

Introduction to Real Analysis, Quiz 11

1. (32 pts) State the *Intermediate Value Theorem*.
2. (28 pts) Let $f : X \rightarrow Y$ be a continuous function between two metric spaces. Prove that $f^{-1}(F)$ is closed in X if F is closed in Y .
3. (28 pts) Let $f : X \rightarrow Y$, $g : Y \rightarrow Z$ be continuous functions between metric spaces. Show that $g \circ f$ is continuous.
4. (26 pts) Describe “continuous function preserves compactness” formally and prove it.
5. (24 pts) Let $f, g : X \rightarrow Y$ be two continuous functions. Suppose that $g(x) = f(x)$ for $x \in E$, where E is dense in X . Prove that $g(x) = f(x)$ for all $x \in X$.