2.8.1 Pure and Impure Altruism

$u_i(x) = x_i + \alpha \sum_{k \neq i} x_k, \alpha \geq 0 \quad \sim u_{Altruism}$

① PD= $T > H > L > S$

	C	**D**
C	**H,H**	**S,T**
D	**T,S**	**L,L**

Find the "utility payoff matrix" of PD if subjects all have utility $u_{Altruism}$.

(sol) $i = 1, k = 2,$

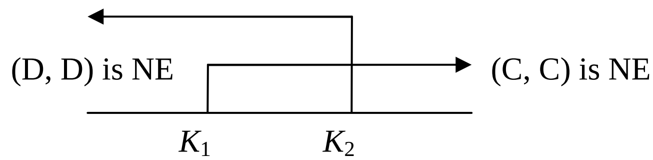
	C	D
C	$H(1+\alpha), H(1+\alpha)$	$S + \alpha T, T + \alpha S$
D	$T + \alpha S, S + \alpha T$	$L(1+\alpha), L(1+\alpha)$

② When will (D,D) still be the only Nash equilibrium? When will (C,C) be NE?

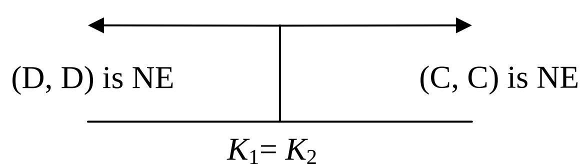
(sol) (D,D) is NE iff $(L-S) + \alpha(L-T) \geq 0$ iff $\alpha \leq \frac{L-S}{T-L} = K_1$

(C,C) is NE iff $(H-T) + \alpha(H-S) \geq 0$ iff $\alpha \geq \frac{T-H}{H-S} = K_2$

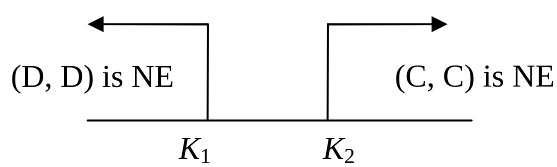
$\therefore$ If $K_1 > K_2$:



If $K_1 = K_2$:



$\therefore$ If $K_1 > K_2$:



$\Rightarrow$ (D,D) is unique Nash Eq. iff $\alpha < \min\{K_1, K_2\}$.

③ How much Altruism explain corporation on [(C, C)] in PD?

(sol) If $\alpha > K_2$, we may sustain (C, C) as NE.

If $K_1 \geq \alpha \geq K_2$, we may sustain (C, C) and (D, D) as NE.

④ **Can $u_{Altruism}$ explain rejection in ultimatum games?**

(sol) Given an offer $(10-a,a)$, $u_r(reject)=0+\alpha\cdot 0=0\geq u_r(accept)=a+\alpha(10-a)$

$\Leftrightarrow \alpha \leq \frac{-a}{10-a}$. This reuires $\alpha<0\left(\because \frac{-a}{10-a}\right)$, which contradicts the assumption that people are altruistic $(\alpha\geq 0)$ not spiteful.

[Homework] Can explain public goods contribution?

2.8.2 Guilt-Equality [Fehr and Schmidt (1999)]

$u_i(x)=x_i-\frac{\alpha_i}{n-1}\sum_{k\neq i}\max(x_k-x_i,0)-\frac{\beta_i}{n-1}\sum_{k\neq i}\max(x_i-x_k,0)\sim u_{FS}$ **,where**

$0\leq\beta_i\leq 1, \beta_i\leq\alpha_i$ **(不好意思)**

① **Find the PD's utility payoff matrix?**

(sol)

	C	D
C	H, H	$S-\alpha_i(S-T), T-\beta_i(T-S)$
D	$T-\beta_i(T-S), S-\alpha_i(S-T)$ S	L, L

② **When will (D, D) still be NE? When will (C, C) be NE?**

(sol) (D, D) is NE iff $(L-S)+\alpha_i(S-T)\geq 0$

iff $\alpha_i\leq\frac{L-S}{T-S}=K_3$ (not envy enough)

(D, D) is NE iff $(H-T)+\beta_i(T-S)\geq 0$

iff $\beta_i\geq\frac{T-H}{T-S}=K_4$ (fell bad enough about getting too much)

③ **How many Inequality Aversion explain corporation in PD?**

(sol) As long as people feel “不好意思” for getting too much $(\beta_i\geq K_4)$,(C, C) is sustained.

