Testing Game Theory in the Field: Swedish LUPI Lottery Games

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Population Uncertainty

- Game theory often assumes fixed-N players
- Not realistic in entry situations:
  - Voter turn-outs,
  - (Travel) congestion games,
  - Online auctions, etc.
- Games with population uncertainty (Myerson, IJGT 1998, GEB 2000, etc.)

Poisson Games

- Poisson Games: Assume N ~ Poisson(n)
  - Environmental Equivalence (EE)
  - Independence of Actions (IA)
- Applied to voting games by Myerson (1998)
- Contests: Myerson and Warneryd (2006)
- Other applications?

Research Questions

1. Where is a Poisson game relevant?
2. How good does Poisson equilibrium fit the data (if there is such application)?
3. How did we get to equilibrium? Or, if it doesn’t, why don’t we get to equilibrium?

Join the Swedish LUPI Game

- 49 games played daily: Jan. 29 – Mar. 18, 07'
- Each choose an integer from 1 to K=99999
- The person that chooses the lowest number that no one else does wins
  - LUPI: Lowest Unique Positive Integer
- Fixed Prize: Earn 10,000 Euros if win, 0 if not.
- Play against approximately 53,783 players
  - Assume "approximately 54k" is Poisson(53783)

Why Care?

- LUPI *is* a part of the economy
- The Swedish Limbo game
- Lowest unique bid auctions (ongoing research by Eichberger & Vinogradov, Raviv & Virag and Rapoport et al)
- Unique opportunity to test the theory
- Close field-laboratory parallel
- Full vs bounded rationality

Solving the LUPI Game

- To win by picking k = I uniquely picked number k and nobody uniquely picked numbers 1~(k-1)
- The mixed equilibrium is characterized by

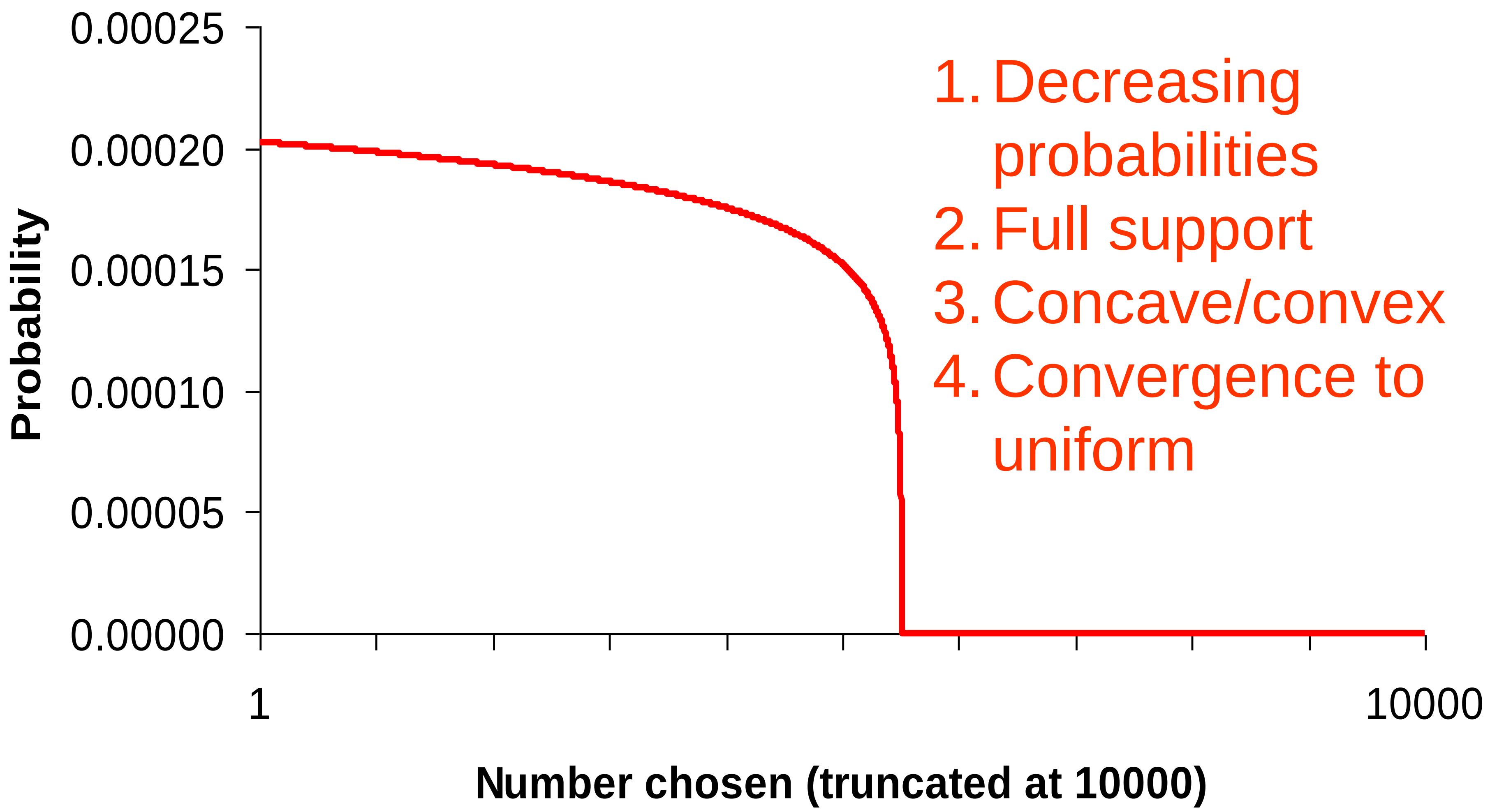
$$e^{-np_1} = \left(1 - np_1 e^{-np_1}\right) \cdot e^{-np_2}$$

