44, 9), \quad (\omega_x^B, \omega_y^B) = (6, 41),$$

$$\Rightarrow \frac{P_y}{P_x} = \frac{(1-\alpha)\omega_x^A + (1-\beta)\omega_x^B}{\alpha \cdot \omega_y^A + \beta \cdot \omega_y^B}$$

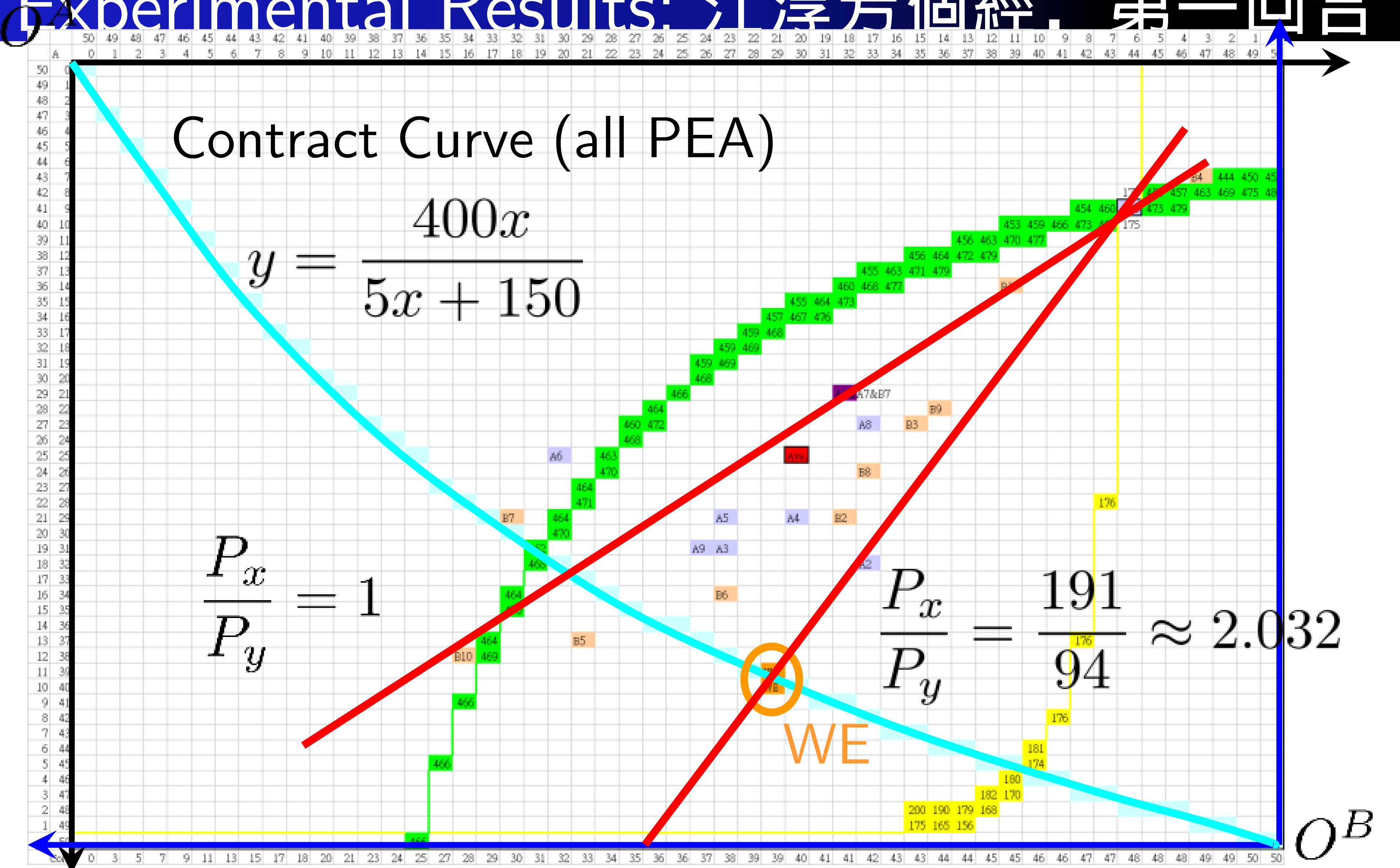
$$= \frac{(0.4)44 + (0.2)6}{0.6 \cdot 9 + 0.8 \cdot 41} = \frac{17.6 + 1.2}{5.4 + 32.8} = \frac{94}{191}$$

