

# General Equilibrium and Efficiency with Production

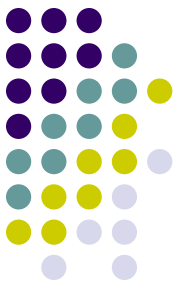
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(Lecture 13, Micro Theory I)



# What We Learned from Exchange Economy and Robinson Crusoe?



- Pareto Efficient Allocation (PEA) – Optimum
  - Cannot make one better off without hurting others
- Walrasian Equilibrium (WE) – Price Vector
  - When Supply Meets Demand
  - Even if there is only one person Robinson Crusoe
- 1<sup>st</sup> Welfare Theorem: WE are PEA
  - Schizophrenic Robinson Crusoe achieves optimum
- 2<sup>nd</sup> Welfare Theorem: PEA supported as WE
- These also apply to the general case as well!

# The General Model: Firms

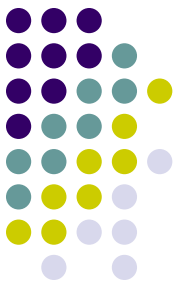


- $F$  Firms:  $f=1, 2, \dots, F$
- Production Plan  $y^f = (y_1^f, \dots, y_n^f) \in \gamma^f$
- Production Set  $\gamma^f \subset \mathbb{R}^n$
- Production Plan for the Economy  $\{y^f\}_{f=1}^F$
- Aggregate Production Plan

$$y = \sum_{f=1}^F y^f$$

- Aggregate Production Set  $\gamma$

# The General Model: Consumers



- $n$  Commodities (are private):  $1, 2, \dots, n$
- $H$  Consumers:  $h = 1, 2, \dots, H$ 
  - Consumption Set:  $X^h \subset \mathbb{R}^n$
  - Endowment:  $\omega^h = (\omega_1^h, \dots, \omega_n^h) \in X^h$
  - Consumption Vector:  $x^h = (x_1^h, \dots, x_n^h) \in X^h$
  - Utility Function:  $U^h(x^h) = U^h(x_1^h, \dots, x_n^h)$
  - Consumption Allocation:  $\{x^h\}_{h=1}^H$
  - Aggregate Consumption and Endowment:  
$$x = \sum_{h=1}^H x^h \text{ and } \omega = \sum_{h=1}^H \omega^h$$

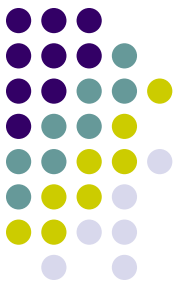
# The General Model: Shareholdings



- All firms are owned by consumers.
- Consumer  $h$  has  $\theta^{hf}$  ownership shares in firm  $f$
- Ownership share must sum up to one:

$$\sum_{h=1}^H \theta^{hf} = 1, f = 1, \dots, F$$

# The General Model: Feasible Allocation



- A allocation is **feasible** if
- The sum of the net demands **doesn't exceed** aggregate production plan:
$$x - \omega \leq y$$
- A feasible plan for the economy  $\{\hat{x}^h\}_{h=1}^H, \{\hat{y}^f\}_{f=1}^F$  is **Pareto efficient** if
- there is no other feasible plan  $\{x^h\}_{h=1}^H, \{y^f\}_{f=1}^F$
- that is
- **strictly preferred** by at least one:  $U^i(x^i) > U^i(\hat{x}^i)$
- and **weakly preferred** by all:  $U^h(x^h) \geq U^h(\hat{x}_6^h)$



# Walrasian Equilibrium

- Price-taking firms and consumers: Prices  $p > 0$
- Consumers:  $h=1, 2, \dots, H$ 
  - Firm Profit:  $p \cdot y^f$
  - Total Dividend Payments:  $\sum \theta^{hf} p \cdot y^f$
  - Wealth:  $W^h = p \cdot \omega^h + \sum_f \theta^{hf} p \cdot y^f$
  - Budget Set:  $\{x^h \in X^h \mid p \cdot x^h \leq W^h\}$
- Consumer interest best served by profit maximizing firm managers



# Walrasian Equilibrium

- Price-taking: Prices  $p > 0$
- Firm  $f$  chooses production plan  $\bar{y}^f$  to max. profit  
 $p \cdot \bar{y}^f \geq p \cdot y^f$ , for all  $y^f \in \gamma^f$ ,  $f = 1, \dots, F$
- Consumer  $h$  chooses most preferred consumption in her budget set:  
 $\bar{x}^h \succ_h x^h$  for all  $x^h$  such that  $p \cdot x^h \leq W^h$
- Vector of Excess Demand:  $\bar{e} = \bar{x} - \omega - \bar{y}$ 
  - Total consumption  $\bar{x} = \sum \bar{x}^h$
  - Total production  $\bar{y} = \sum \bar{y}^f$



# Definition:

## Walrasian Equilibrium Prices



- The price vector  $p \geq 0$  is a **Walrasian Equilibrium price vector** if
  - there is no market in excess demand ( $\bar{e} \leq 0$ ),
  - and  $p_j = 0$  for any market that is in excess supply ( $\bar{e}_j < 0$ ).
- 
- We are now ready to state and prove the “Adam Smith Theorem” (WE  $\rightarrow$  PEA)...

# Proposition 5.2-1: First Welfare Theorem



- If preferences of each consumer satisfies LNS, then the Walrasian Equilibrium allocation is Pareto efficient.
- Proof:

# Proposition 5.2-3: Second Welfare Theorem



- Let  $\{\hat{x}^h\}_{h=1}^H, \{\hat{y}^f\}_{f=1}^F$  be a PEA. Suppose:
  1. Consumption vectors are private
  2. Consumption sets  $X^h$  are convex
  3. Utility functions  $U^h(\cdot)$  are continuous, quasi-concave, and satisfies LNS
  4. There is some  $\underline{x}^h \in X^h$  such that  $\underline{x}^h < \hat{x}^h$
  5. Production sets  $\gamma^f$  convex and of free disposal
- Then there exist a price vector  $p \geq 0$  such that
 
$$x^h \succ_h \hat{x}^h \Rightarrow p \cdot x^h > p \cdot \hat{x}^h \quad (\text{Consumers Max Utility})$$

$$y^f \in \gamma^f \Rightarrow p \cdot y^f \leq p \cdot \hat{y}^f \quad (\text{Producers Max Profit})$$



# Summary of 5.2

- Pareto Efficiency:
  - Cannot make one better off without hurting others
- Walrasian Equilibrium: market clearing prices
- Welfare Theorems:
  - First: Walrasian Equilibrium is Pareto Efficient
  - Second: Pareto Efficient allocations can be supported as Walrasian Equilibria (with transfer)
- Homework: Exercise 5.2-1~3, J/R -