Nobody chose 1

Nobody uniquely chose 1

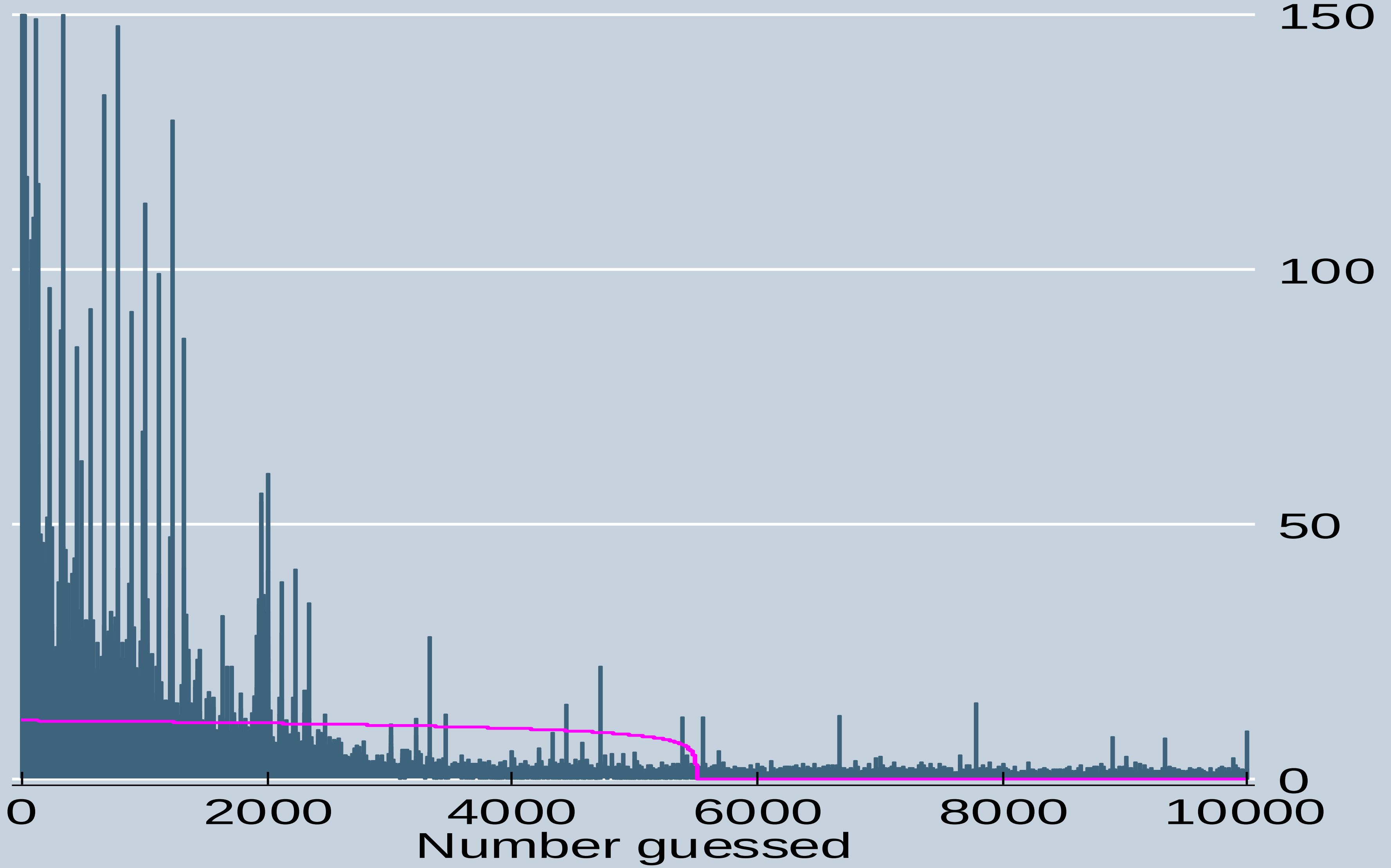
Nobody chose 2

The Unique Poisson Equilibrium

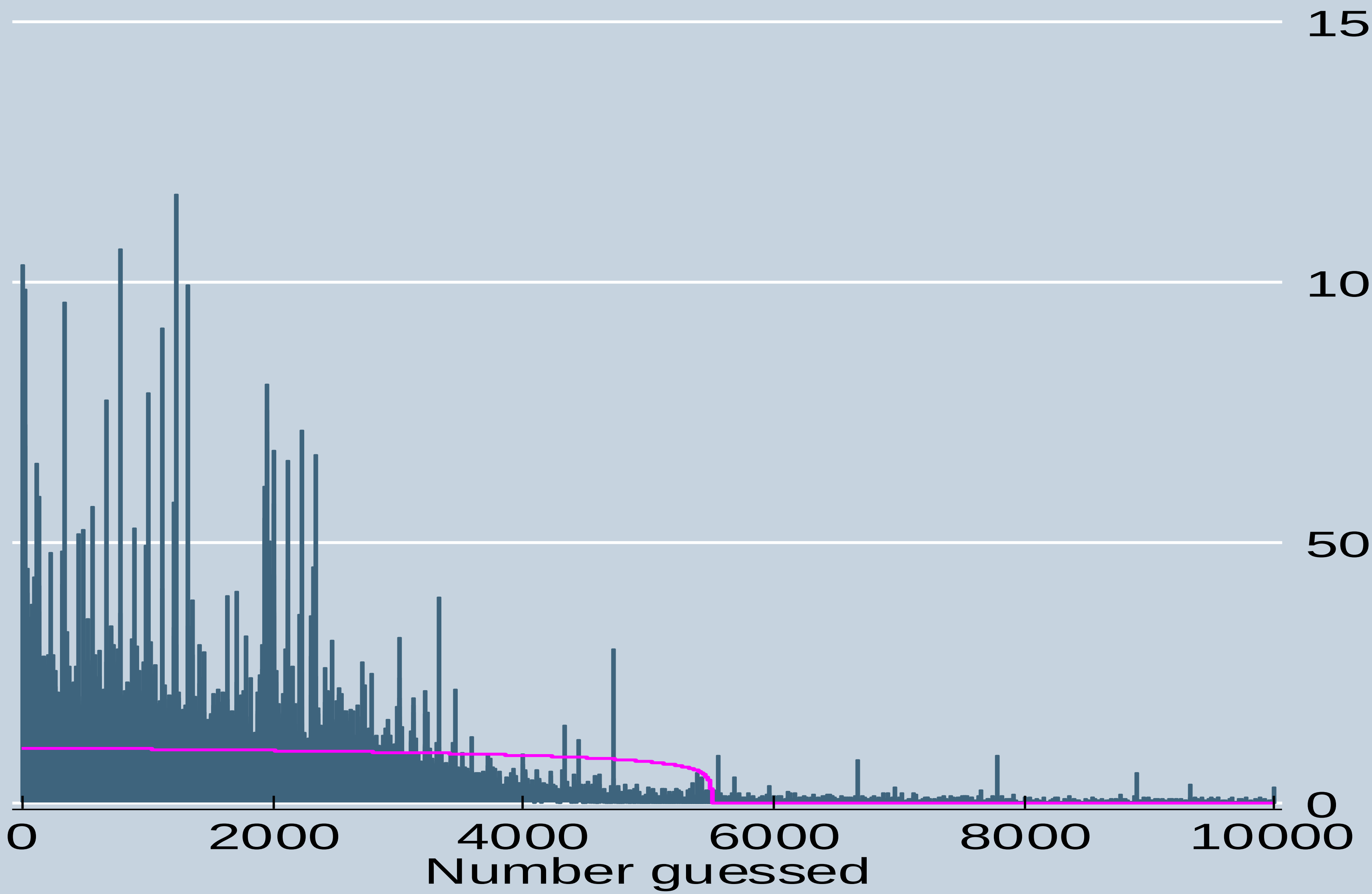


1. Decreasing probabilities
2. Full support
