Theory of Choice

Joseph Tao-yi Wang
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(Lecture 4, Micro Theory I)

Preferences, Utility and Choice

- Empirically, we see people make choices
- Can we come up with a theory about "why" people made these choices?
- Preferences: People choose certain things instead of others because they "prefer" them
  - As an individual, preferences are primitive; my choices are made based on my preferences
- Can we do some reverse engineering?

- Revealed Preferences: Inferring someone's preferences by his/her choices
  - As an econometrician, choices are primitive; preferences are "revealed" by observing them
- Not formally discussed in Riley's book, but the idea of revealed preferences is everywhere...
- Can we do further reverse engineering?

Choices ←→ Preferences ←→ Utility

- Can we describe preferences with a function?
- Utility: A function that "describes" preferences
  - Someone's true utility may not be the same as what economists assume, but they behave as if
  - Reverse engineering: Program a robot that makes the same choice as you do...
- What are the axioms needed for a preference to be described by a utility function?

Why do we care about this?

- Need objective function to constrain-maximize
- Cannot observe one's real utility (objective)
  - Neuroeconomics is trying this, but "not there yet" (Except places that ignore human rights...)
- Can we find an as if utility function (economic model) to describe one's preferences?
  - Can elicit preferences by asking people to make a lot of choices (= revealed preference!)
- If yes, we can use it as our objective function

Preferences: How alternatives are ordered?

- A binary relation for household h : $\succsim_h$

  $x^1 \succsim_h x^2$ (x^1 is ordered as least as high as x^2)

  – But order may not be defined for all bundles…
- Weak inequality order:

  $x^1 \succsim_h x^2$ if and only if $x^1 \geq x^2$

  – Cannot define order between (1,2) and (2,1)…

Preferences: Completeness and Transitivity

- To represent preferences with utility function, consumers have to be able to compare all bundles

- Complete Axiom: (Total Order)

  For any consumption bundle $x^1, x^2 \in X$,

  either $x^1 \succsim_h x^2$ or $x^2 \succsim_h x^1$.

  - Also need consistency across pair-wise rankings…

- Transitive Axiom:

  For any consumption bundle $x^1, x^2, x^3 \in X$,

  if $x^1 \succsim_h x^2$ and $x^2 \succsim_h x^3$, then $x^1 \succsim_h x^3$.

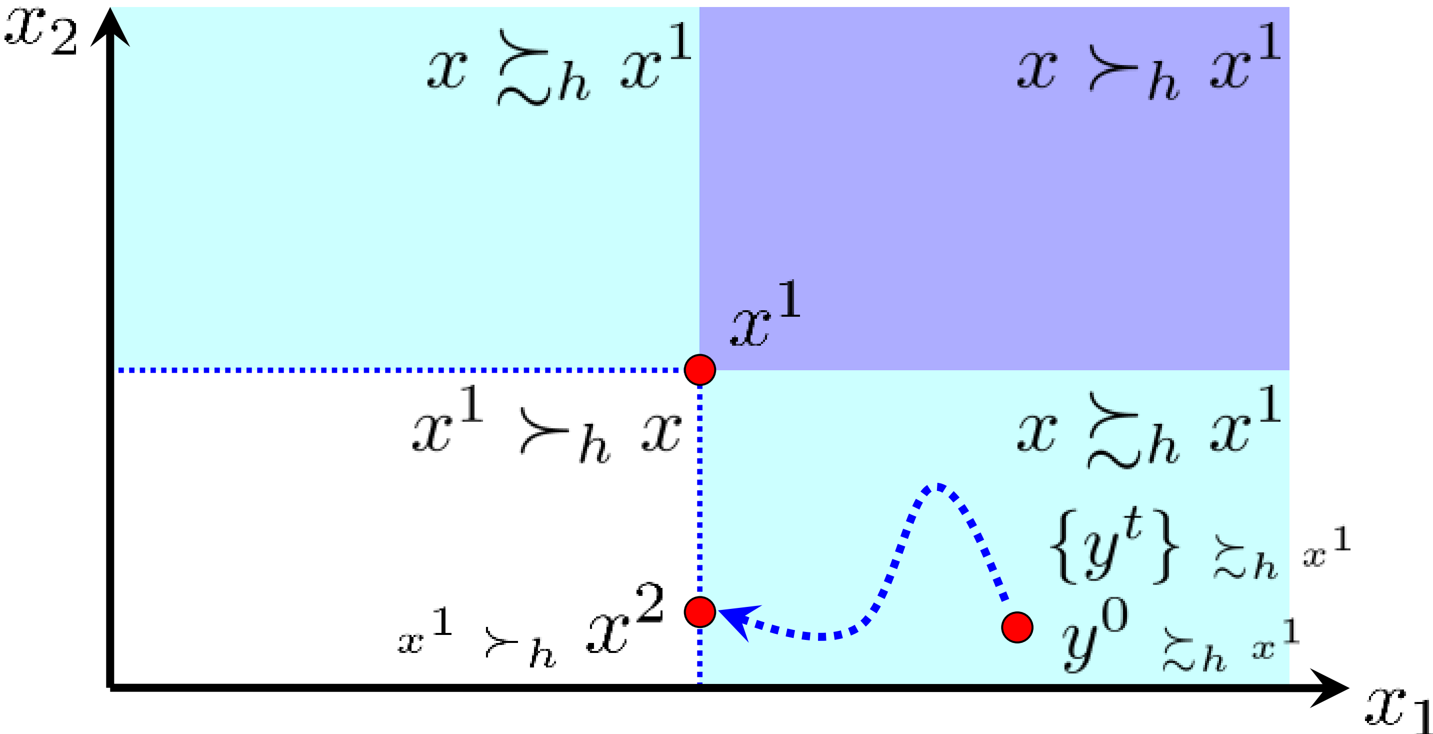
Preferences: Indifference; Strictly Preferred

