Introduction to Real Analysis, Quiz 5

1. (15 pts each) Give formal definitions to the following statements.

   (i) U is an *open set* in a metric space X.

   (ii) F is an *closed set* in a metric space X.

2. (12 pts each) Is the set S open in X? Is it closed? Explanations are needed.

   (i) $X = \mathbb{R}^2$. S is some open ball $N_r(x)$ for $r > 0$.

   (ii) S is X itself.

   (iii) S is an empty set.

   (iv) $X = \mathbb{R}^5$. S is a non-empty finite set.

3. (30 pts) Prove that S is open in X if and only if S^c is closed in X.

4. (30 pts) Show that the union of any collection of open sets is open.

5. (**0 pts, don't do this unless you have time**) Prove that a bounded closed set of real numbers contains its supremum and infimum.