3. Concave/convex
4. Convergence to uniform

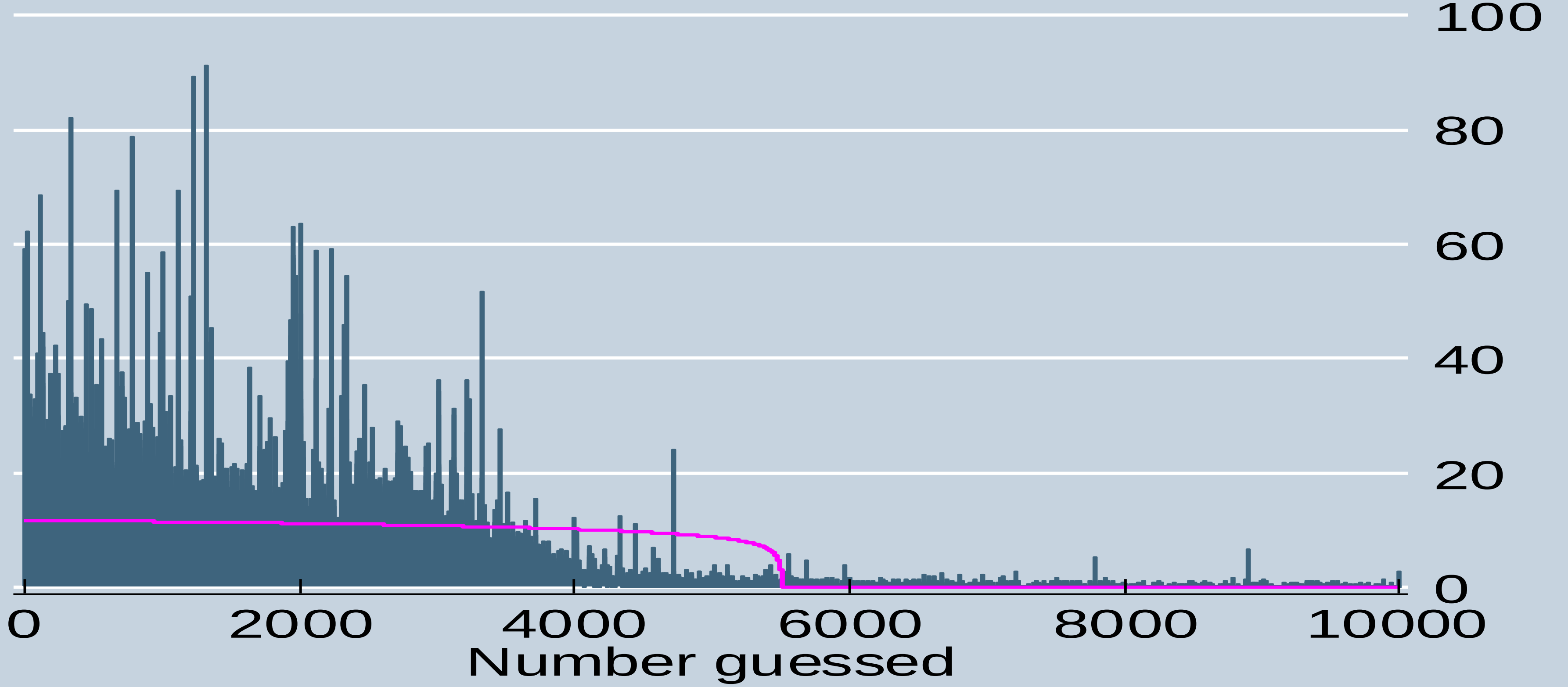
Average Daily Frequencies (Wk 1)



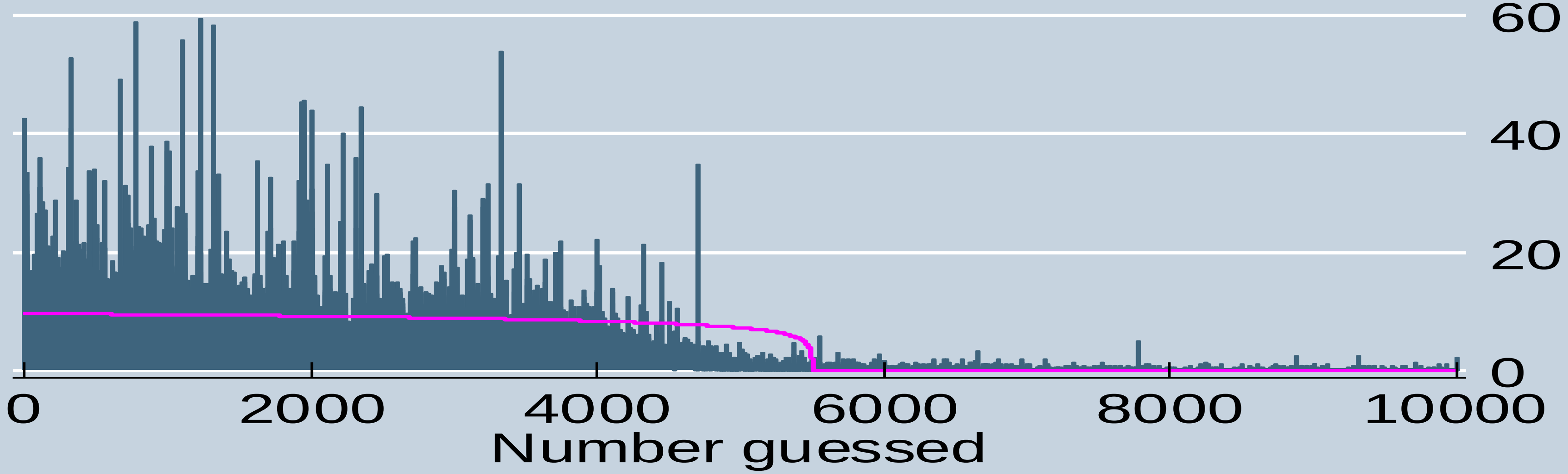
Average Daily Frequencies (Wk 3)



Average Daily Frequencies (Wk 5)