- Indifference:
  $x^1 \sim_h x^2$ if and only if $x^1 \succsim_h x^2$ and $x^2 \succsim_h x^1$
- Strictly Preferred:
  $x^1 \succ_h x^2$ if and only if $x^1 \succsim_h x^2$, but $x^2 \not\succsim_h x^1$
  $x^2 \succ_h x^1$ if and only if $x^2 \succsim_h x^1$, but $x^1 \not\succsim_h x^2$
- Indifference order and strict preference order are both transitive, but not complete (total)
- The two axioms above are not enough…

Example: "Not-Less-Than" Order

- "Not-less-than" order: (Complete & Transitive)

$$x^1 \succsim_h x^2 \text{ if and only if } x^1 \not< x^2$$



Continuous Preferences

- Why is non-continuous order a problem?

$$y^t(\sim_h x^1) \to x^2, \text{ but } x^1 \succ_h x^2$$

- Corresponding utility also not continuous!

$$U(y^t) = U(x^1) \to U(x^2) < U(x^1)$$

- Continuous Order:

Suppose $\{x^t\}_{t=1,2,\cdots} \to x^0$. For any bundle y,

If for all t, $x^t \succsim_i y$ then $x^0 \succsim_i y$.

If for all t, $y \succsim_i x^t$ then $y \succsim_i x^0$.

Preferences: Where Do These Postulates Apply?

- More applicable to daily shopping (familiar…)
  - Can you rank things at open-air markets in Turkey?
- What if today's choice depends on past history or future plans? Consider: $x_t = (x_{1t}, x_{2t}, \cdots, x_{nt})$

  Then use $x = (x_1, x_2, \cdots, x_t, \cdots, x_T)$
- What if there is uncertainty about the complete bundle? Consider: $(x_1, x_2^g, x_2^b; \pi^g, \pi^b)$
- Would adding time and uncertainty make the commodities less "familiar"?

Preferences: LNS (rules out "total indifference")

- Back to full information, static (1 period) case:
- An "everything-is-as-good-as-everything" order satisfies all other postulates so far
  - But this isn't really useful for explaining choices...
- Local non-satiation (LNS):

For any consumption bundle $x \in C \subset \mathbb{R}^n$
and any δ-neighborhood $N(x, \delta)$ of x,
there is some bundle $y \in N(x, \delta)$ s. t. $y \succ_h x$

Preferences: Strict Monotonicity

- Another strong assumption is "More is always strictly preferred."
  - Natural for analyzing consumption of commodity groups (food, clothing, housing...)
- Strict Monotonicity:

$$\text{If } y > x, \text{ then } y \succ_h x.$$

Preferences: Convexity

- Final postulate: "Individuals prefer variety."
- Convexity:

  Let C be a convex subset of $\mathbb{R}^n$

  For any $x^0, x^1 \in C$, if $x^0 \succsim_h y$ and $x^1 \succsim_h y$,

  then $x^\lambda = (1-\lambda)x^0 + \lambda x^1 \succsim_h y$, $0 < \lambda < 1$.
- Strict Convexity:

  For any $x^0, x^1, y \in C$, if $x^0 \succsim_h y$ and $x^1 \succsim_h y$,

  then $x^\lambda \succ_h y$, $0 < \lambda < 1$.