④ **Can u_{FS} explain “rejection in ultimatum games”?**

(sol) Given an offer $(10-a,a)$,where

$a\leq\frac{10}{2}=5$,

$u_r(reject) = 0 \geq u_r(accept) = a - \alpha_R(10-2a) = (1+2\alpha_R)a - 10\alpha_R$

$\Leftrightarrow \alpha_R \geq \dfrac{a}{10-2a}$ or $a \leq 10\left(\dfrac{\alpha_R}{1+2\alpha_R}\right)_{\#}$

$$\left(\text{If } 10 \geq a \geq 5\text{, need } 0 \leq a - \beta_R(2a-10) \Leftrightarrow \beta_R \geq \frac{a}{2a-10} = \frac{1}{2-\frac{10}{a}} \geq \frac{1}{2-1} = 1 \text{ impossible}\right)$$

⑤ **Can u_{FS} explain "fair offers proposed in ultimatum games"?**

(sol) Given α_R, belief is $u_P(10-a,a) = \begin{cases} 0 \text{ ,if } a < 10\left(\dfrac{\alpha_R}{1+2\alpha_R}\right) \leq 5 \\ (10-a) - \beta_P(10-2a)\text{,if } 10\left(\dfrac{\alpha_R}{1+2\alpha_R}\right) \leq a \leq 5 \\ (10-a) - \alpha_P(2a-10)\text{,if } a > 5 \end{cases}$

$\because \alpha_P \geq \beta_P$, won't offer $a > 5; (10-a) - \beta_P(10-2a)$, won't pick $a < \dfrac{10\alpha_R}{1+2\alpha_R}$

$\Rightarrow$ Pick α to $\max(2\beta_P - 1)a + 10(1-\beta_P)$

If $\beta_P > \frac{1}{2}$, Pick largest $a = 5$ (If you feel guilty for getting too much, propose 5-5)

If $\beta_P < \frac{1}{2}$, Pick smallest $a = \dfrac{10\alpha_R}{1+2\alpha_R}(1+\varepsilon)$ (Squeeze out as much as possible, but still accept.)

2.8.2 Fehr & Schmidt(1999)[Continued]

⑥ **Can u_{FS} also explain "contribution in public goods games"?**

(Sol) Player *i* contribution $g_i \in [0,y], G = \{g_1, \cdots, g_n\}$

Earnings $x_i(G) = y - g_i + m\sum_{k=1}^{n} g_k$, where $m < 1$

[A] i th player is free ride iff for given g_i

$$0 \ge (I)_i = u_i(G) - u_i(0, g_{-i})$$
$$= \left[\left(y - g_i + m\sum_{k=1}^{n} g_k \right) - \frac{\alpha_i}{n-1}\sum_{k \ne i}\max(g_i - g_k, 0) - \frac{\beta_i}{n-1}\sum_{k \ne i}\max(g_k - g_i, 0) \right]$$
$$- \left[\left(y + m\sum_{k \ne i} g_k \right) - 0 - \frac{\beta_i}{n-1}\sum_{k \ne i} g_k \right]$$
$$= (m-1)g_i - \frac{\alpha_i}{n-1}\sum_{k \ne i}\max(g_i - g_k, 0) - \frac{\beta_i}{n-1}\sum_{k \ne i}\max(g_k - g_i, 0) + \frac{\beta_i}{n-1}\sum_{k \ne i} g_k$$
$$(\text{Wolog, assume } g_1 \ge g_2 \ge \cdots \ge g_i \ge \cdots \ge g_n)$$
$$\Leftrightarrow (I)_i = (m-1)g_i - \frac{\alpha_i}{n-1}\sum_{k=i+1}^{n} g_i - g_k + \frac{\beta_i}{n-1}\sum_{k=1}^{i-1} g_i + \frac{\beta_i}{n-1}\sum_{k=i+1}^{n} g_k \le 0$$

If $(m-1) + \beta_i \le 0$, then $(I) \le (m-1)g_i - 0 + \beta_i g_i = [(m-1) + \beta_i]g_i \le 0$

$\therefore \beta_i \le 1 - m$ supports free-riding always.