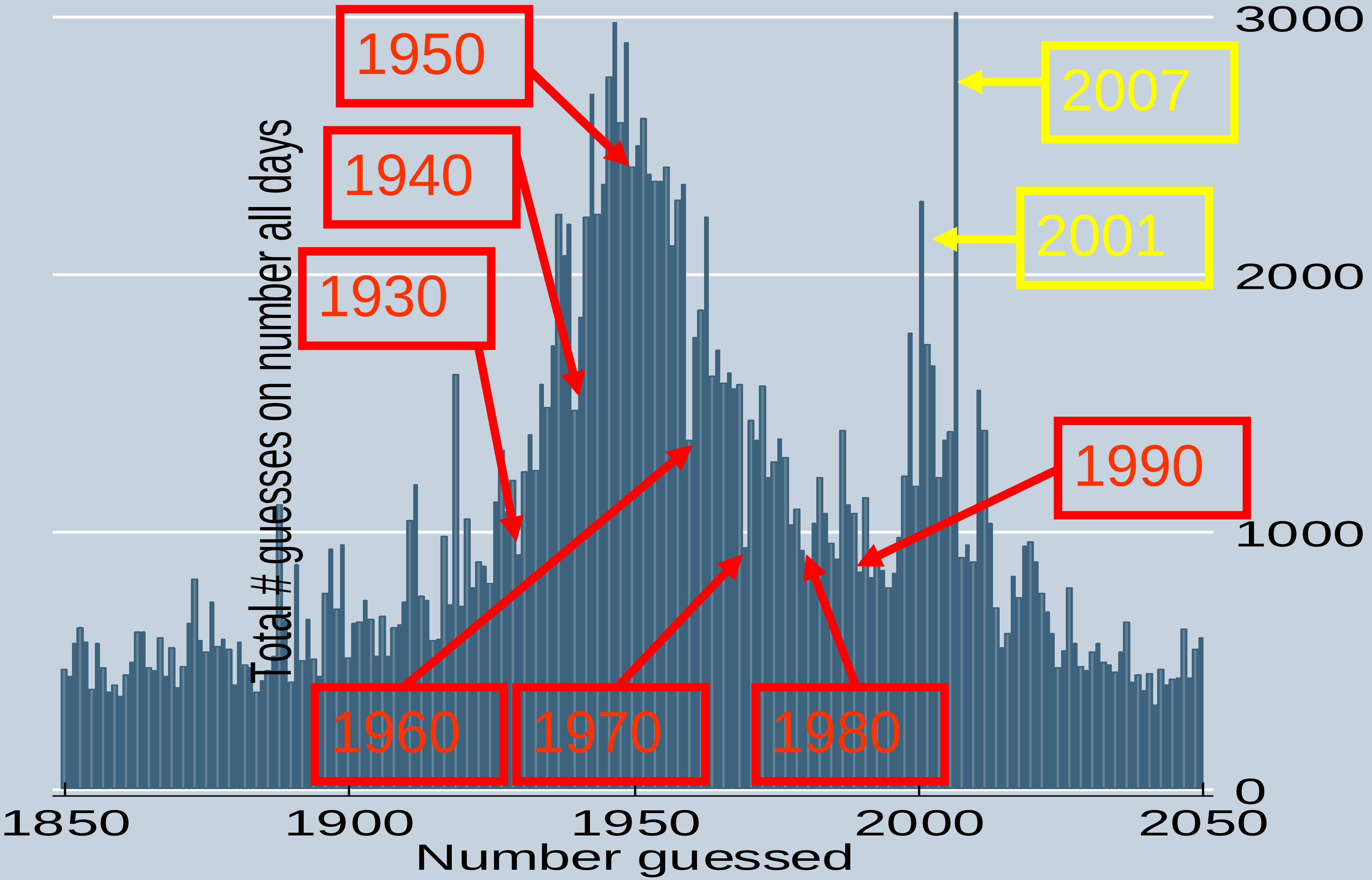
Average Daily Frequencies (Wk 7)



Details about the Swedish Game

- Players can bet (1 Euro each) up to 6 numbers from (1, 2, 3,…, 99999)
- The (first) prize fluctuates slightly (guaranteed >10,000 Euro until 3/18/07)
- Share prize if there is a tie
- Smaller second and third prizes offered
- Do people really think it's Poisson?