$$\Rightarrow \frac{P_x}{P_y} = \frac{191}{94} \approx 2.032$$

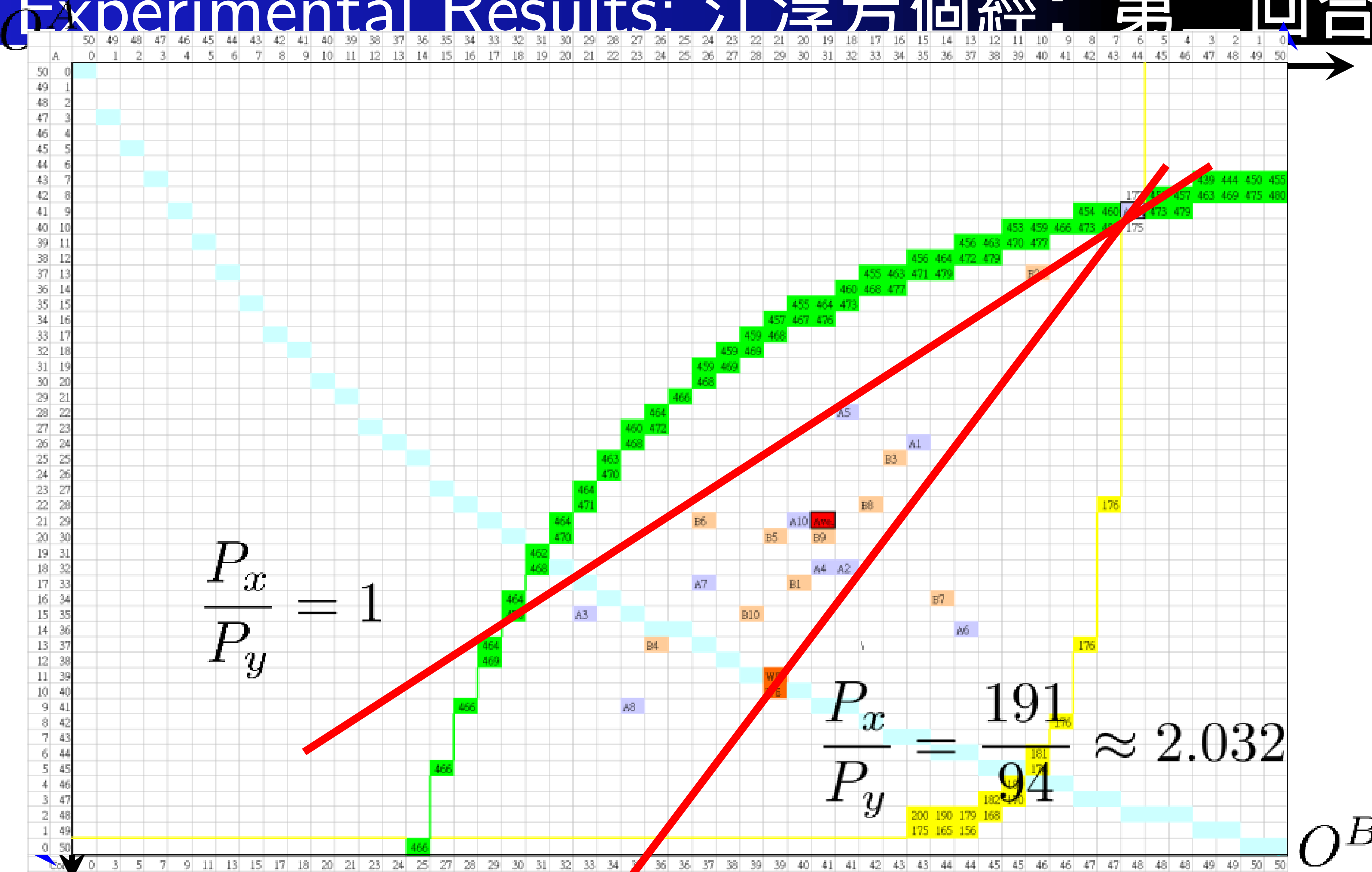
Walras. Equil. in Edgeworth Box Experiment