[B] Show that if k th players free ride, $k > m \cdot \frac{n-1}{2}$, then everyone free rides.

(Sol) Assume $g_1 \ge \cdots \ge g_{n-k} \ge g_{n-k+1} = \cdots = g_n = 0$, for some $k > m\left(\frac{n-1}{2}\right)$. Then consider $i = n-k$,

$$(I)_{n-k} = (m-1)g_{n-k} - \frac{\alpha_{n-k}}{n-1}\sum_{j=n-k+1}^{n}(g_{n-k} - 0) - \frac{\beta_{n-k}}{n-1}\sum_{j=1}^{n-k-1}(g_j - g_{n-k}) + - \frac{\beta_{n-k}}{n-1}\sum_{j=1}^{n-k-1} g_j$$
$$= \left[(m-1) - \frac{k}{n-1}\cdot\alpha_{n-k} + \frac{n-k-1}{n-1}\cdot\beta_{n-k} \right]\cdot g_{n-k}$$
$$= \left[m - (1-\beta_{n-k}) - \frac{k}{n-1}\cdot(\alpha_{n-k} + \beta_{n-k}) \right]\cdot g_{n-k}$$
$$\overset{-\alpha_{n-k} \le -\beta_{n-k}}{\le} \left[-(1-\beta_{n-k}) + m - \frac{2k}{n-1}\cdot\beta_{n-k} \right]\cdot g_{n-k}$$
$$\overset{m < \frac{2k}{n-1}}{\le} \left[-(1-\beta_{n-k}) + m - m\beta_{n-k} \right]\cdot g_{n-k}$$
$$= -(1-m)(1-\beta_{n-k})\cdot g_{n-k} \le 0$$

$\Rightarrow g_{n-k} = 0$. By mathematic induction, $g_1 = \cdots = g_n = 0$.

[C] Show "k people have $\beta_i < 1 - m$ (free ride), others have $\beta_i > 1 - m$ & $\frac{m + \beta_i - 1}{\alpha_i + \beta_i} > \frac{i}{n-1}$ will condition $g_i \in [0, y]$" is an equilibrium.

(Sol) Assume $0 = g_1 = \cdots = g_k \le g_{k+1} = \cdots = g_n$, Consider $i \ge k+1$, ($g_1 - g_k$ trivial by [A]),

$$(I)_i = (m-1)g_i - \frac{\alpha_i}{n-1}\sum_{j=1}^{i-1}(g_i - g_j) + \frac{\beta_i}{n-1}\sum_{j=i+1}^{n} g_i + \frac{\beta_i}{n-1}\sum_{j=1}^{i-1} g_j$$

$$= \left[(m-1) + \beta_j - \frac{i-1}{n-1}(\alpha_i + \beta_i)\right] g_i + \frac{\alpha_i + \beta_i}{n-1}\sum_{j=i+1}^{n} g_i > 0$$

since $\frac{(m+\beta_i - 1)}{\alpha_i + \beta_i} > \frac{i}{n-1} > \frac{i-1}{n-1}$ #

2.8.2 Other (Homework)

1. Show that G-E (Fehr-Schmidt 99') predicts, under proposer competition:

(1) Proposers offer almost everything to Responder (independent of # of proposers)

Under Responder competition, we have:

(2) Responders accept any offer; Proposer offers 0 iff $\beta_P < \frac{n-1}{n}$

highest equation offer is $\min_{k\in\{1,\cdots,n\}} \frac{\alpha_k}{\beta_k + 2\alpha_k + (n-1)(1-\beta_k)} \to 0$ as $n \to \infty$

2. ERC: $u_i(x) = u\left(x_i, \frac{x_i}{\sum_{k=1}^{n} x_k}\right)$. Show that

(1) Offer between 0 and 50% in dictator game

(2) How can G-E (F-S) get this concavity?

