Introduction to Real Analysis, Quiz 7

1. (25 pts) Define "C is a *connected set* in the metric space X".

2. (1) (25 pts) State *Heine-Borel theorem*.
   (2) (20 pts) Is $([a, b], d)$ compact where d denotes the discrete metric? Why you cannot use Heine-Borel in this case?

3. (24 pts) Prove that if a set is compact, then every infinite subset has a limit point.

4. (24 pts) Show that the Cantor set is perfect, that is, closed and with no isolated point.

5. (20 pts) Prove that, if C is connected, then $\overline{C}$ is also connected. How about the inverse?