Experimental Results: 江淳芳個經：第一回合



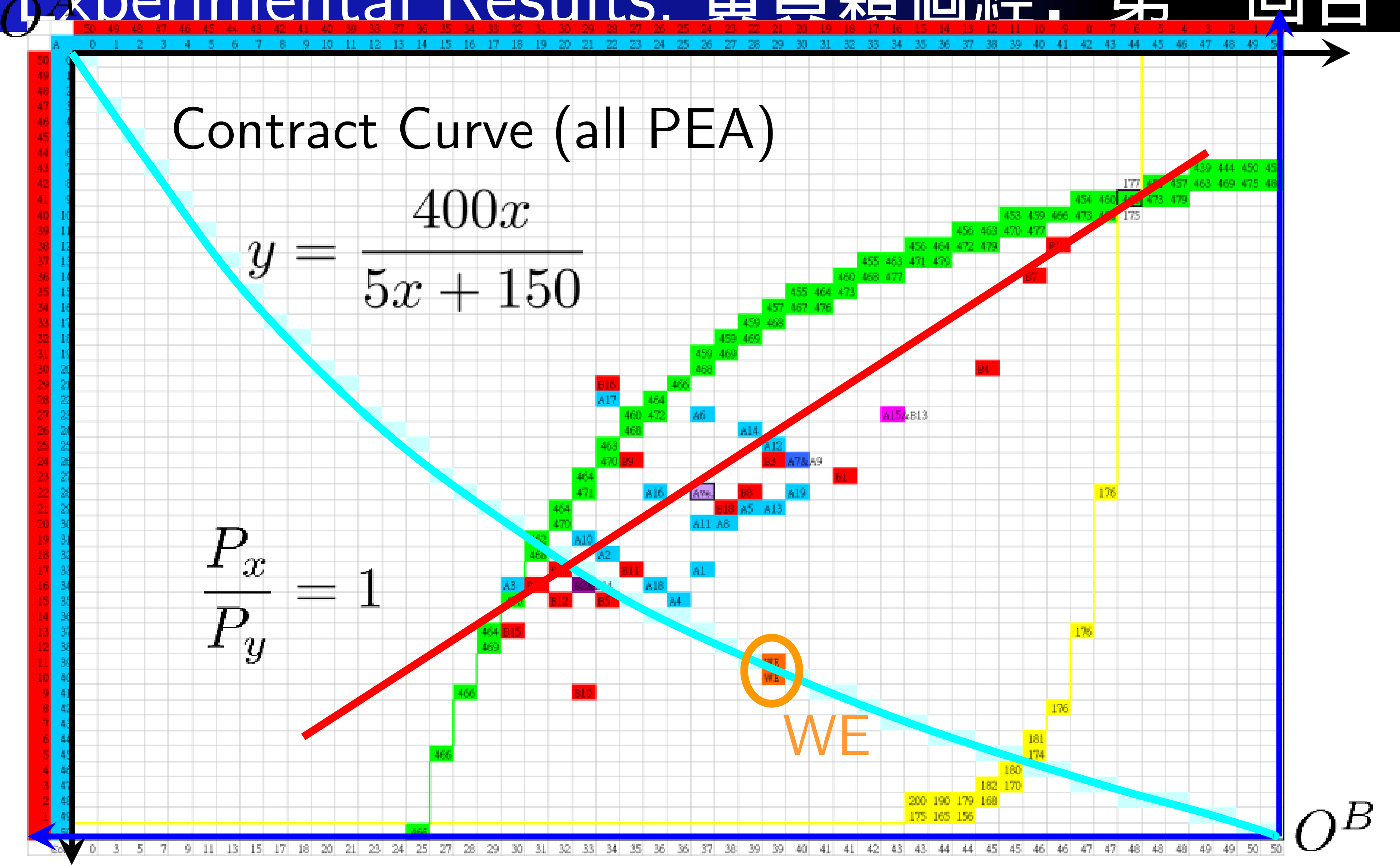
Experimental Results: 汀淳芳個經：第二回合



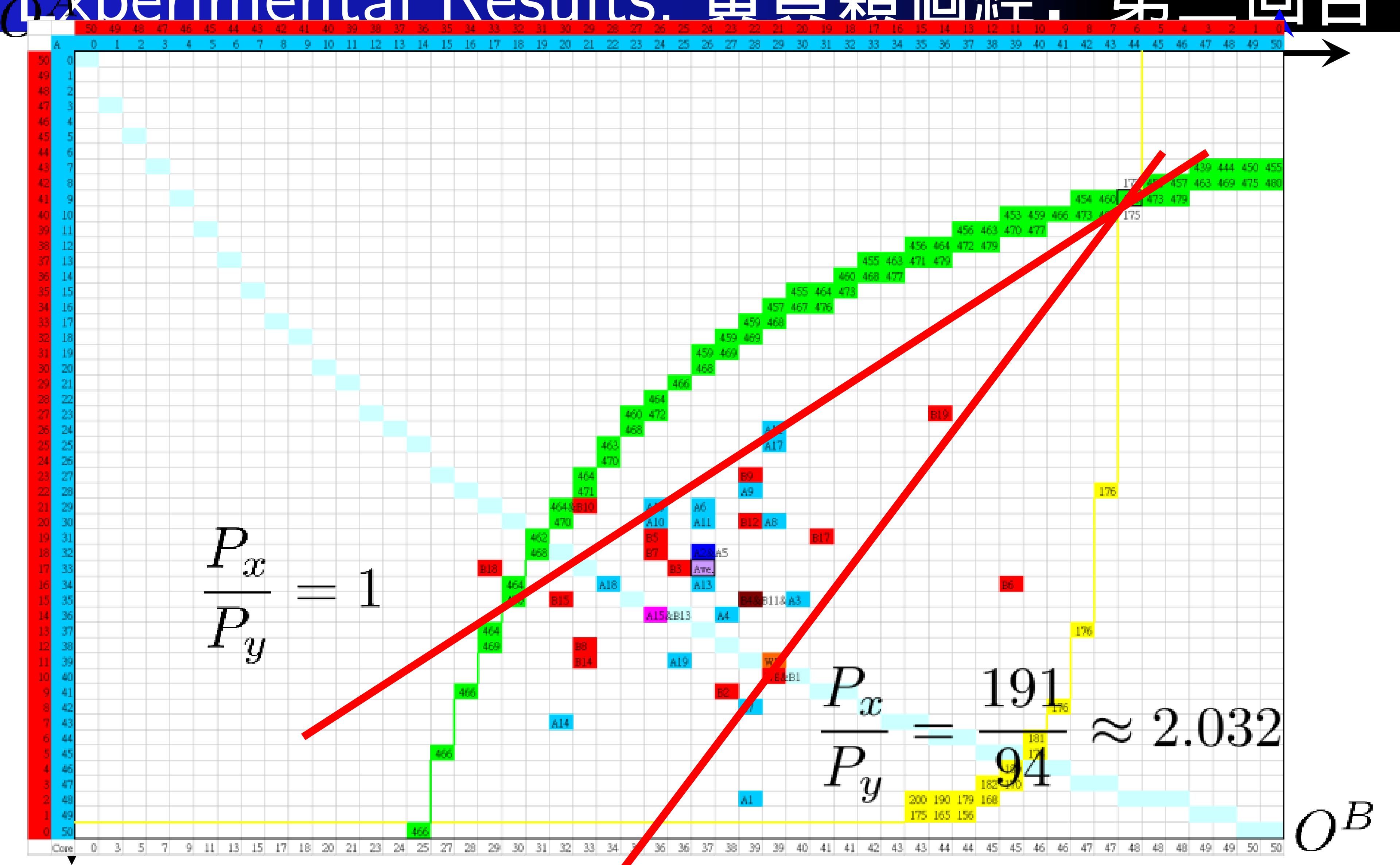
$$\frac{P_x}{P_y} = 1$$

$$\frac{P_x}{P_y} = \frac{191}{94} \approx 2.032$$

