Online Math Camp (23S)

TA Session Notes 3/20

Sets

Example 1: $S_1 = \{\triangle, ☺, X\}$

$S_2 = \{X, O\}$

$\Rightarrow$ $S_1 \cap S_2 = \{X\}$

$S_1 \cup S_2 = \{\triangle, ☺, X, O\}$

Example 2: $S_1 = \{x \in \mathbb{R} \mid x > 0\} = \mathbb{R}_+$

$S_2 = \{x \mid x \text{ is an integer}\} = \mathbb{Z}$

$\Rightarrow$ $\mathbb{R}_+ \cap \mathbb{Z} = \{1, 2, 3, \cdots\} = \mathbb{N}$

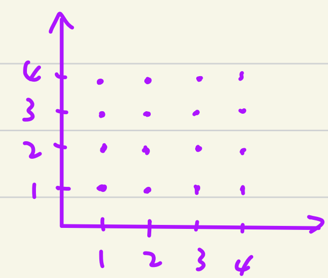
$\mathbb{Z} \setminus \mathbb{R}_+ = \{0, -1, -2, -3, \cdots\}$

Relation:

Def: R is a relation on S

$S \times S = \{(a, b) \mid a \in S, b \in S\}$

$\mathbb{N} \times \mathbb{N}$:



Example: S: the set of all human beings

aRb: a is the father of b

$R = \{(a, b) \mid a \text{ is the father of } b\}$

Order ($>$)

A kind of relation:

① $a>b,\ b>c \Rightarrow a>c$ (Transitivity)

② One & only one of the following holds: $a>b$, $a=b$, $\underset{\text{a<b}}{b>a}$ (三一律)

Example:

① "$>$" on $\mathbb{R}$: "$>$" $= \{(5,3), (\pi, 0), \cdots\}$

② We say "$x>y$" if $|x|<|y|$ or "$|x|=|y|$ and $x>0, y<0$" ("weird" order since "$>$" means smaller in absolute value)

No Rational Number Satisfies $x^2 = 2$

(pf) Let $x^2 = 2$ and $x \in \mathbb{Q}$. $\Rightarrow x = \frac{p}{q}$ where $(p, q) = 1$, & $p, q \in \mathbb{Z}$

No common diviser

$\Rightarrow x^2 = \frac{p^2}{q^2} = 2 \Rightarrow p^2 = 2q^2 \Rightarrow p$ is a multiple of 2 (i.e. even number)

Hence, set $p = 2p'$, $p' \in \mathbb{Z}$ $\Rightarrow p^2 = (2p')^2 = 2q^2 \Rightarrow 2(p')^2 = q^2$

i.e. q is a multiple of 2. Contradiction to $(p, q) = 1$! $(\rightarrow\times\leftarrow)$

(pf) Alternatively, you can use 牛頓一次因式檢驗法 from high school:

$\Rightarrow$ The only possible rational roots for $x^2 - 2 = 0$ are $\pm 1, \pm 2$. $(\rightarrow\times\leftarrow)$

Rudin 1.14 - 1.16

Example 1: Prove $-(-x) = x$ (i.e. Rudin 1.14(d))

i.e. Since $x + (-x) = 0 \ \forall x$ and apply to $(-x) \Rightarrow (-x) + (-(-x)) = 0$ #

(pf) $x + (-x) = 0$

$\underline{(-x) + (-(-x)) = 0} = -(-x) + (-x)$ (since $a+b = b+a$)

$\Rightarrow x + \underbrace{(-x) + [-(-x)]}_{=0} = -(-x) + \underbrace{(-x) + [-(-x)]}_{=0}$ (all $[-(-x)]$ to both sides)

$\Rightarrow x = -(-x)$ #

Rudin 1.14 - 1.16

Example 2: Prove $-x = (-1)\cdot x$ Strategy: Show that $(-x) + (-1)\cdot x = 0$

(pf) $x + (-1)\cdot x = 1\cdot x + (-1)\cdot x = [1 + (-1)]\cdot x = 0\cdot x = 0$

So, if $0\cdot x = 0$, we are done: $\underline{x} + (-1)\cdot x = 0 = \underline{x} + (-x)$

$\Rightarrow (-1)\cdot x = -x$.

But why $0\cdot x = 0$?

Because $x + 0 = x = 1\cdot x = (1+0)\cdot x$ (since $1 = 1+0$)

$= x + 0\cdot x$

$\Rightarrow \cancel{x} + 0 = \cancel{x} + 0\cdot x$

$\Rightarrow 0 = 0\cdot x$ #