Introduction to Real Analysis, Quiz 6

1. (25 pts) Define "K is a *compact set* in the metric space X".

2. (15 pts each) Is the set S compact in X? Proofs are needed.

   (i) $X = \mathbb{R}^2$. S is some open ball $N_r(x)$ for $r > 0$.

   (ii) S is an empty set.

   (iii) $X = \mathbb{R}^5$. S is a non-empty finite set.

3. (24 pts) Given X being a metric space and $K \subset Y \subset X$. Prove that K is compact in Y if and only if K is compact in X.

4. (24 pts) Let F be a closed set and K be a compact set. Prove that $F \cap K$ is a compact set.

5. (20 pts) Let $K = \{0\} \cup \left\{ \frac{1}{n} \middle| n \in \mathbb{N} \right\}$. Prove that K is compact without using Heine-Borel Theorem.