Experimental Results: 黃貞穎個經: 第一回合



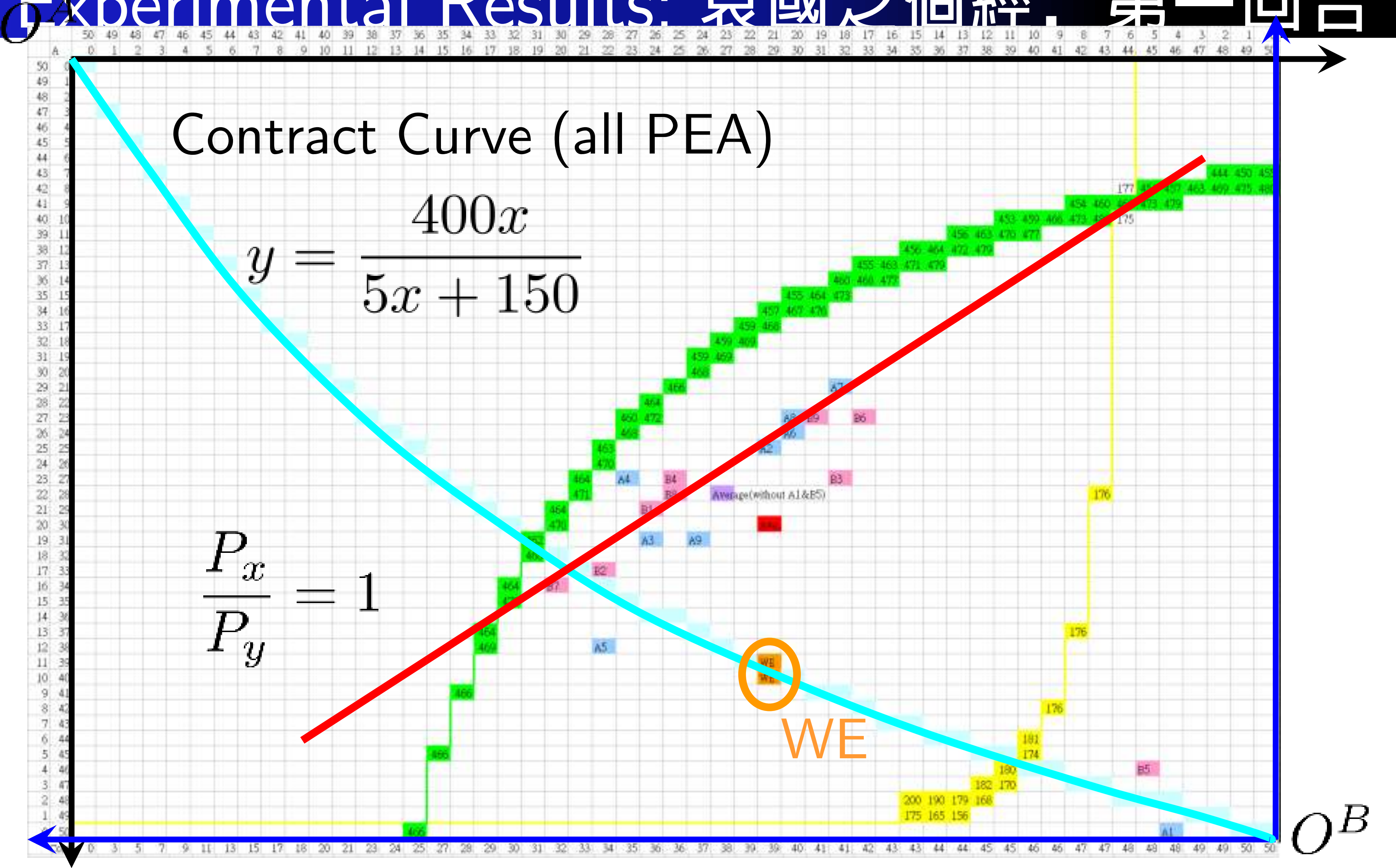
Experimental Results: 黃貞穎個經：第二回合



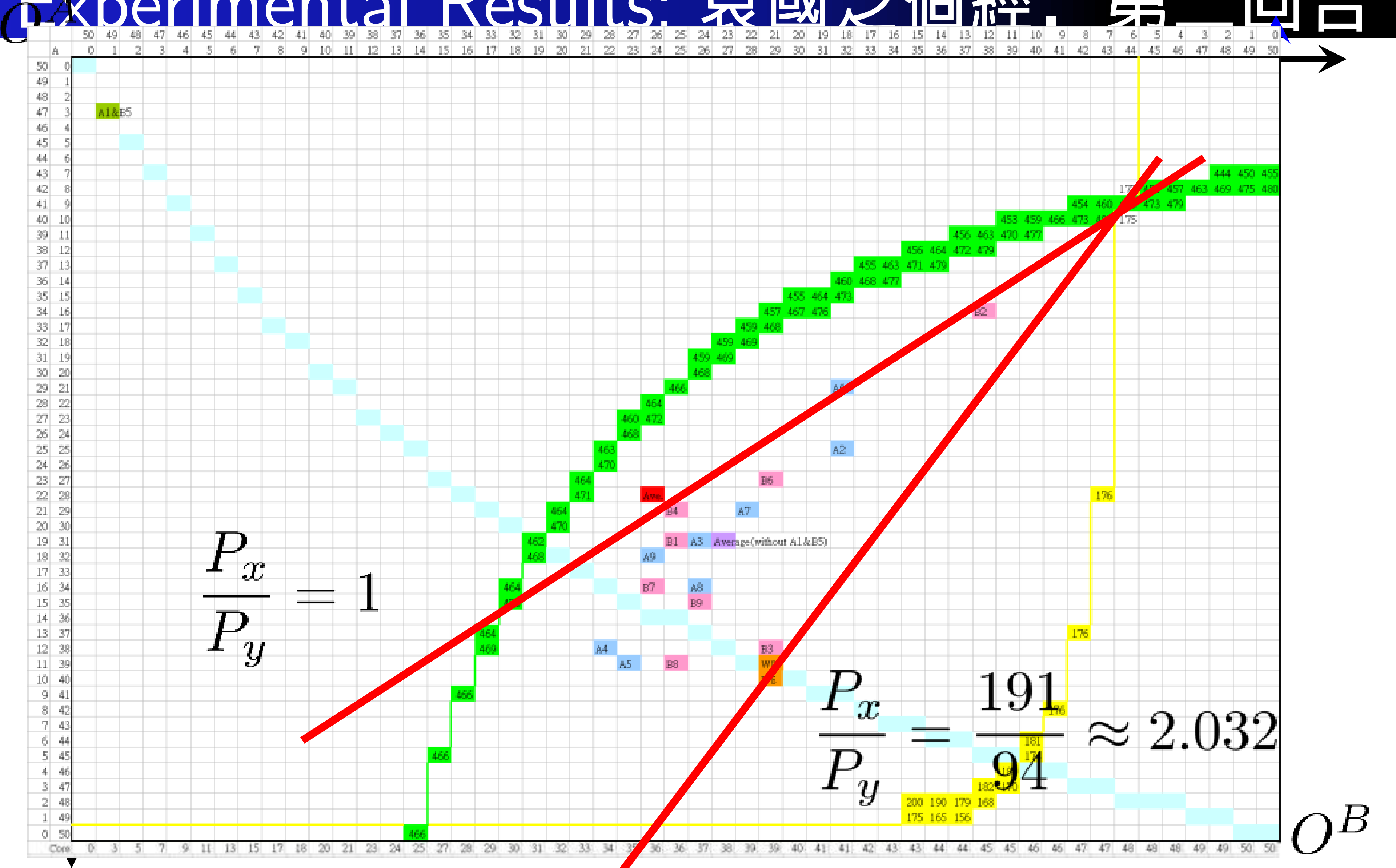
$$\frac{P_x}{P_y} = 1$$