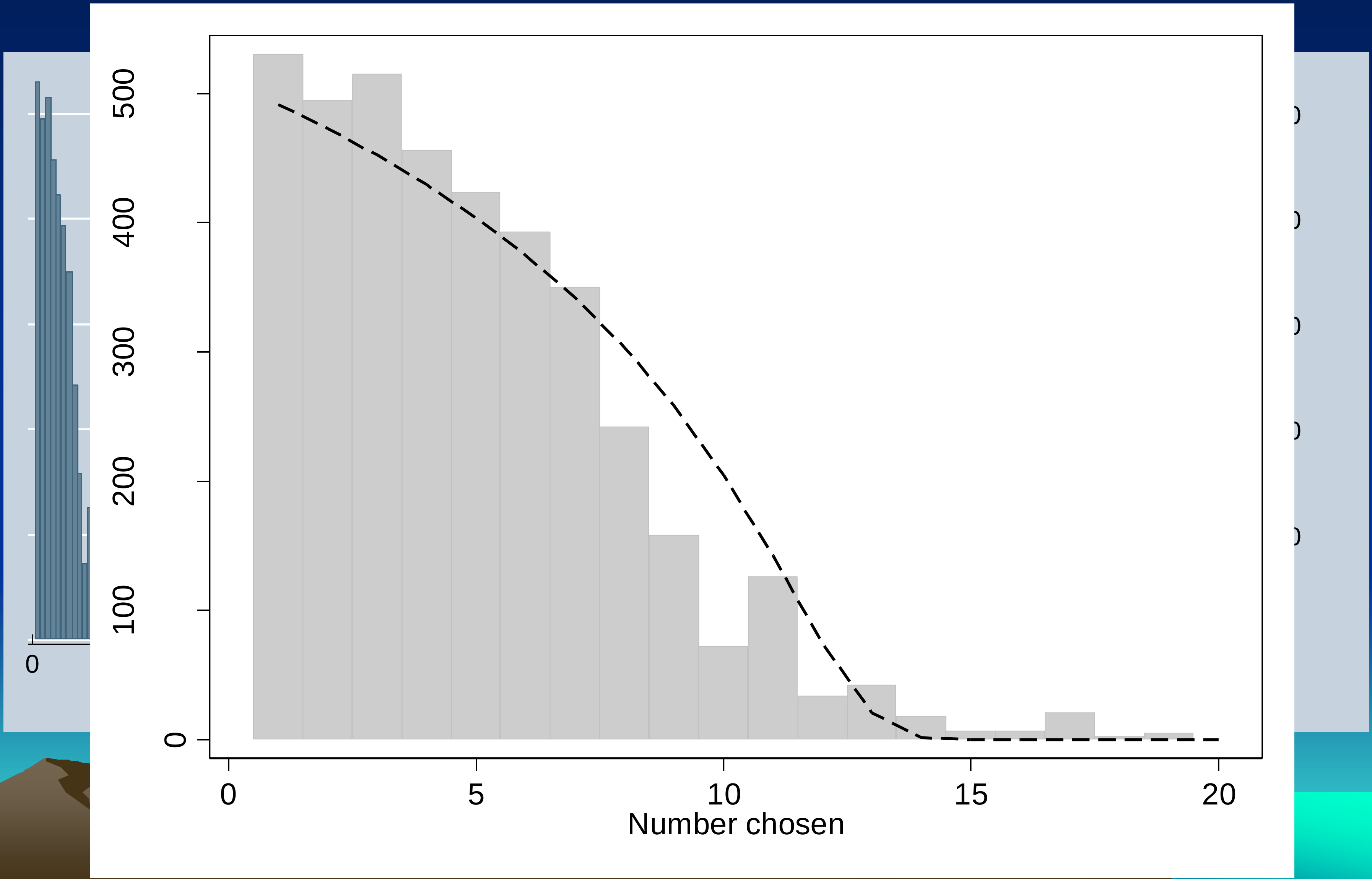
Birth/Current Year Effects



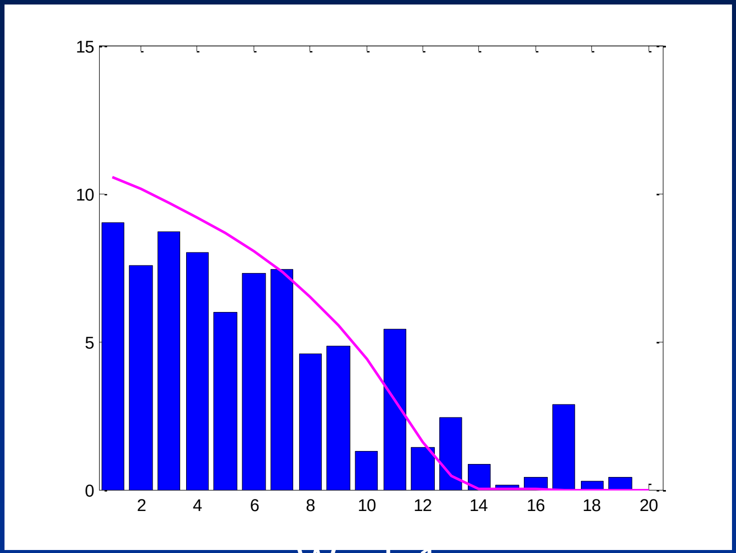
Lab Experimental Design

- CASSEL at UCLA
- Choose between *1* and *K=99*
- 49 rounds, w/ winning number announced
- Scale down prize and population by 2,000:
  - Winning prize = USD $7.00
  - *n=26.9 (=53,783 / 2,000)*
  - Variance is smaller than Poisson (due to a technical error; could have made it Poisson)

