The 2x2 Exchange Economy

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(Lecture 7, Micro Theory I)

Road Map for Chapter 3

- Pareto Efficiency
  - Cannot make one better off without hurting others
- Walrasian (Price-taking) Equilibrium
  - When Supply Meets Demand
  - Focus on Exchange Economy First
- 1st Welfare Theorem: Walrasian Equilibrium is Efficient (Adam Smith Theorem)
- 2nd Welfare Theorem: Any Efficient Allocation can be supported as a Walrasian Equilibrium

2x2 Exchange Economy

- 2 Commodities: Good 1 and 2
- 2 Consumers: Alex and Bev $h = A, B$
  - Endowment: $\omega^h = (\omega_1^h, \omega_2^h),\ \omega_i = \omega_i^A + \omega_i^B$
  - Consumption Set: $x^h = (x_1^h, x_2^h) \in \mathbb{R}_+^2$
  - Strictly Monotonic Utility Function: $U^h(x^h) = U^h(x_1^h, x_2^h)$
- Edgeworth Box
- These consumers could be representative agents, or literally TWO people (bargaining)

Why do we care about this?

- The Walrasian (Price-taking) Equilibrium (W.E.) is (a candidate of) Adam Smith's "Invisible Hand"
  - Are real market rules like Walrasian auctioneers?
  - Is Price-taking the result of competition, or competition itself?
- Illustrate W.E. in more general cases
  - Hard to graph "N goods" as 2D
- Two-party Bargaining
  - This is what Edgeworth really had in mind

Why do we care about this?

- Consider the following situation: You company is trying to make a deal with another company
  - Your company has better technology, but lack funding
  - Other company has plenty of funding, but low-tech
- There are “gives” and “takes” for both sides
- Where would you end up making the deal?
  - Definitely not where “something is left on the table.”
- What are the possible outcomes?
  - How did you get there?

Social Choice and Pareto Efficiency

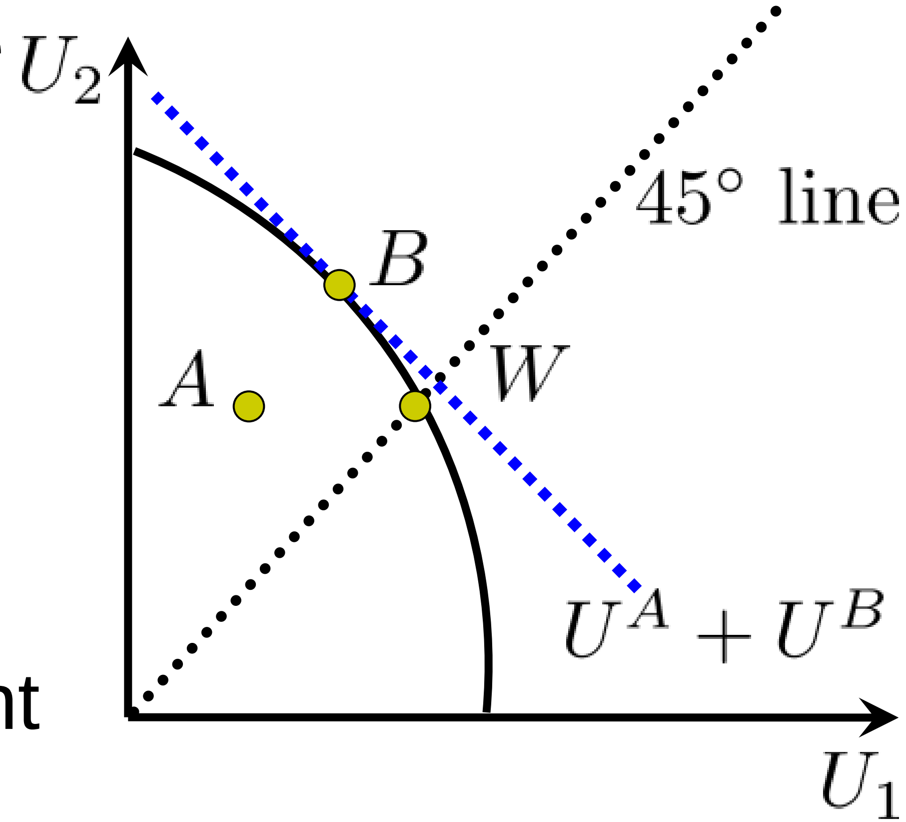
- Benthamite:
  - Behind Veil of Ignorance
  - Assign Prob. 50-50

$$\max \tfrac{1}{2}U^A + \tfrac{1}{2}U^B$$

- Rawlsian:
  - Extremely Risk Averse

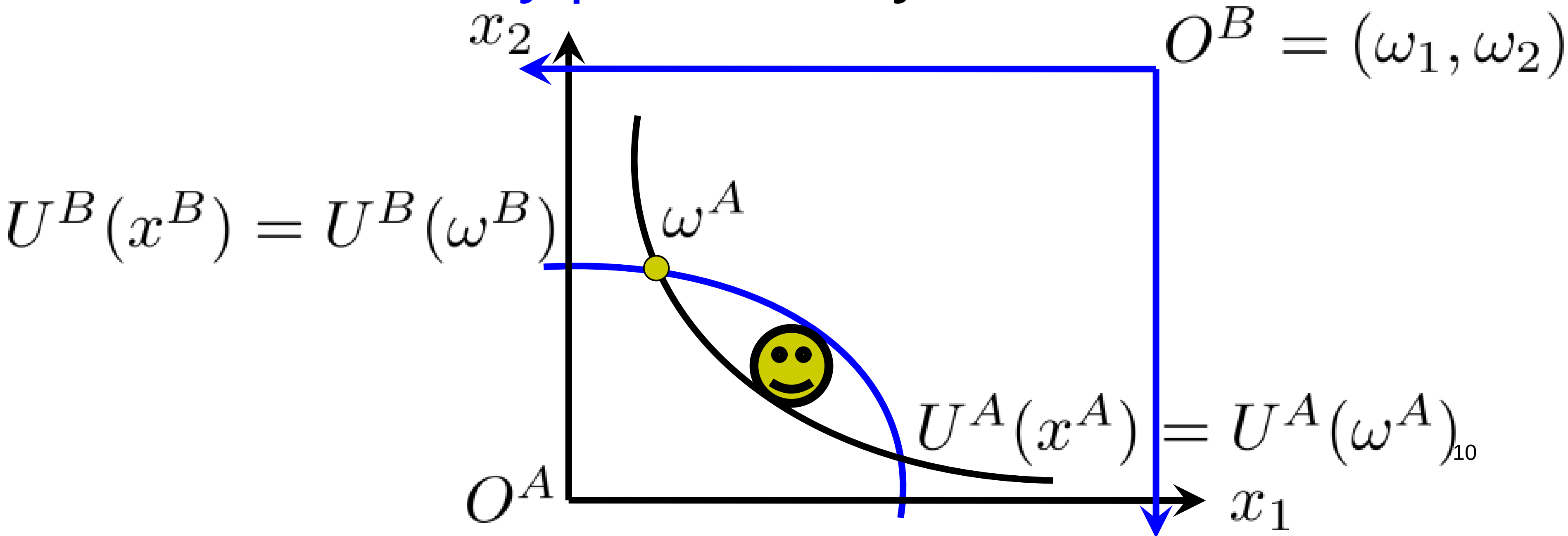
$$\max \min\{U^A, U^B\}$$

- Both are Pareto Efficient
  - But *A* is not



Pareto Efficiency

- A feasible allocation is Pareto efficient if
- there is no other feasible allocation that is
- strictly preferred by at least one consumer
- and is weakly preferred by all consumers.