$$\frac{P_x}{P_y} = \frac{191}{94} \approx 2.032$$

Experimental Results: 袁國芝個經：第一回合



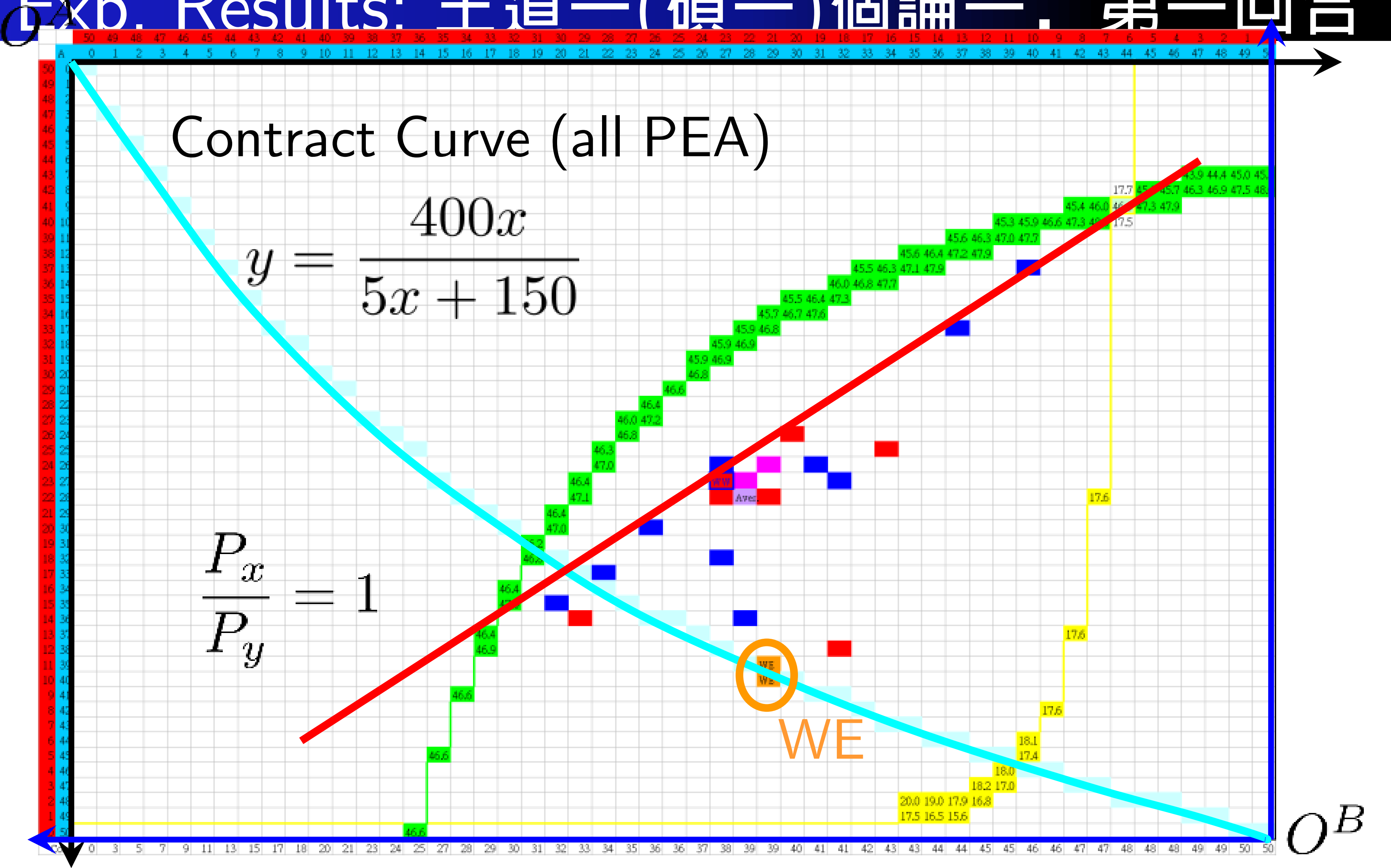
Experimental Results: 袁國芝個經：第二回合



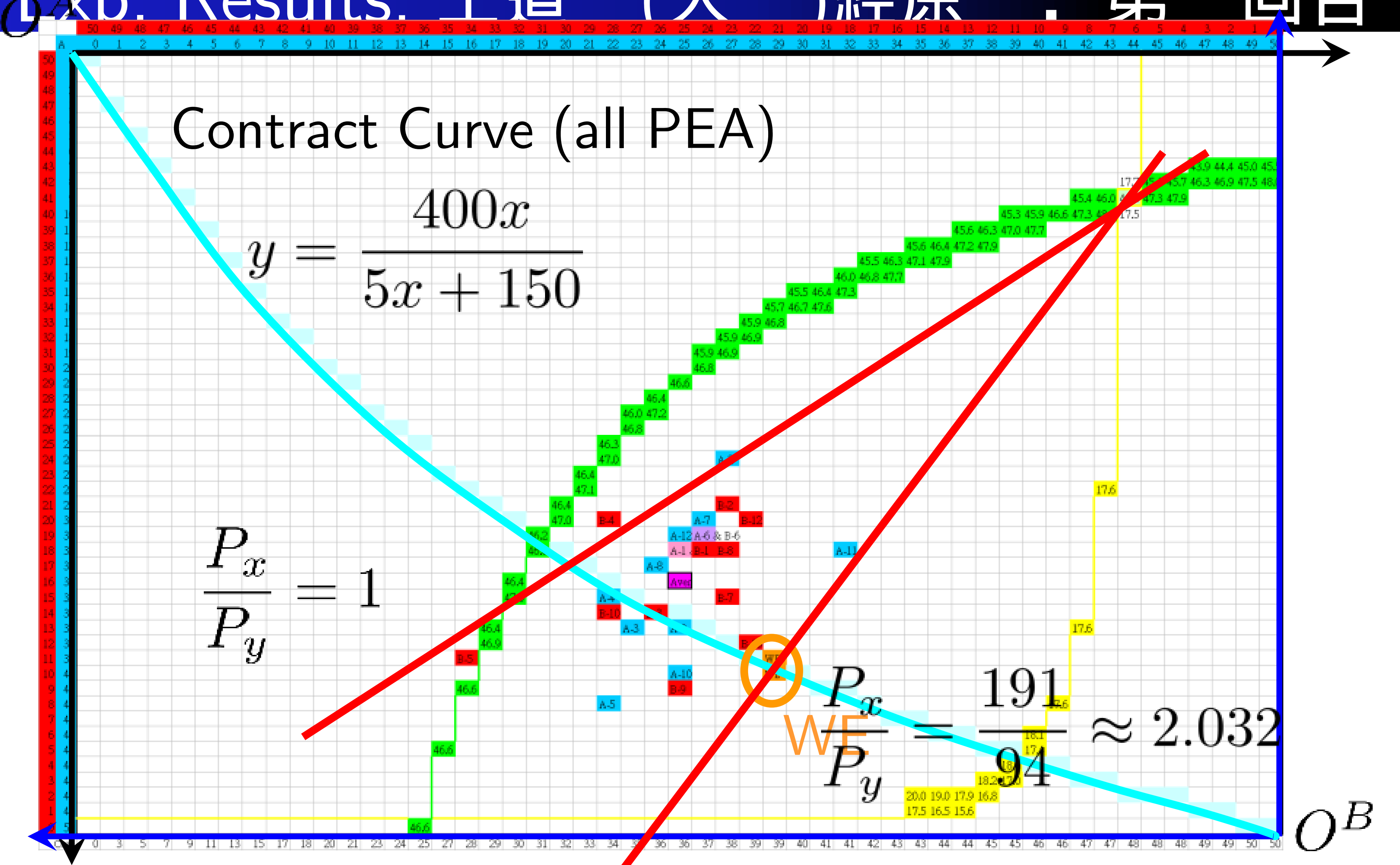
$$\frac{P_x}{P_y} = 1$$

$$\frac{P_x}{P_y} = \frac{191}{94} \approx 2.032$$

Exp. Results: 王道—(碩—)個論—： 第一回合



Exp. Results: 王道一(大一)經原一：第一回合



What Have We Learned?

- Bilateral trade happens in the Eye
- Prices converge toward WE prices
- Final positions converge toward core and WE
  - Average closer in 2^{nd} round; variance decreases
- Still a lot of noise (but does not effect results)
- Markets work without full information (Hayek)
- What provided the force of competition?
  - Existence of perfect substitute (other A and Bs)
- How can we get further converge?
  - Experience? Larger space? Other trading rules?