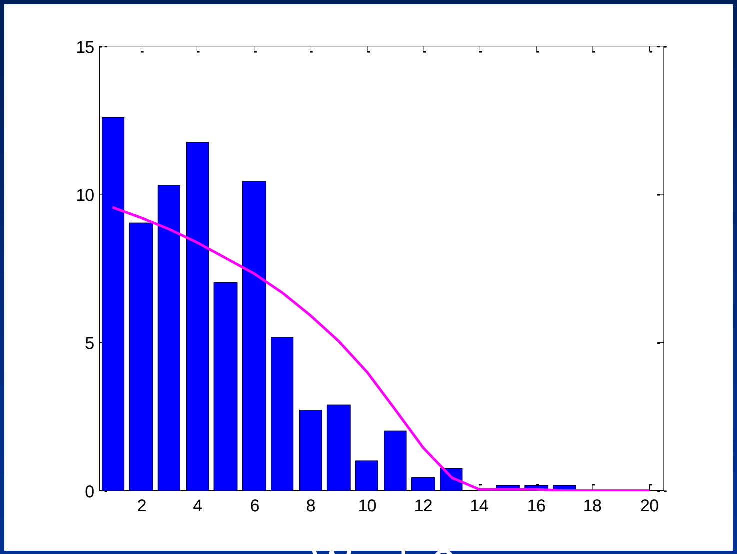
Aggregate Data in the Lab



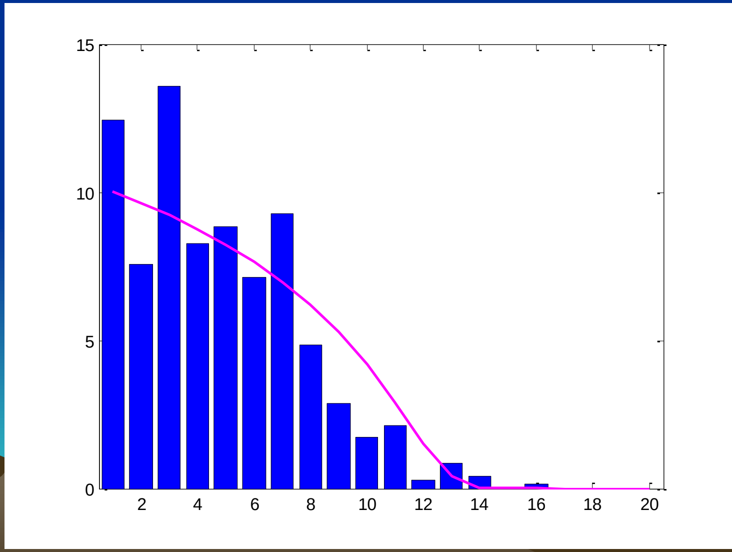
Week-by-Week data in the Lab



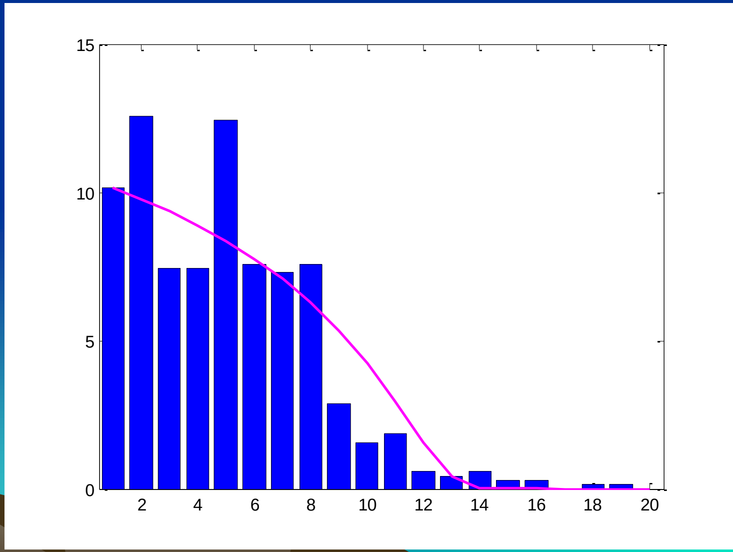
Week 1



Week 3

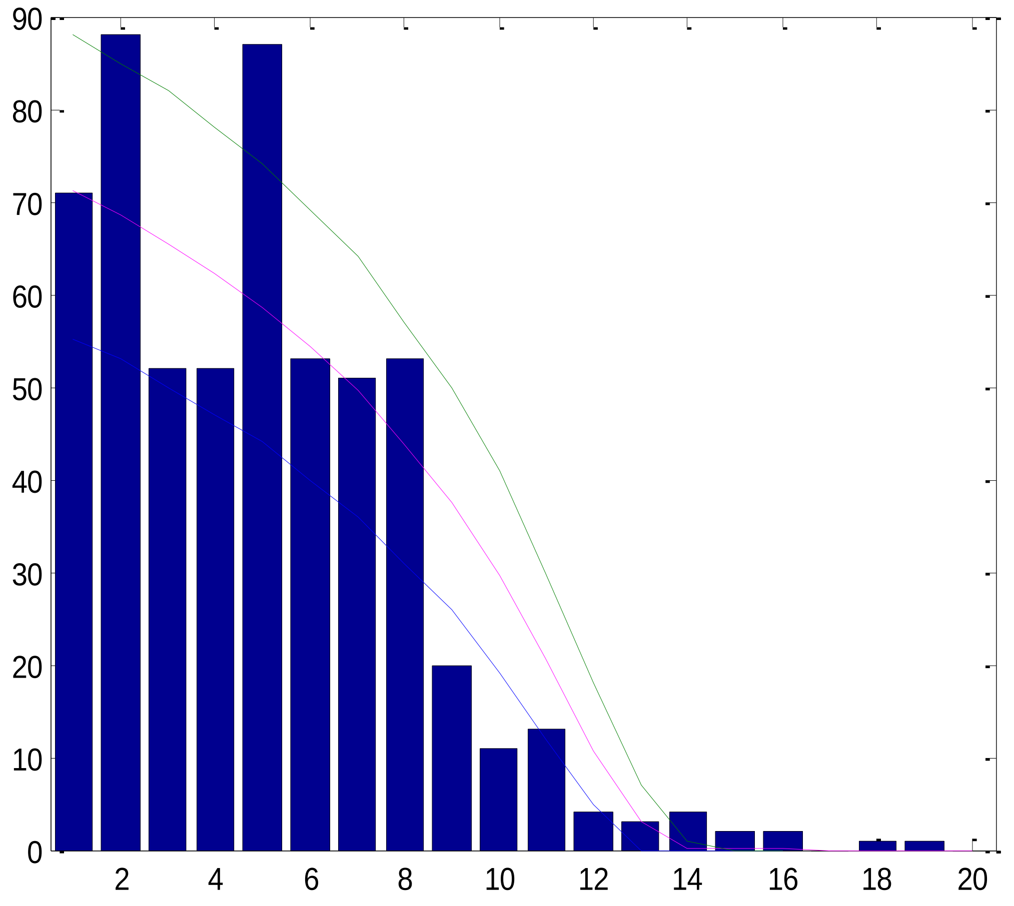


Week 5



Week 7

Week-by-Week data in the Lab



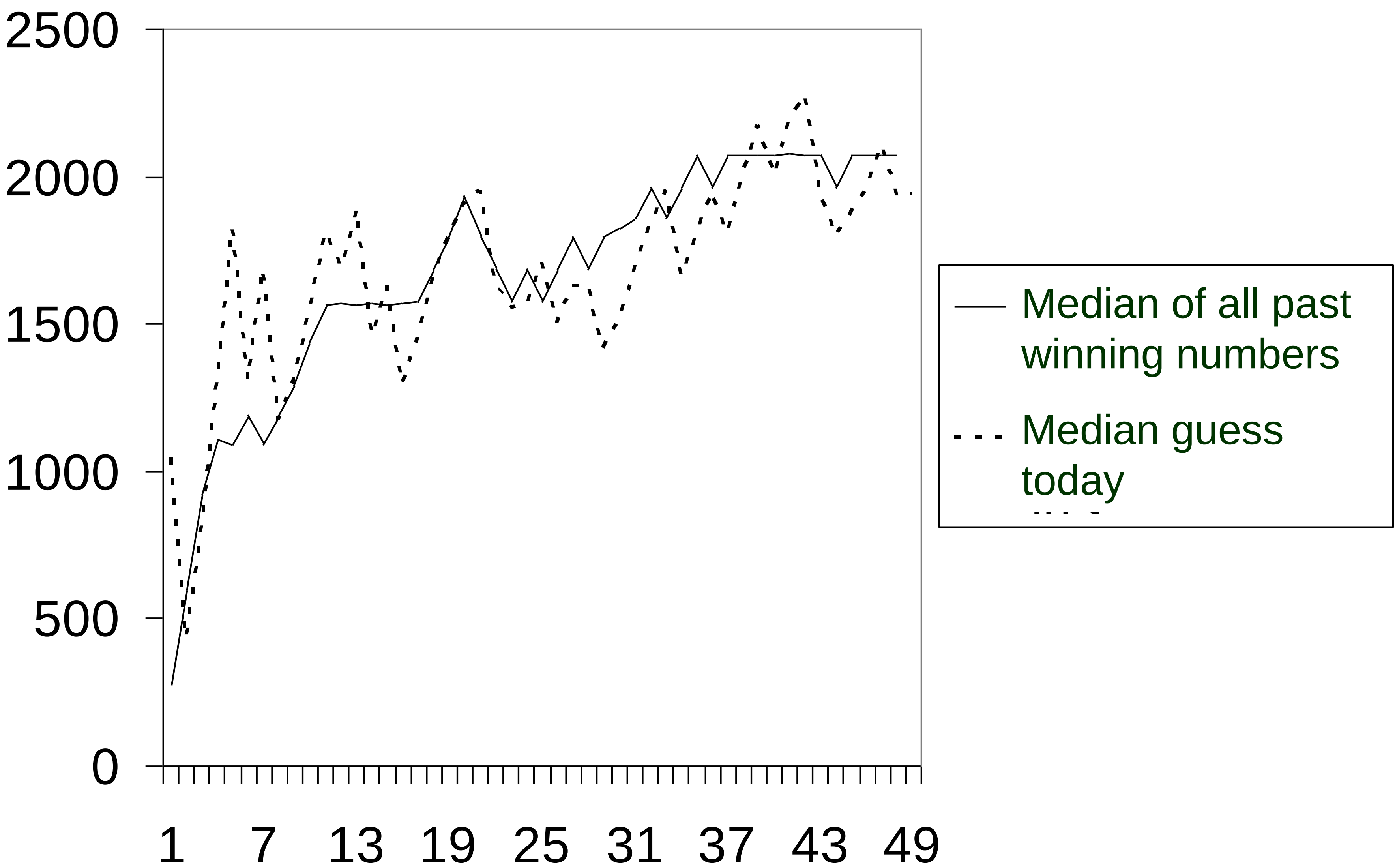
- Not quite in equilibrium
- 95 percent confidence intervals for last week in the lab

Learning in the Field

- Winning numbers are the only feedback
- Nobody except the winner is reinforced
- Can update beliefs about other's strategy since they don't see the frequencies

- But, people do respond to winning numbers!