(3) Ultimatum: Reject 0 always. Never reject 50-50.

Rejection rate $\downarrow$ as %$\uparrow$ and as pie size $\downarrow$ (fixing %)

Offer $<$50%, $>$ Dictator results.

(4) 3 players ultimatum-dictator combo: Allocation to the inactive Recipient is ignored

(5) What's ERC's prediction for PD and PG?

3. Consider PG with punishment: After contribution, announces G Player i can punish k one unit with a punishment P_{ik} at cost c<1.

(1) Standard Game Theory predicts P_{ik}=0.

(2) If are sufficiently guilty $(\beta_i \le 1-m)$ and sufficiently

envions: $\alpha_i > c(n-1)(1+\alpha_i) - c(n^* - 1)(\alpha_i + \beta_i) \Rightarrow \exists$ eq. with $g_k > 0$

2.8.3 Rabin (1993): Fairness Equilibrium

Two Players: 1 & 2

Strategy: a_i

i 'th belief about other's strategy : $b_j (= b_{3-j})$

i 'th belief about other's belief : c_i

Kindness (of 1 toward 2) : $f_1(a_1, b_2) = \frac{\pi_2(b_2, a_1) - \pi_2^{fair}(b_2)}{\pi_2^{\max}(b_2) - \pi_2^{\min}(b_2)}$, $\pi_2(b_2)$=possible payoffs of 2 given b_2 (belief about 2's action) equitable(ex: $\frac{\pi^{\max}+\pi^{\min}}{2}$ excluding Pareto dominated ……)

$f_1 > 0$:kind; $f_1 < 0$:mean

Perceived kindness (of what 1 think about 2's kindness to her):

$$\tilde{f}_2(b_2,c_1)=\frac{\pi_1(c_1,b_2)-\pi_1^{fair}(c_1)}{\pi_1^{\max}(c_1)-\pi_1^{\min}(c_1)}$$

Social Utility function(for 1)

$$u_1(a_1,b_2,c_1)=\pi_1(a_1,b_2)+\alpha\cdot\tilde{f}_2(b_2,c_1)+\alpha\cdot\tilde{f}_2(b_2,c_1)\cdot f_1(a_1,b_2)$$

Eq. requires $a_i=b_i=c_i\left(a_1=b_1=c_1,a_2=b_2=c_2\right)$

1 PD

	C	D
C	4, 4	$-\varepsilon$, 6
D	6, $-\varepsilon$	0, 0

⇨

	C	D
C	4+¾α, 4+¾α	-½α, 6
D	6, -½α	0, 0

if $\varepsilon=0$

$$\pi_i^{\max}(C)=4,\pi_i^{\min}(C)=-\varepsilon,\pi_i^{\max}(D)=0,$$

$$\pi_i^{\min}(D)=0,\pi_i^{fair}(C)=\frac{4+\varepsilon}{2},\pi_i^{fair}(D)=6\left(0\text{ is excluded}\right)$$

$$f_1(C,C)=\frac{4-\frac{4+\varepsilon}{2}}{4+\varepsilon}=\frac{2-\frac{\varepsilon}{2}}{4+\varepsilon},\ \tilde{f}_2(C,C)=\frac{4}{4+\varepsilon}-\frac{1}{2},$$

$$u_1(C,C,C)=4+\alpha\left(\frac{4}{4+\varepsilon}-\frac{1}{2}\right)\left(\frac{4}{4+\varepsilon}+\frac{1}{2}\right)=4+\alpha\left[\frac{4^2}{(4+\varepsilon)^2}-\frac{1}{4}\right]$$

$$f_1(D,D)=\frac{0-6}{6-0}=-1,\ \tilde{f}_2(D,D)=-1,u_1(D,D,D)=0-\alpha+\alpha=0=u_2(D,D,D)$$