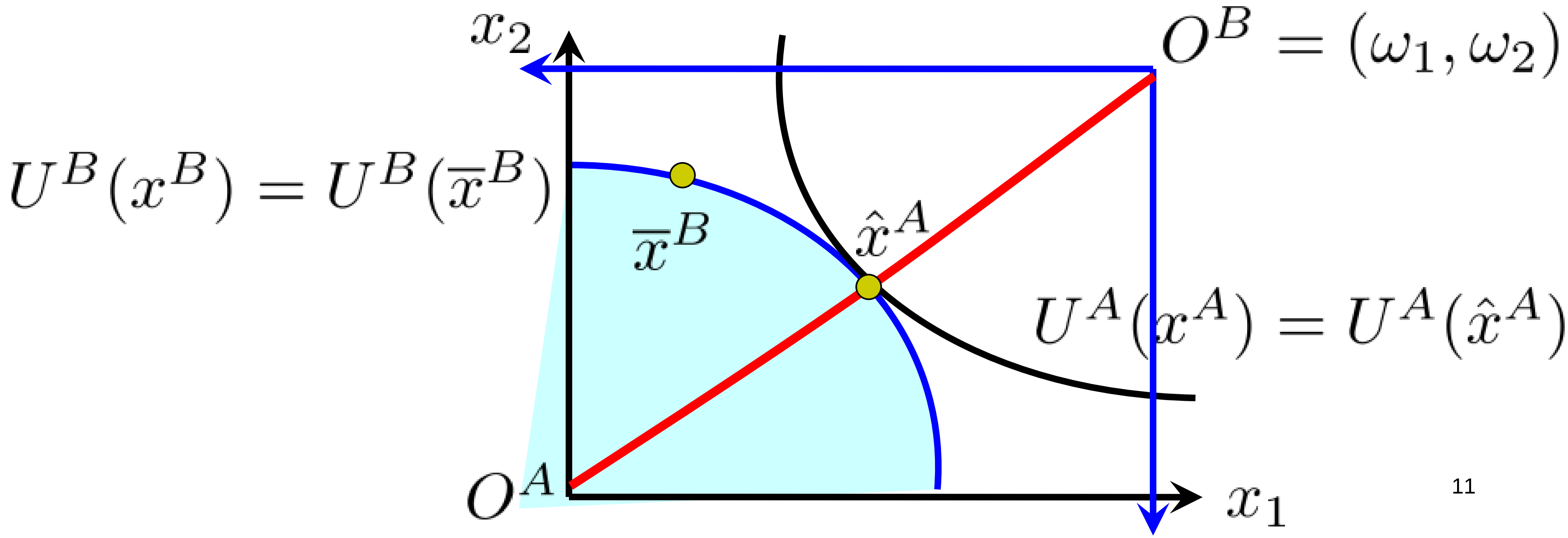


Pareto Efficient Allocations

For $\omega = (\omega_1, \omega_2)$, consider

$$\max_{x^A, x^B} \{U^A(x^A) | U^B(x^B) \geq U^B(\overline{x}^B), x^A + x^B \leq \omega\}$$

Need $MRS^A(\hat{x}^A) = MRS^B(\overline{x}^B)$ (interior solution)



Walrasian Equilibrium (in 2x2 Exchange Economy)

- All Price-takers: $\text{Prices } p \geq 0$
- 2 Consumers: Alex and Bev $h = A, B$
  - Endowment: $\omega^h = (\omega_1^h, \omega_2^h),\ \omega_i = \omega_i^A + \omega_i^B$
  - Consumption Set: $x^h = (x_1^h, x_2^h) \in \mathbb{R}_+^2$
  - Wealth: $W^h = p \cdot \omega^h$
- Market Demand: $x(p) = \sum_h x^h(p, p \cdot \omega^h)$
- Vector of Excess Demand: $e(p) = x(p) - \omega$
  - Vector of total Endowment: $\omega = \sum_h \omega^h$

Definition: Market Clearing Prices

- Let excess demand for commodity *j* be $e_j(p)$
- The market for commodity *j* clears if

$$e_j(p) \leq 0 \text{ and } p_j \cdot e_j(p) = 0$$

- Why is this important?
- Walras Law
  - The last market clears if all other markets clear
- Market clearing defines Walrasian Equilibrium

Walras Law

- LNS implies consumer must spend all income
- If not, we have $p \cdot x^h < p \cdot \omega^h$
- But then there exist δ-neighborhood $N(x^h, \delta)$
- In the budget set for sufficiently small $\delta > 0$
  - Contradicting LNS

