

Introduction to Real Analysis, Quiz 9

- (30 pts) Define “ X is a *complete metric space*”.
- (10 pts each) What are the lim sup and lim inf for the following sequences?

(i) $a_n = \frac{(-1)^n}{1 + \frac{1}{n}}$

(ii) $a_n = \frac{1 - 2 + 3 - 4 + \cdots + (-1)^{n-1}n}{n}$

(iii) $a_n = \frac{n^2 + 4n - 3}{2n^2 + 3n + 5}$

- (10 pts each) Discuss if the following series converge or diverge.

(i) $\sum_{n=0}^{\infty} \frac{n}{2n+1}$

(ii) $\sum_{n=0}^{\infty} \frac{1}{2^{\frac{n}{2}}}$

(iii) $\sum_{n=0}^{\infty} \frac{1}{n!}$

- (24 pts) Say $|a_n| < 1$ for all $n \in \mathbb{N}$. Prove that the series $\sum a_n x^n$ converges for all x with $|x| < 1$.

- (24 pts) Calculate

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)(n+4)}$$