

Introduction to Real Analysis, Quiz 8

- (30 pts) Let $\{p_n\}$ be a sequence. What does it mean by $\lim_{n \rightarrow \infty} p_n = p$?
- (20 pts each) Give a rigorous $N-\varepsilon$ argument to calculate the limits of the following sequences.
 - $\sqrt{n+1} - \sqrt{n}$
 - $\frac{3n}{n+3}$
- (24 pts) Prove that a sequence can have at most one limit.
- (24 pts) Show that, if a sequence is Cauchy in \mathbb{R} , then it converges.
- (20 pts) Given $a_1 > b_1 > 0$. Define $a_n = \frac{a_{n-1} + b_{n-1}}{2}$, $b_n = \frac{2a_{n-1}b_{n-1}}{a_{n-1} + b_{n-1}}$ for $n \geq 2$. Prove that $a_n > a_{n+1} > b_{n+1} > b_n$. Deduce that two sequences $\{a_n\}$ and $\{b_n\}$ converge and share the same limit.