Introduction to Real Analysis, Quiz 1

1. (30 pts, 15pts each) Give formal definitions to the following statements.

    (a) R is a *relation* between sets A and B.

    (b) $\succ$ is an *order* on the set S.

2. (32 pts, 8pts each) Let $A = \{1, 2, 3\}, B = \{3, 4\}$. What are $A \cap B, A \cup B, A \setminus B, A \times B$?

3. (28 pts) Define the addition of rational numbers and check that your definition is well-defined, that is, if $\dfrac{a}{b} = \dfrac{a'}{b'}, \dfrac{c}{d} = \dfrac{c'}{d'}$, then $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{a'}{b'} + \dfrac{c'}{d'}$.

4. (28 pts) Prove that there is no rational number whose square is 12.

5. (20 pts) Let F be an ordered field and $0 \in F$ be the additive identity. Prove that if $x \neq 0$, then $x^2 > 0$. (Consequently, the multiplicative identity is positive. Moreover, $\mathbb{C}$ is not an ordered field since $i^2 = -1$.)
   Note: You should carefully prove $0x = 0$ first if you need to use this fact.