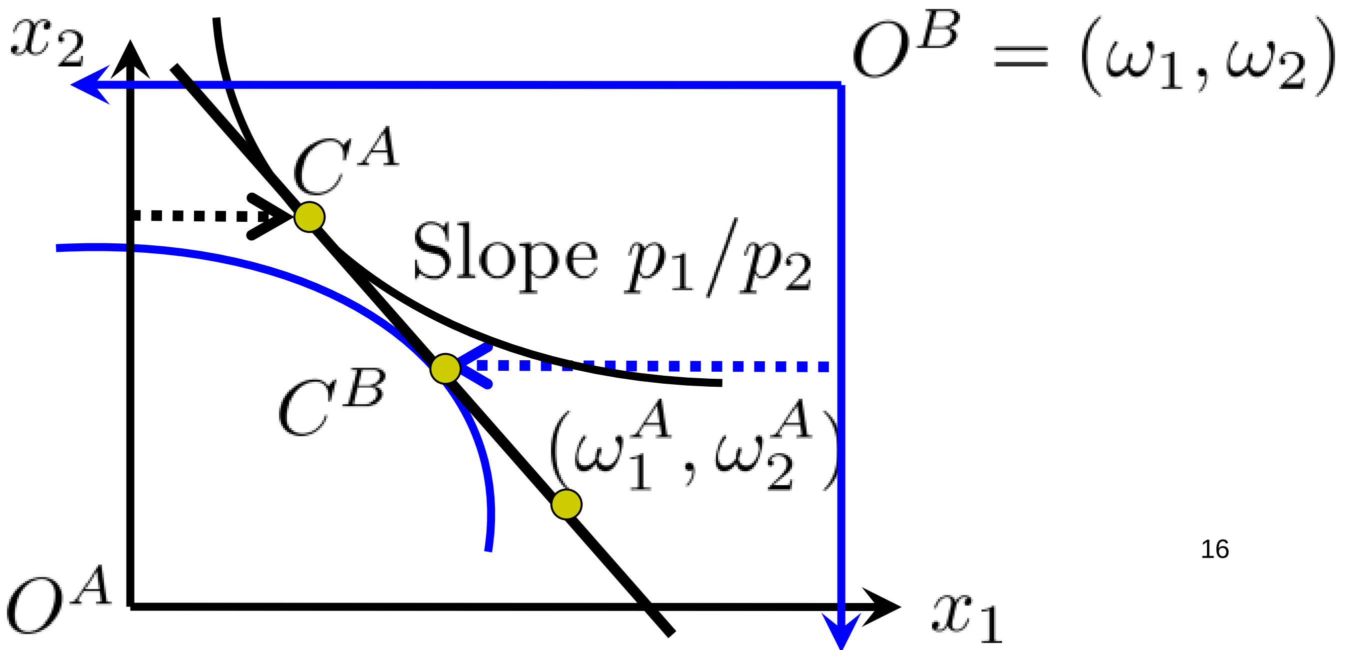
$$\sum_h (p \cdot x^h - p \cdot \omega^h) = 0 = p \cdot \left(\sum_h (x^h - \omega^h) \right)$$

$$= p \cdot (x - \omega) = p \cdot e(p) = p_1 e_1(p) + p_2 e_2(p) = 0$$

- If one market clears, so must the other.

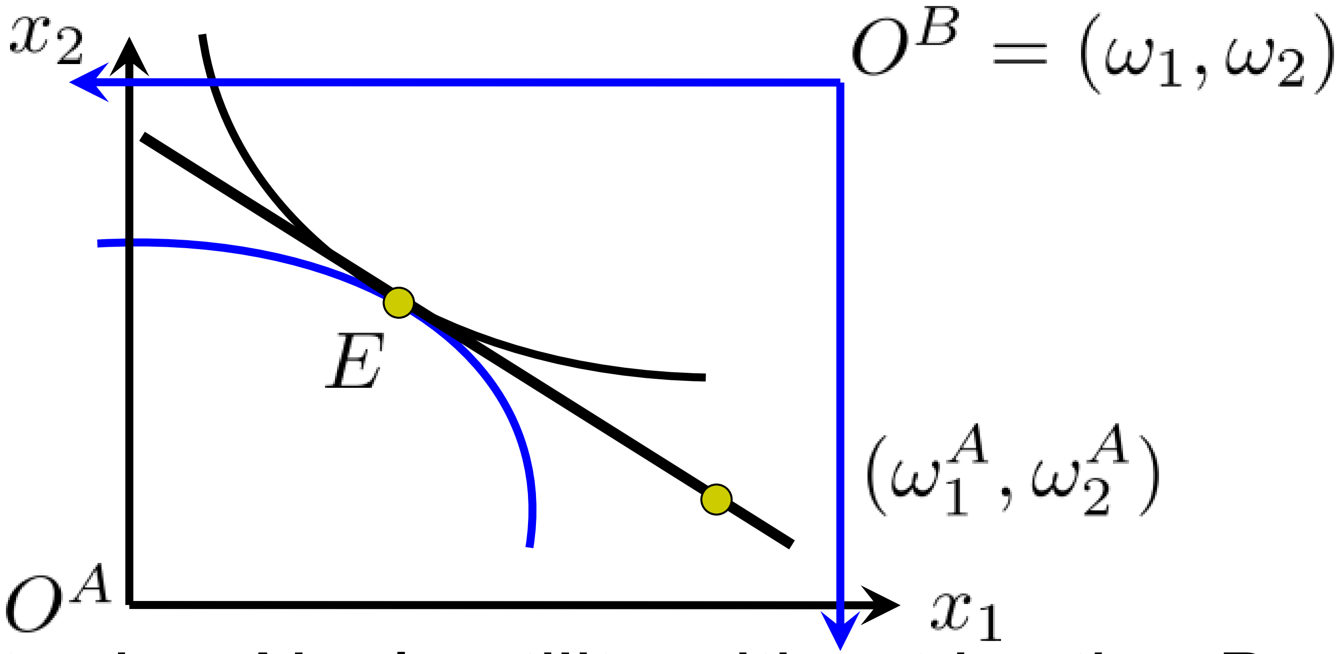
Definition: Walrasian Equilibrium

- The price vector $p \geq 0$ is a Walrasian Equilibrium price vector if all markets clear.
  - WE = price vector!!!
- EX: Excess supply of commodity 1…



Definition: Walrasian Equilibrium

- Lower price for commodity 1 if excess supply
  - Until Markets Clear



- Cannot raise Alex's utility without hurting Bev
  - Hence, we have…

First Welfare Theorem: WE → PE

- If preferences satisfy LNS, then a Walrasian Equilibrium allocation (in an exchange economy) is Pareto efficient.
- Sketch of Proof:

1. Any weakly (strictly) preferred bundle must cost at least as much (strictly more) as WE
2. Markets clear
   → Pareto preferred allocation not feasible

First Welfare Theorem: WE → PE

1. Since WE allocation $\overline{x}^h$ maximizes utility, so
   $$U^h(x^h) > U(\overline{x}^h) \Rightarrow p \cdot x^h > p \cdot \overline{x}^h$$
   Now need to show that
   $$U^h(x^h) \geq U(\overline{x}^h) \Rightarrow p \cdot x^h \geq p \cdot \overline{x}^h$$

- If not, we have $p \cdot x^h < p \cdot \overline{x}^h$
- But then there exist $\delta\text{-neighborhood } N(x^h, \delta)$
- In the budget set for sufficiently small $\delta > 0$
- Contradicts LNS that requires a point $\tilde{x}^h$ such that $U^h(\tilde{x}^h) > U^h(x^h) \geq U(\overline{x}^h)$