Proposition 2.1-1: When's Utility Function Continuous?

Utility Function Representation of Preferences

If preferences are complete, reflective ($x \succsim_h x$),
transitive and continuous on $C \subset \mathbb{R}^n$,
they can be represented by a function $U(x)$
which is continuous over X.

→ Can use utility function to represent preferences

→ Use it as objective in constraint maximization

- Special Case: Strict Monotonicity

Special Case: Strict Monotonicity

Consider $x^0, x^1 \in X$, $x^1 > x^0 \Rightarrow x^1 \succ_h x^0$

For $T = \{x \in X | x^1 \succsim_h x \succsim_h x^0\}$,

Claim:

For any $y \in T$, there exists some weight $\lambda \in [0,1]$ such that $y \sim_h x^\lambda$ where $x^\lambda = (1-\lambda)x^0 + \lambda x^1$

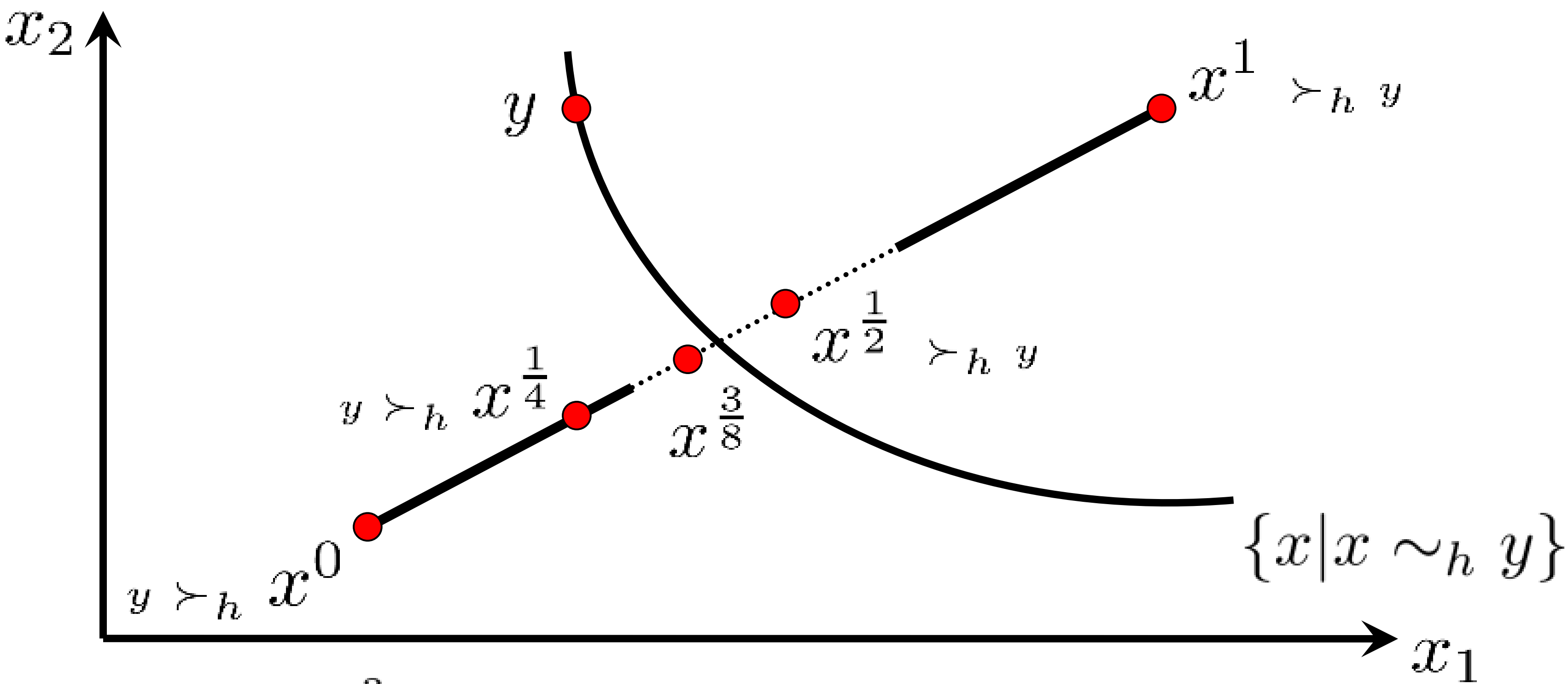
Moreover, $\lambda(y) : T \to [0,1]$ is continuous.