Learning in the Field

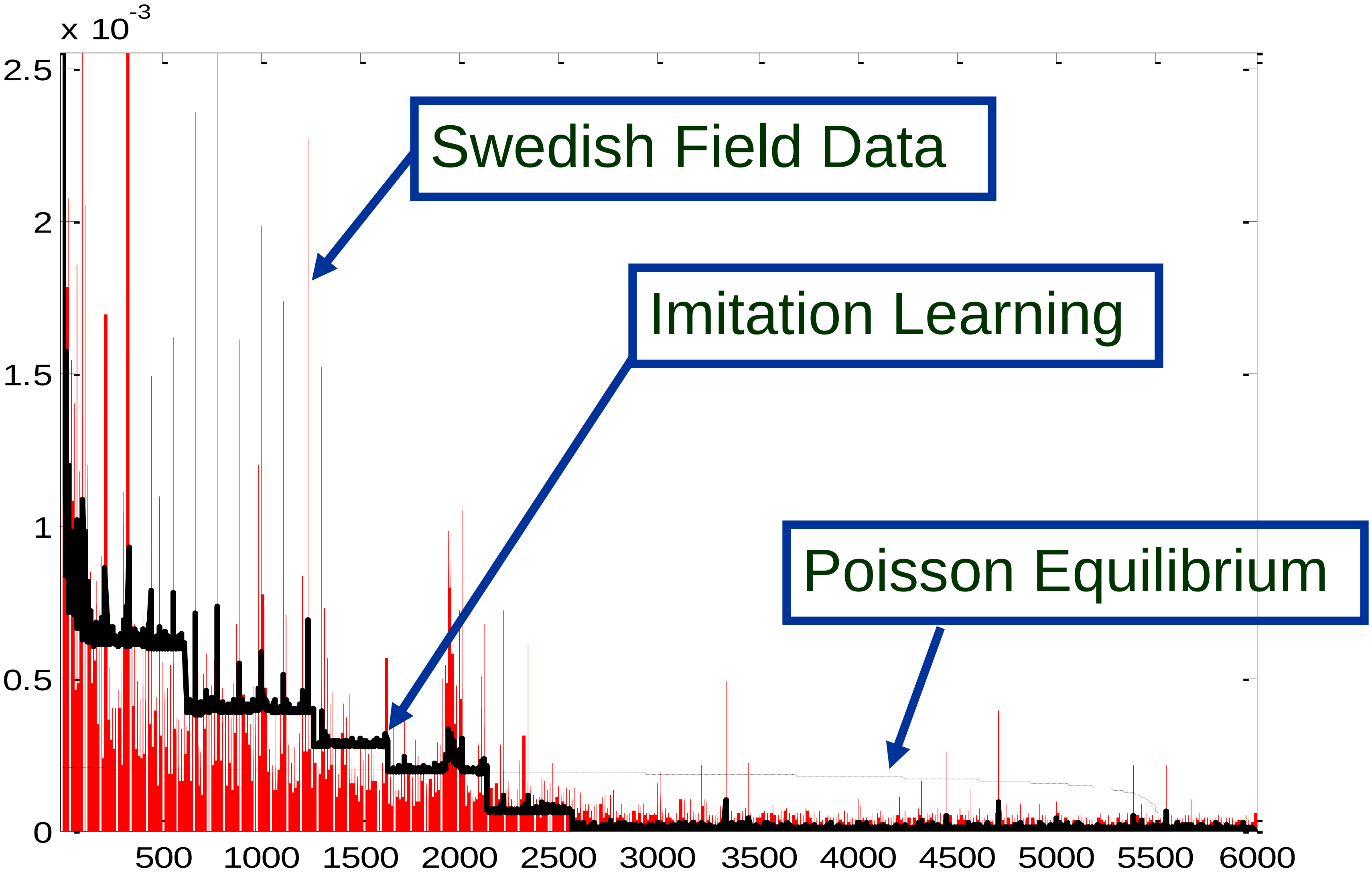


Imitation Learning

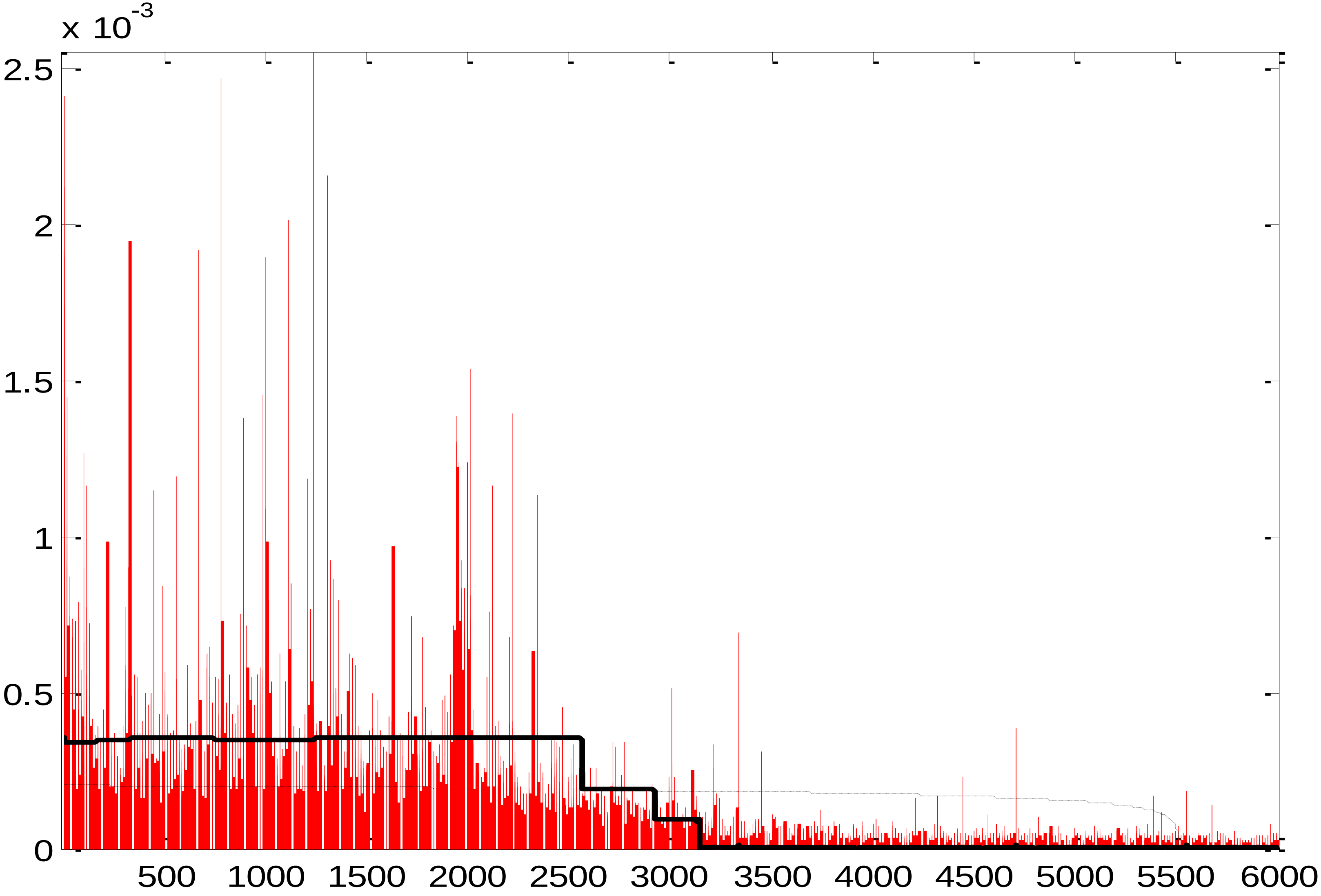
- Start with initial attractions A(1)
  - Backed out by empirically using initial data
- Update attractions for a window (size W) close to the previous winning number
- Why would this work at all?