First Welfare Theorem: WE → PE

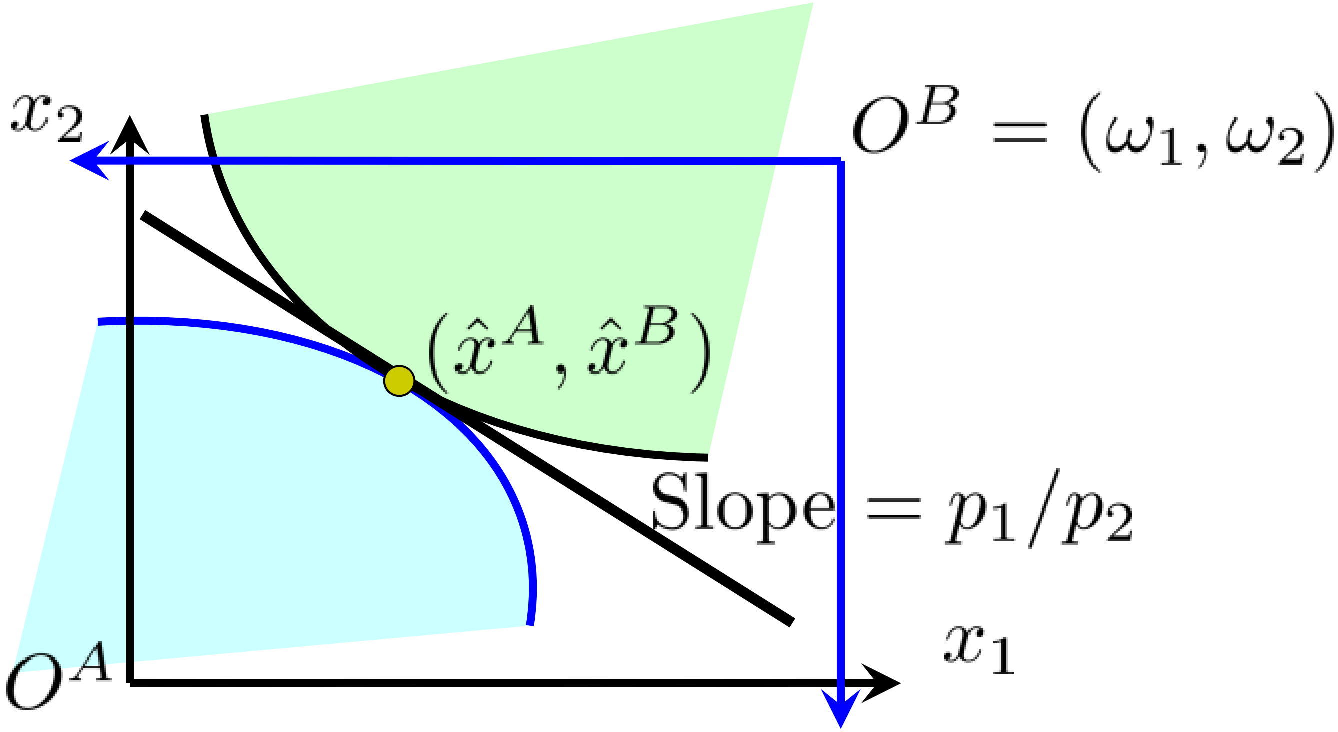
1. $U^h(x^h) > U(\overline{x}^h) \Rightarrow p \cdot x^h > p \cdot \overline{x}^h$

   $U^h(x^h) \geq U(\overline{x}^h) \Rightarrow p \cdot x^h \geq p \cdot \overline{x}^h$
2. True for Pareto preferred allocation (x^A, x^B)

- Hence, $p \cdot x^h > p \cdot \overline{x}^h$ for at least one, and
- $p \cdot x^h \geq p \cdot \overline{x}^h$ for all others (by PEA)
- Thus, $p \cdot \sum_h x^h > p \cdot \sum_h \overline{x}^h = p \cdot \sum_h \omega^h$
- Since $p \geq 0$, at least one j → $\sum_h x_j^h > \sum_h \omega_j^h$
  - Not feasible!

Second Welfare Theorem: PE → WE

- For a Pareto efficient allocation $(\hat{x}^A, \hat{x}^B)$
- Convex preferences imply convex regions
  - Separating hyperplane theorem generates prices

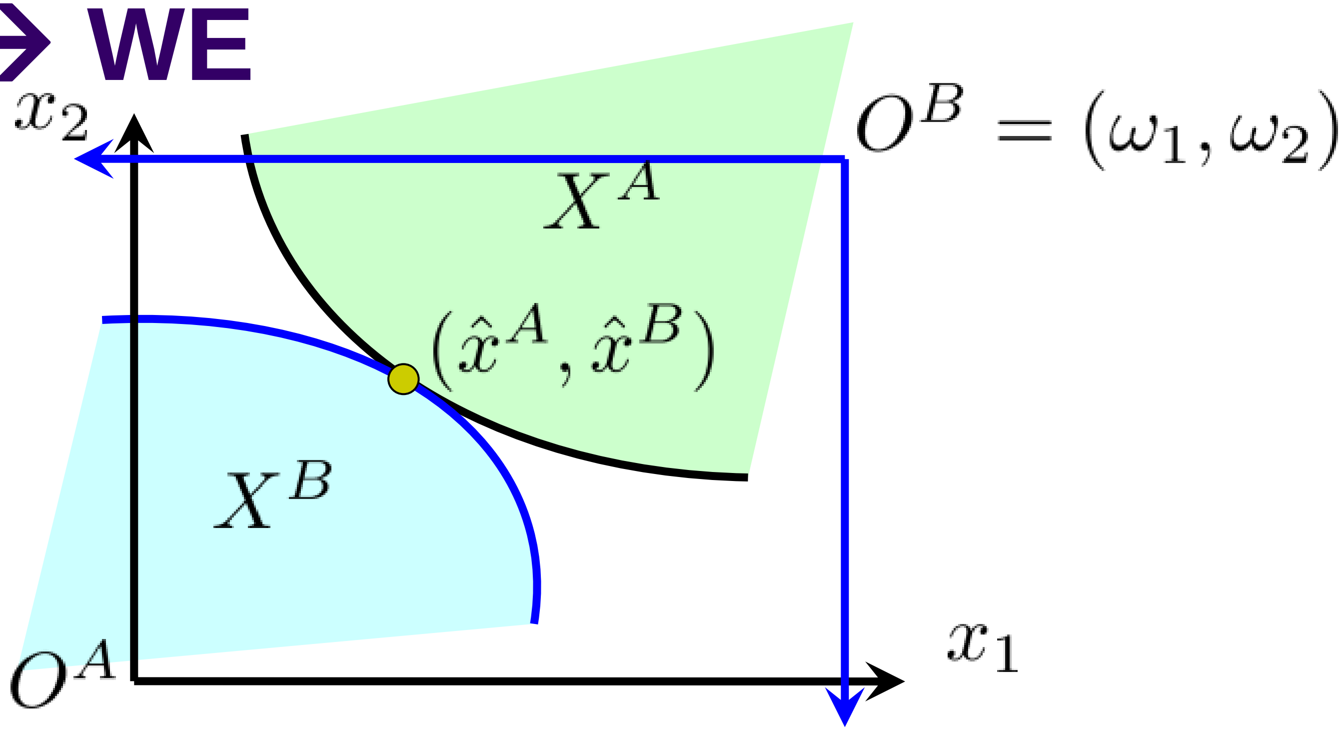


Second Welfare Theorem: PE → WE

- If preferences are convex & strictly increasing, then any Pareto efficient allocation (of an exchange economy) can be supported by a price vector $p \geq 0$ (as a Walrasian Equilibrium).
- Sketch of Proof:

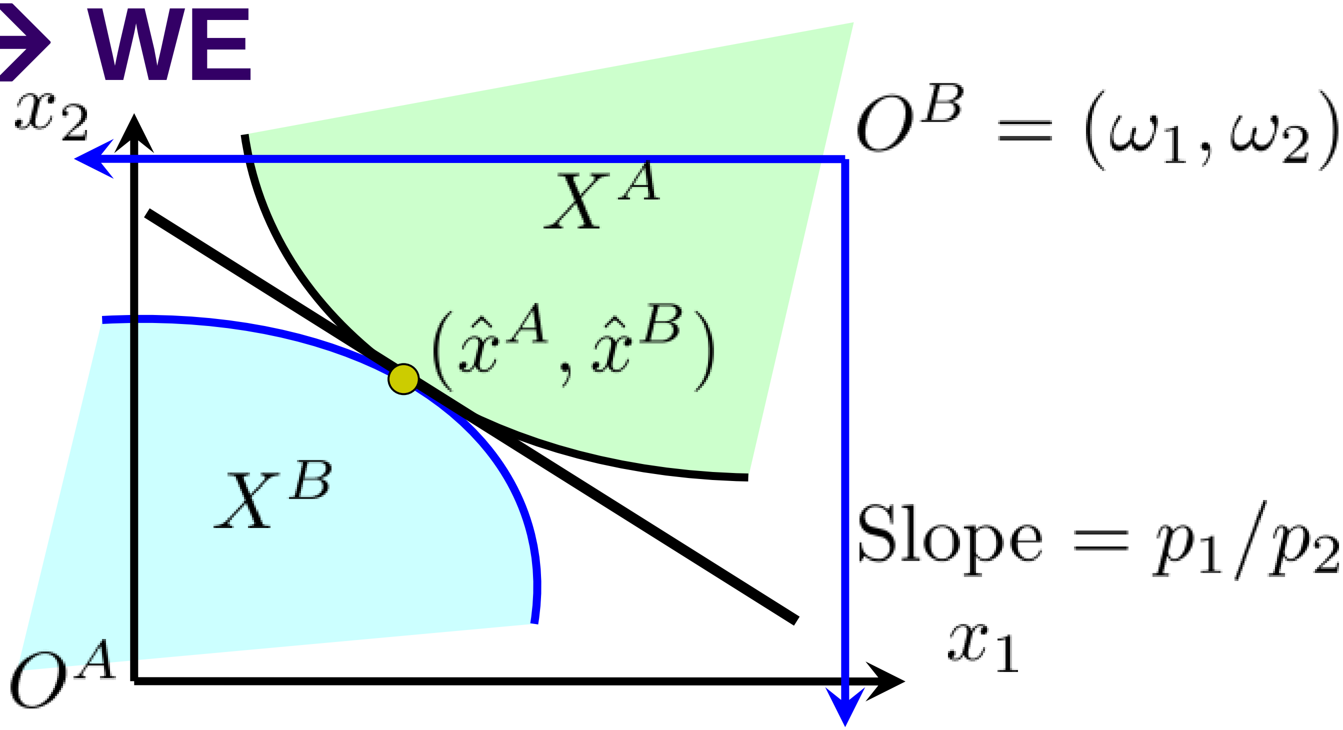
1. The at-least-as-good-as sets are convex
2. Supporting Hyperplane Theorem → $p \geq 0$
3. Alex and Bev are both maximizing at $p \geq 0$

Second Welfare Theorem: PE → WE



- Alex's "at-least-as-good-as" set X^A is convex
- Bev's "at-least-as-good-as" set X^B is convex
- $X^A \cap X^B$ has no interior since $(\hat{x}^A, \hat{x}^B)$ is PEA