$$f_1(C,D)=\frac{6-6}{6-0}=0,\ \tilde{f}_2(D,C)=\frac{-\varepsilon-\frac{4+\varepsilon}{2}}{4+\varepsilon}=\frac{-\varepsilon}{4+\varepsilon}-\frac{1}{2},$$

$$u_1(C,D,C)=-\varepsilon+\alpha\left(\frac{-\varepsilon}{4+\varepsilon}-\frac{1}{2}\right)=\frac{-(1+\alpha)\varepsilon}{4+\varepsilon}-\frac{1}{2}\alpha=u_2(C,D,C)$$

$$f_1(D,C)=\frac{-\varepsilon-\frac{4+\varepsilon}{2}}{4+\varepsilon}=\frac{-\varepsilon}{4+\varepsilon}-\frac{1}{2},\ \tilde{f}_2(C,D)=\frac{6-6}{6-0}=0,u_1(D,C,D)=6=u_2(D,C,D)$$

⊛ (C,C) is a Fairness Eq. if $\left(4+\frac{3}{4}\alpha\right)-6=\frac{3}{4}\alpha-2\ge 0 \Leftrightarrow \alpha \ge \frac{8}{3}$

(For $\varepsilon \ne 0, (C,C)$ is F.E. if $\left\{4+\alpha\left[\frac{4^2}{(4+\varepsilon)^2}-\frac{1}{4}\right]\right\}-6\ge 0 \Leftrightarrow \alpha \ge \frac{2}{\frac{4^2}{(4+\varepsilon)^2}-\frac{1}{4}}$)

2.8.3 Rabin (1993):F. E. (Continued)

2 Chicken

	C	D
C	-2, -2	2, 0
D	0, 2	1, 1

⇨

	C	D
C	-2, -2	2, -½α
D	-½α, 2	1+¾α, 1+¾α

$$\pi_i^{\max}(C)=1, \pi_i^{\min}(C)=0, \pi_i^{\max}(D)=2, \pi_i^{\min}(D)=-2, \pi_i^{fair}(C)=\frac{1}{2}, \pi_i^{fair}(D)=2$$

$$f_1(C,C)=\frac{1-\frac{1}{2}}{1-0}=\frac{1}{2}, \tilde{f}_2(C,C)=\frac{1}{2},$$

$$u_1(C,C,C)=1+\alpha\left(\frac{1}{2}\right)+\alpha\left(\frac{1}{2}\right)^2=1+\frac{3}{4}\alpha=u_2(C,C,C)$$

$$f_1(D,D)=\frac{-2-2}{2-(-2)}=-1, \tilde{f}_2(D,D)=-1,$$

$$u_1(D,D,D)=-1+\alpha(-1)+\alpha(-1)^2=-2=u_2(D,D,D)$$

$$f_1(C,D)=\frac{2-2}{2-(-2)}=0=\tilde{f}_2(C,D), \tilde{f}_2(D,C)=\frac{0-\frac{1}{2}}{1-0}=-\frac{1}{2}=f_1(D,C),$$

$$u_1(C,D,C)=0-\frac{1}{2}\alpha=u_2(C,D,C)$$

$$\Rightarrow u_1(D,C,D)=2=u_2(D,C,D)$$

1. (D, D) is F. E. if $(-2)-\left(\frac{-1}{2}\alpha\right)=-2+\frac{\alpha}{2}\ge 0 \Leftrightarrow \alpha \ge 4$
2. (C, C) is F. E. if $\left(1+\frac{3}{4}\alpha\right)-2=\frac{3}{4}\alpha-1\ge 0 \Leftrightarrow \alpha \ge \frac{4}{3}$
3. (C, D) is F. E. if $\alpha \le \frac{4}{3}$ (So is (D, C) …)

2.8.4 Extensive Form Fairness Equilibrium: Falk & Fischbncher(1998) (TBA)

i 's strategy s_i

belief about j 's choice: s_i'

belief about j 's belief about i 's choice: s_i''