Proof:

Consider the sequence of intervals $\{x^{\nu_t}, x^{\mu_t}\}$,

Appeal to the completeness of real numbers…

Special Case: Strict Monotonicity



Either $x^{\frac{3}{8}} \sim_h y$ (done),
$x^{\frac{3}{8}} \succ_h y$ (consider $x^{\frac{3}{16}}$), or $y \succ_h x^{\frac{3}{8}}$ (consider $x^{\frac{7}{16}}$).

Special Case: Strict Monotonicity

Goal: Find $x^{\hat{\lambda}} \sim_h y$ as the limiting point of

Sequences $x^{\nu_t} (\succsim_h y)$ and $(y \succsim_h) x^{\mu_t}$

Start with $\nu_0 = 1,\ \mu_0 = 0$. Let $\lambda_{t+1} = \frac{1}{2}(\nu_t + \mu_t)$

If $y \sim_h x^{\lambda_t}$, we are done.

If $y \succ_h x^{\lambda_t},\ \nu_{t+1} = \nu_t,\ \mu_{t+1} = \lambda_{t+1}$

If $x^{\lambda_t} \succ_h y,\ \nu_{t+1} = \lambda_{t+1},\ \mu_{t+1} = \mu_t$

$$x^1 = x^{\nu_0} \succ_h \cdots \succ_h x^{\nu_n} \succ_h y$$

$$y \succ_h x^{\mu_n} \succ_h \cdots \succ_h x^{\mu_0} = x^0$$

Completeness of real numbers → $\hat{\lambda}(y)$ exists.

Convex Preferences = Quasi-Concave Utility

- Quasi-Concave Utility Function:
- U is quasi-concave on X if for any $x^0, x^1 \in X$
- and convex combination $x^\lambda = (1-\lambda)x^0 + \lambda x^1$

$\lambda \in [0,1]$

$$U(x^\lambda) \geq \min\{U(x^0), U(x^1)\}$$

- Convex Preferences:

Let C be a convex subset of $\mathbb{R}^n$

For any $x^0, x^1 \in C$, if $x^0 \succsim_h y$ and $x^1 \succsim_h y$,

then $x^\lambda = (1-\lambda)x^0 + \lambda x^1 \succsim_h y,\ 0 < \lambda < 1.$

Convex Preferences → Quasi-Concave Utility

- For any $x^0, x^1 \in X$ and convex combination
$$x^\lambda = (1-\lambda)x^0 + \lambda x^1, \lambda \in [0,1]$$
- Since preferences are convex, represented by U
- Without loss of generality, assume $x^0 \succsim_h x^1$
- Then,
$$x^\lambda = (1-\lambda)x^0 + \lambda x^1 \succsim_h x^1.$$
- Hence,
$$U(x^\lambda) \geq U(x^1) = \min\{U(x^0), U(x^1)\}$$

Convex Preferences ← Quasi-Concave Utility

- For any $x^0, x^1 \in X$ and convex combination
$$x^\lambda = (1-\lambda)x^0 + \lambda x^1, \lambda \in [0,1]$$
- Preferences are represented by U
- If $x^0 \succsim_h y$ and $x^1 \succsim_h y$, we have
$$U(x^1) \geq U(y), U(x^0) \geq U(y)$$
- Since U is quasi-concave,
$$U(x^\lambda) \geq \min\{U(x^0), U(x^1)\} \geq U(y)$$
- Hence, $x^\lambda \succsim_h y$.

Summary of 2.1

- Preference Axioms
  - Complete
  - Transitive
  - Continuous
    - Monotonic
    - Convex / Strictly Convex
- Utility Function Representation
- Homework: Exercise 2.1-4 (Optional: 2.1-2)