Second Welfare Theorem: PE → WE

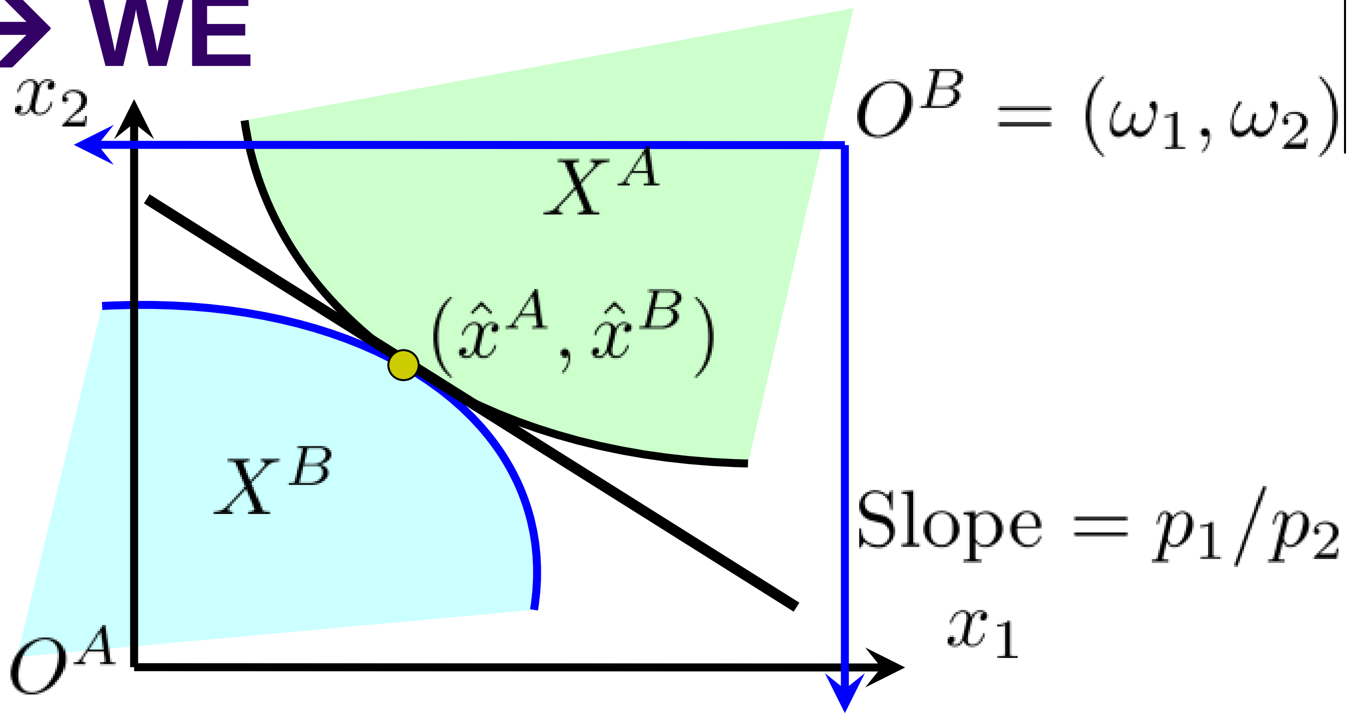


- By Supporting Hyperplane Theorem, exists p,
- Such that

$$p \cdot x^A > p \cdot \hat{x}^A \quad \forall x^A \in \text{int} X^A$$

$$p \cdot x^A < p \cdot \hat{x}^A \quad \forall x^A \in \text{int} X^B$$

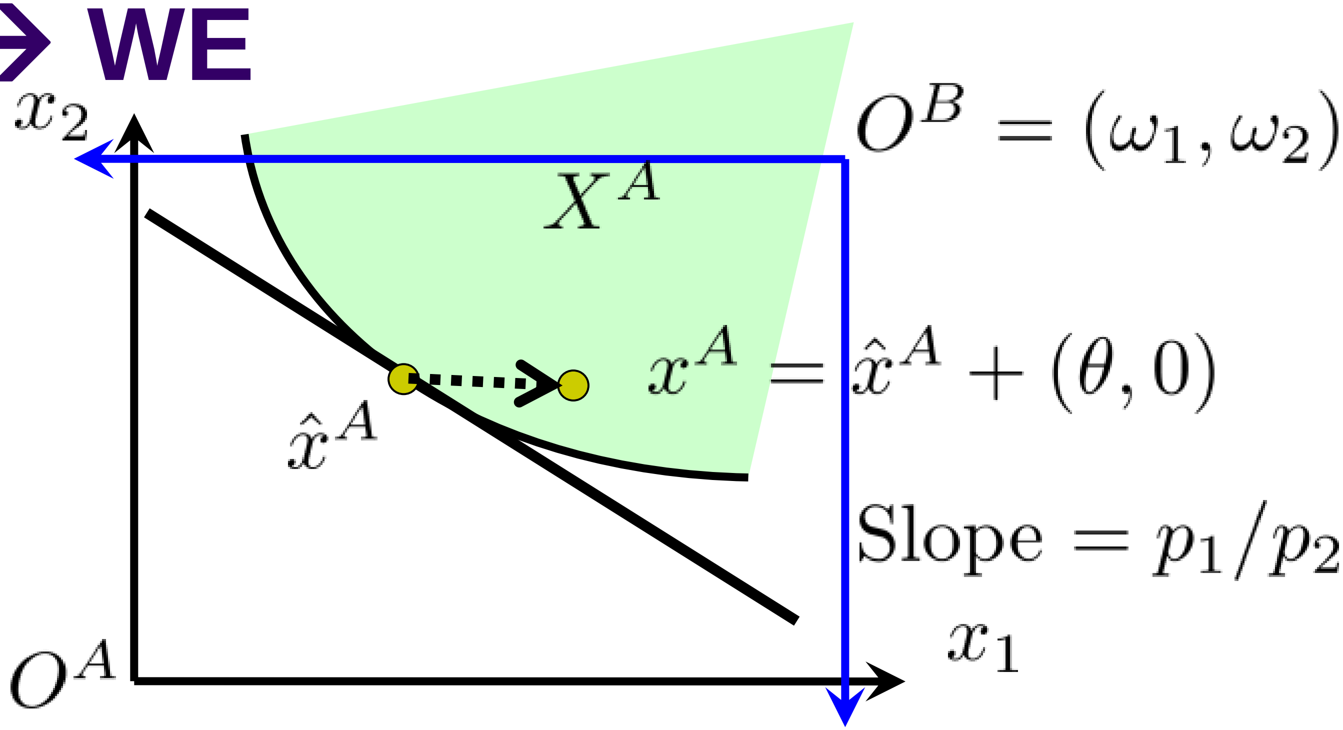
Second Welfare Theorem: PE → WE



- Claim $p \geq 0$, then $p \cdot x^A > p \cdot \hat{x}^A \quad \forall x^A \in \text{int} X^A$
- Implies Alex is maximizing. Similarly,
- Bev is maximizing since $x^A = \omega - x^B \in \text{int} X^B$

$$p \cdot x^B = p \cdot (\omega - x^A) > p \cdot (\omega - \hat{x}^A) = p \cdot \hat{x}^B$$

Second Welfare Theorem: PE → WE



Why $p \geq 0$?

- Suppose $p_1 < 0$, then add $\theta > 0$ to 1st:

$x^A = \hat{x}^A + (\theta, 0) \in \mathrm{int} X^A$ ($U^A(\cdot)$ strictly increasing)

- But $p \cdot x^A = p \cdot \hat{x}^A + p_1 \cdot \theta < p \cdot \hat{x}^A$

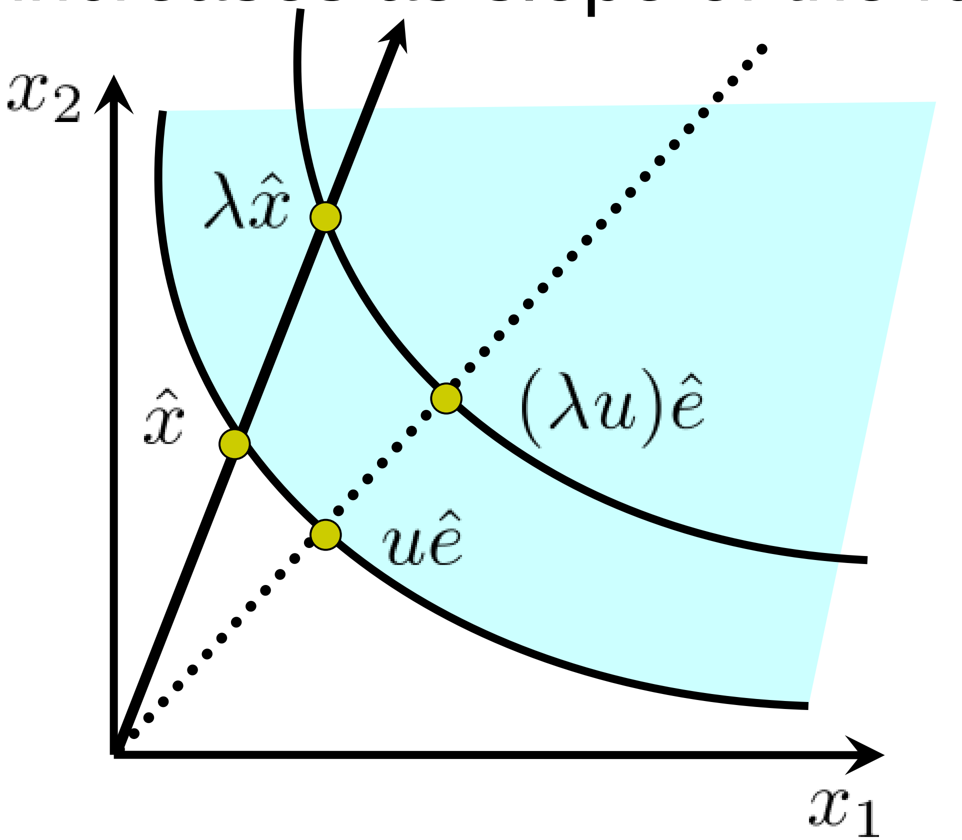
Contradiction!

