Introduction to Real Analysis, Quiz 3

1. (30 pts) State and prove *the Cauchy-Schwarz inequality.*

2. (30 pts) Let $z_1, z_2 \cdots, z_n$ be complex numbers, prove that

$$|z_1 + \cdots + z_n| \leq |z_1| + \cdots + |z_n|$$

   (Hint. Use Induction and prove the base case as detailed as you can.)

3. (27 pts) Prove the following statement, "Principle of Induction $\Rightarrow$ Well-Ordering Principle."

4. (27 pts) Let $z = a + ib, w = u + iv$ and $z^2 = w$. Calculate a, b in terms of u, v. (Reminder. There are two roots.)

5. (24 pts) Suppose z is a complex number with $|z| = 1$, calculate

$$|1 + z|^2 + |1 - z|^2,$$

   and interpret it geometrically. (Hint. What is the geometric interpretation of $|a - b|$?)