# Online Math Camp (235)

## TA Session Note (4/10)

## (Quiz 6 Solution)

1. (25 pts) Define "$K$ is a *compact set* in the metric space $X$".

(pf) For any open

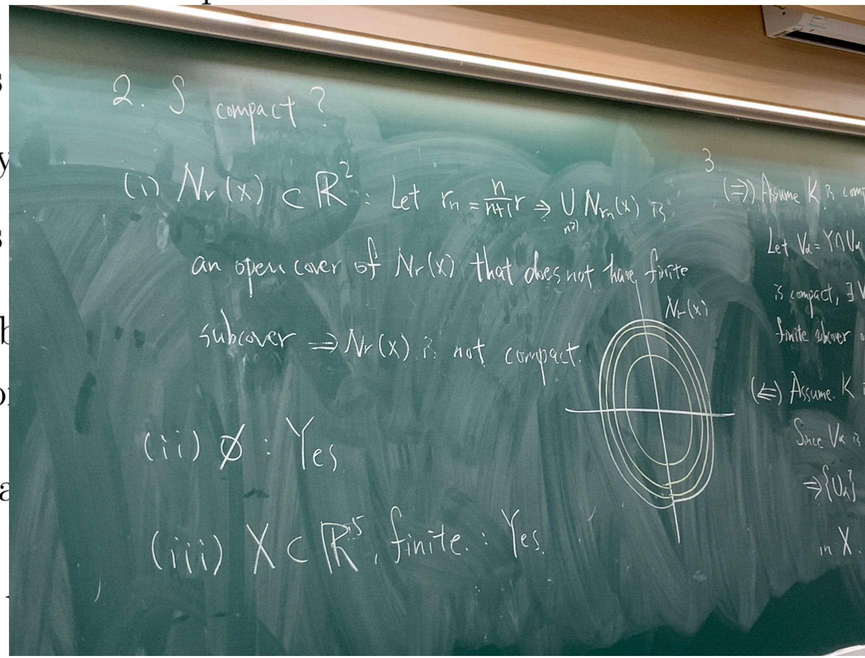
1. (25 pts) Define "$K$ is a *compact set* in the metric space $X$".

2. (15 pts each) Is the set $S$ compact in $X$? Proofs are needed.

   (i) $X = \mathbb{R}^2$. $S$ is

   (ii) $S$ is an empty

   (iii) $X = \mathbb{R}^5$. $S$ is




3. (24 pts) Given $X$ b … $K$ is compact in $Y$ if and only if $K$ is co…

4. (24 pts) Let $F$ be a … $\cap K$ is a compact set.

5. (20 pts) Let $K =$ … out using Heine-Borel Theorem.

3. (24 pts) Given $X$ being a metric space and $K \subset Y \subset X$. Prove that $K$ is compact in $Y$ if and only if $K$ is compact in $X$.

(pf) ($\Rightarrow$) Assume $K$ is compact in $Y \subseteq X$.

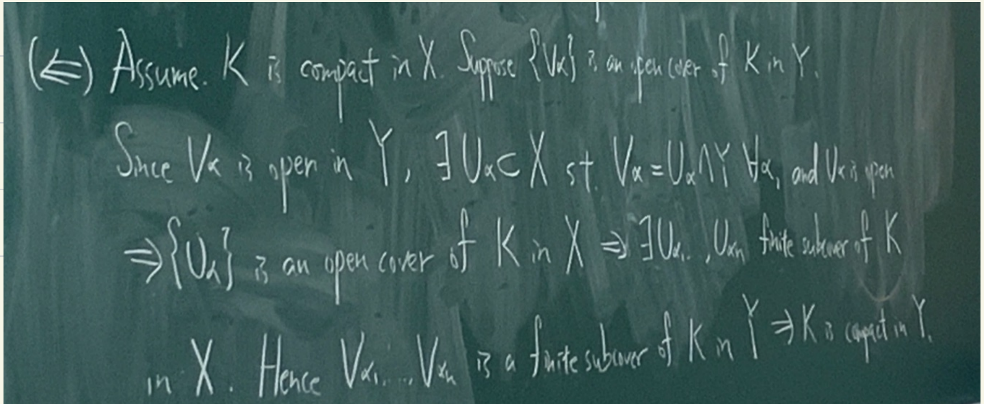
Suppose $\{U_\alpha\}$ is an open cover of $K$ in $X$.

Let $V_\alpha = Y \cap U_\alpha$, then $\{V_\alpha\}$ is an open cover of $K$ in $Y$.

Since $Y$ is compact, $\exists \{V_{\alpha_1}, \cdots, V_{\alpha_n}\}$, a finite subcover of $K$ in $Y$.

$\Rightarrow \{U_{\alpha_1}, U_{\alpha_2}, \cdots, U_{\alpha_n}\}$ is a finite subcover of $K$ in $X$ $\Rightarrow$ $K$ is compact in $X$.

($\Leftarrow$) Similar (See Notes).



4. (24 pts) Let $F$ be a closed set and $K$ be a compact set. Prove that $F \cap K$ is a compact set.

(pf) $K$ is compact $\Rightarrow$ $K$ is closed.

$\Rightarrow$ $F \cap K$ is closed, & in a compact set $K$.

$\Rightarrow$ $F \cap K$ is compact (by Proposition).

5. (20 pts) Let $K = \{0\} \cup \left\{ \frac{1}{n} \middle| n \in \mathbb{N} \right\}$. Prove that $K$ is compact without using Heine-Borel Theorem.

(pf) Let $\{U_\alpha\}$ be an open cover of $K$

$\exists\, G \in \{U_\alpha\}$ such that $0 \in G$, and

$\exists$ sufficient large $m$ such that $N_{\frac{1}{m}}(0) \subseteq G$.

Let $G_n$ be a set such that $G_n \in \{U_\alpha\}$, $\frac{1}{n} \in G_n$

Then $\{G, G_1, G_2, \cdots, G_{m-1}\}$ is a finite subcover of $K$.

$\Rightarrow K$ is compact. #