Proposition 3.1-3 PEA with Homothetic Preferences

- 2x2 Exchange Economy
- Consumers have homothetic preferences
- At aggregate endowment, consumer A has a stronger preference for commodity 1.
- Consumption ratio: $\frac{x_2^A}{x_1^A} < \frac{x_2^B}{x_1^B}$
- And, as $U^A(x^A)$ rises, consumption ratio $\frac{x_2^A}{x_1^A}$ and MRS both rise.

Homothetic Preferences: Radial Parallel

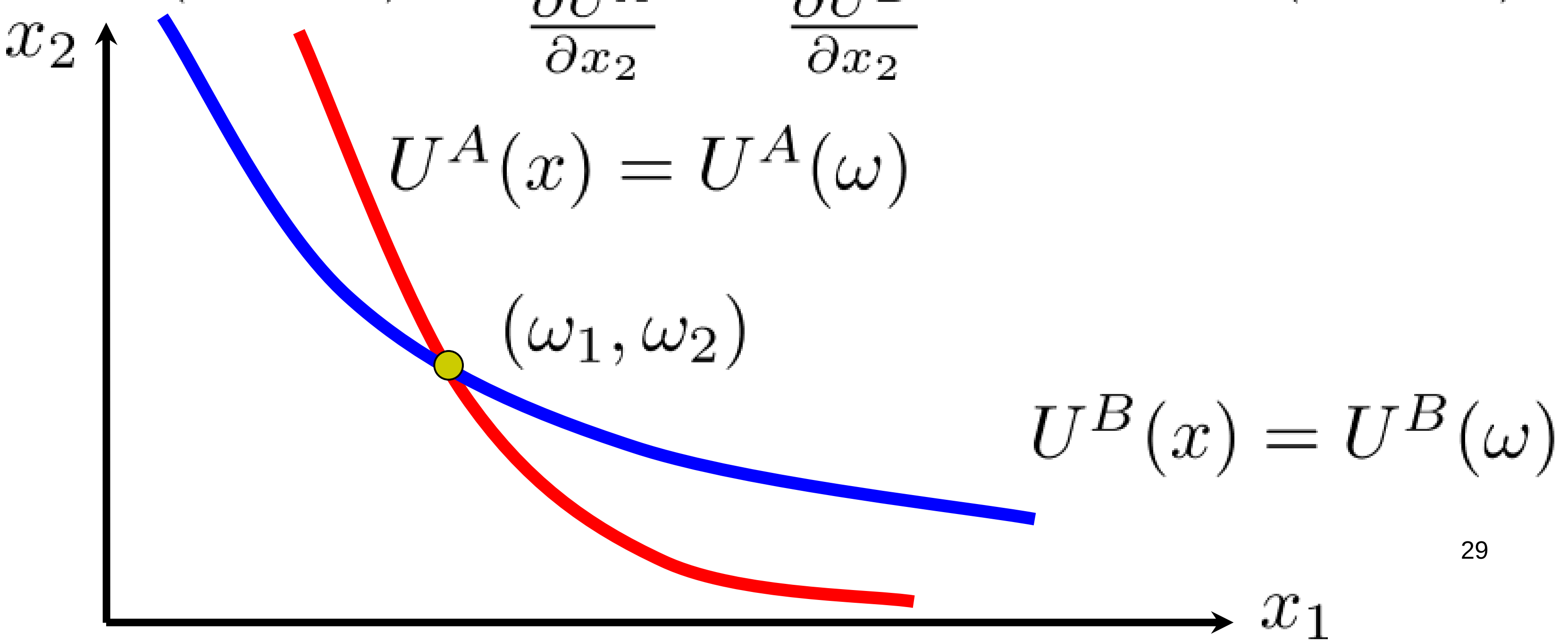
- MRS same on each ray
- MRS increases as slope of the ray increase



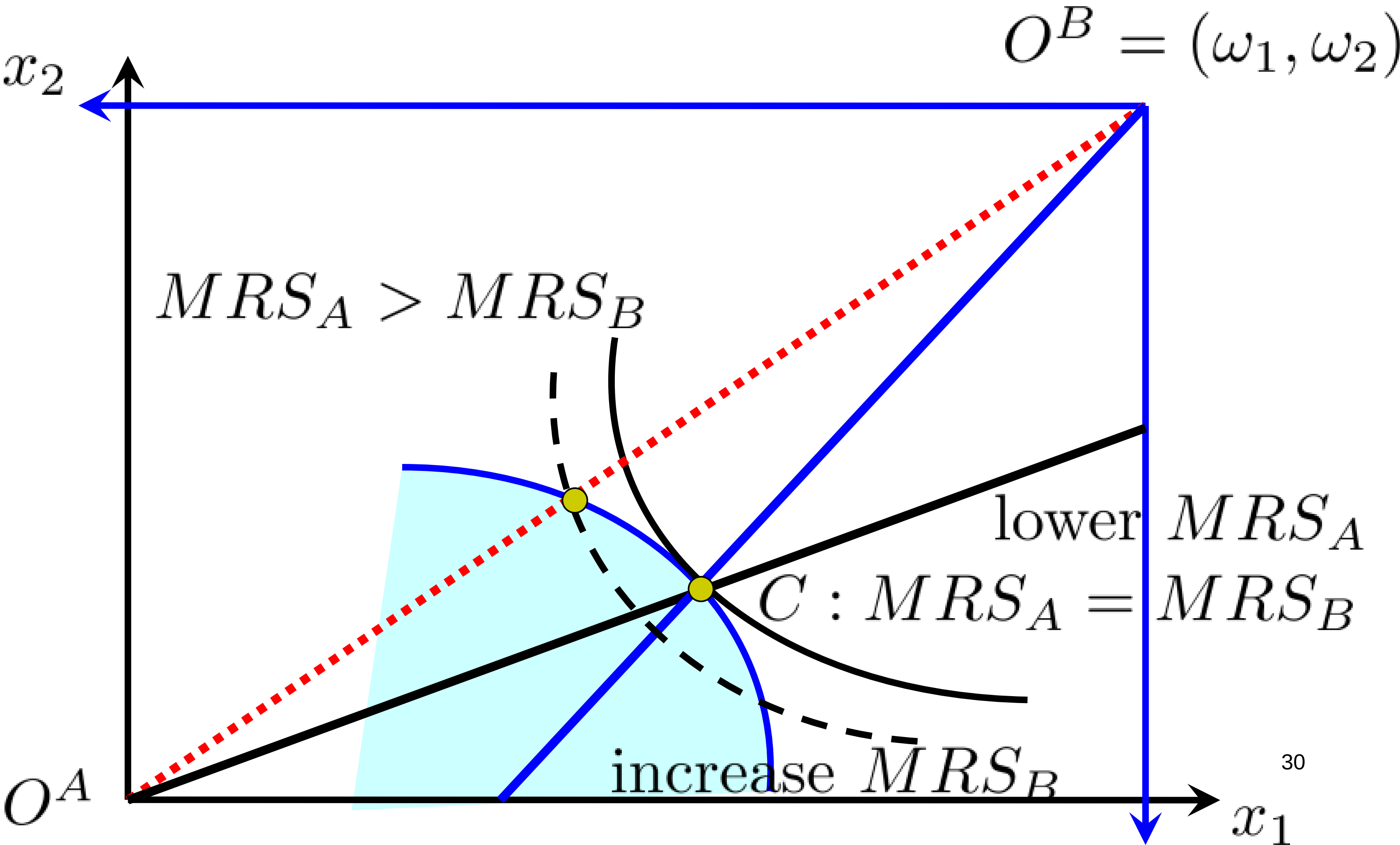
Assumption: Intensity of Preferences

- Alex and Bev: convex, homothetic preferences
- Alex has stronger preferences for commodity 1

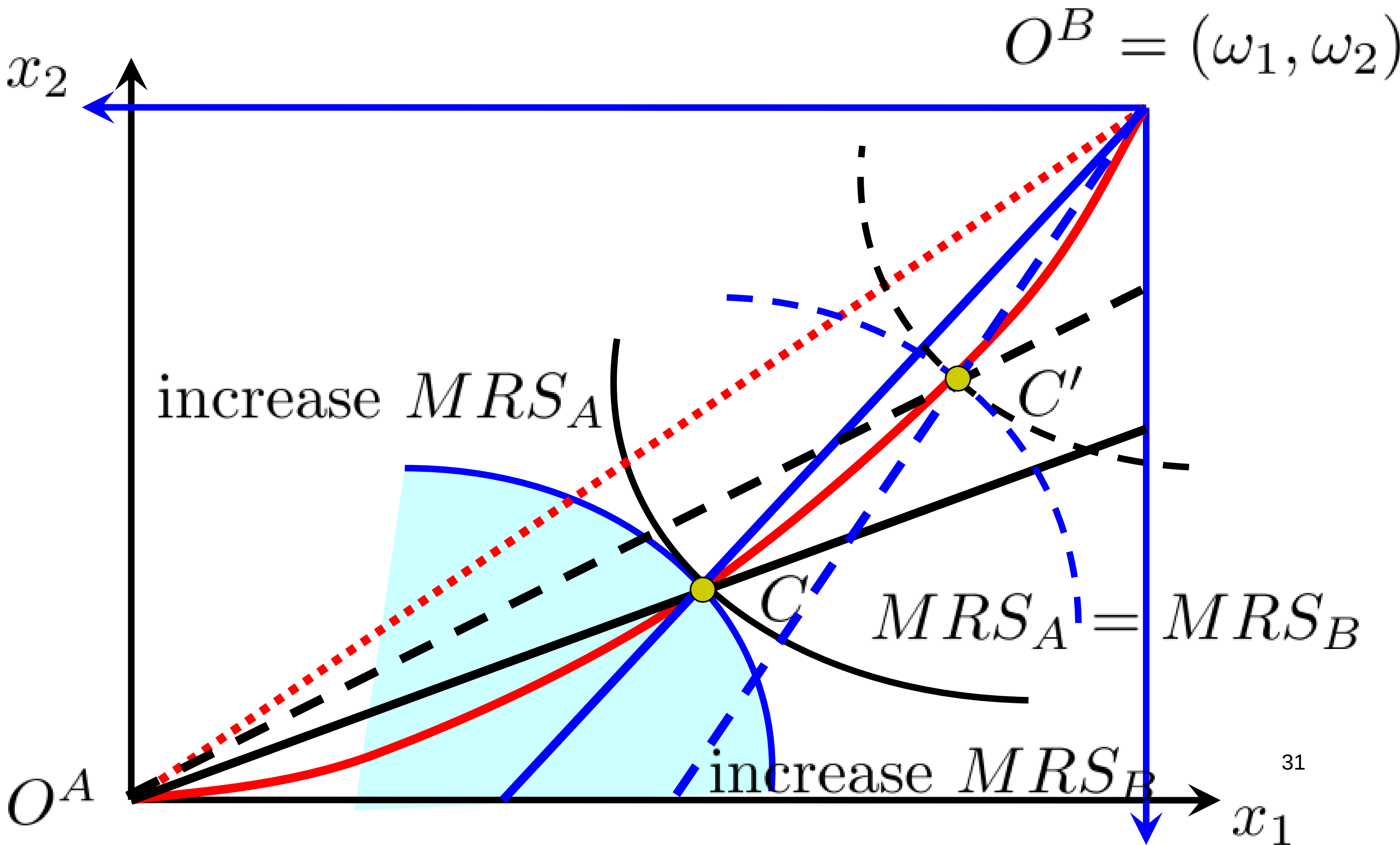
$$MRS_A(\omega_1, \omega_2) = \frac{\frac{\partial U^A}{\partial x_1}}{\frac{\partial U^A}{\partial x_2}} > \frac{\frac{\partial U^B}{\partial x_1}}{\frac{\partial U^B}{\partial x_2}} = MRS_B(\omega_1, \omega_2)$$



Pareto Efficient Allocations



Pareto Efficient Allocations



Proposition 3.1-3 PEA with Homothetic Preferences

- 2x2 Exchange Economy
- Consumers have homothetic preferences
- At aggregate endowment, consumer A has a stronger preference for commodity 1.
- Then at any interior PEA, $\frac{x_2^A}{x_1^A} < \frac{x_2^B}{x_1^B}$
- Moreover, as $U^A(x^A)$ rises, consumption ratio $\frac{x_2^A}{x_1^A}$ and MRS both rise.

Summary of 3.1

- Pareto Efficiency:
  - Cannot make one better off without hurting others
- Walrasian Equilibrium: market clearing prices
- Welfare Theorems:
  - First: Walrasian Equilibrium is Pareto Efficient
  - Second: Pareto Efficient allocations can be supported as Walrasian Equilibria (with transfer)
- Homework: Riley – 3.1-1, 2, 4
- J/R – 5.11, 5.12, 5.15, 5.17, 5.18