  Lab Video
  - The winning number indicates undershooting!
- MLE estimates W=344 for field data

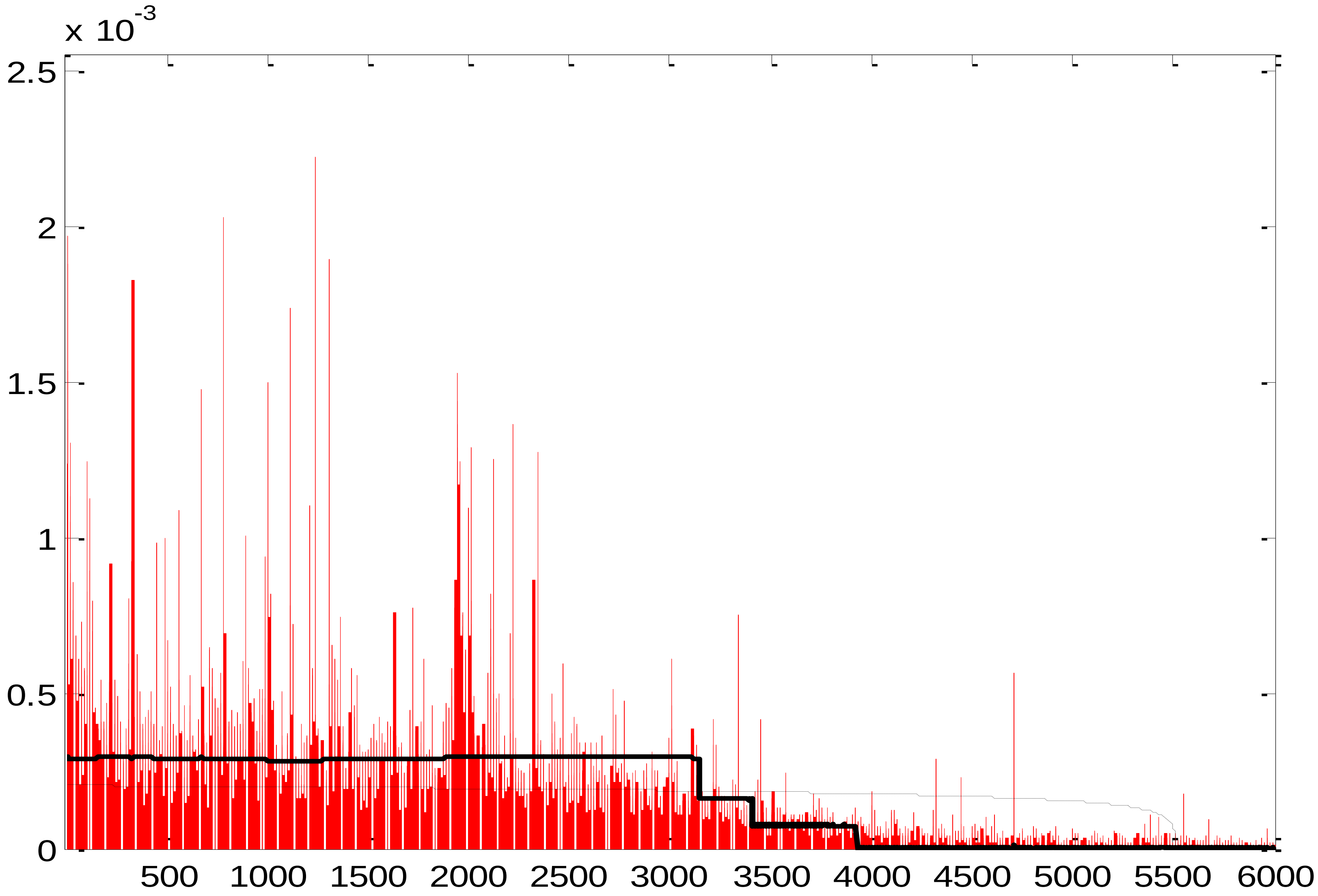
Learning in the Field (Week 1)



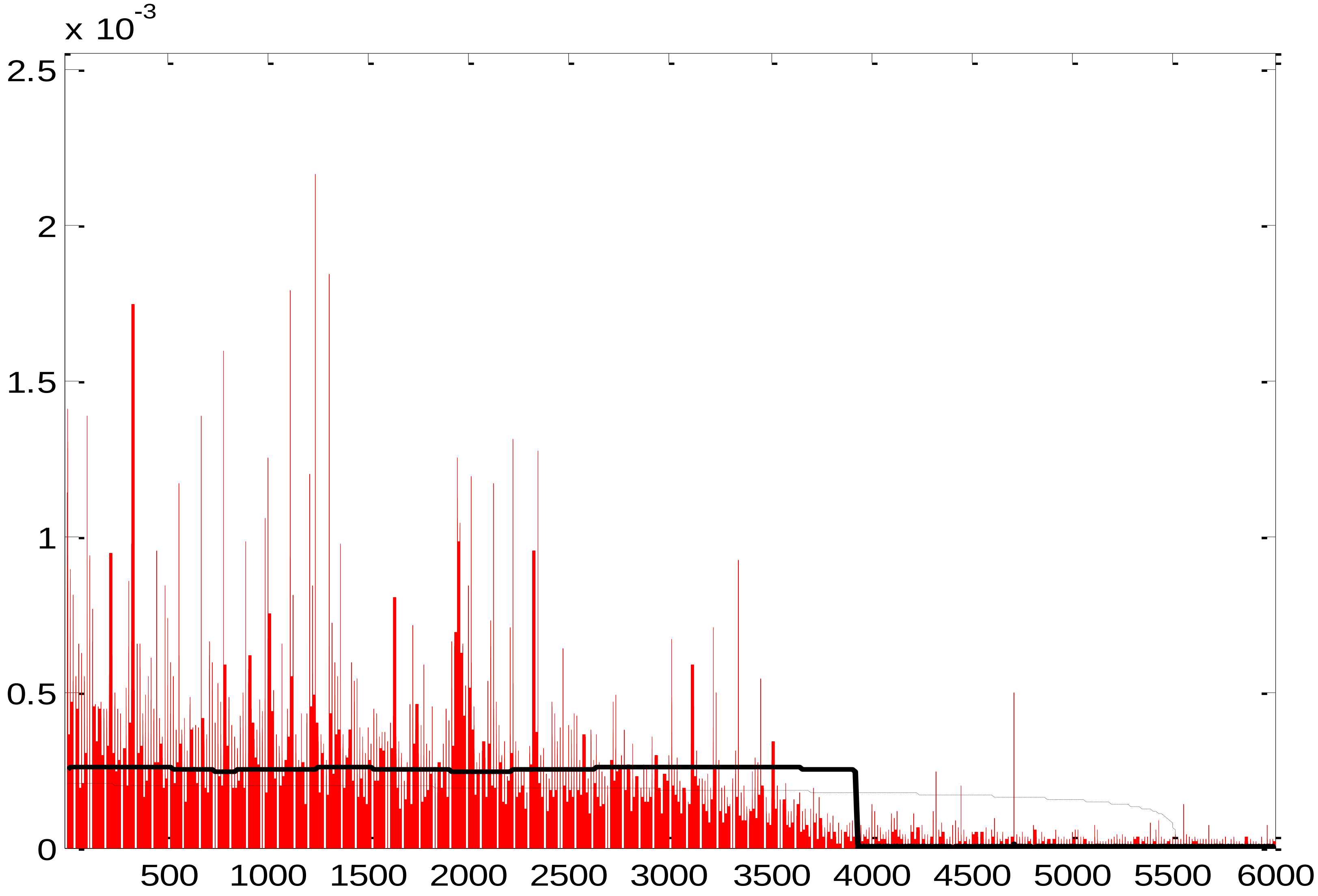
Learning in the Field (Week 2)



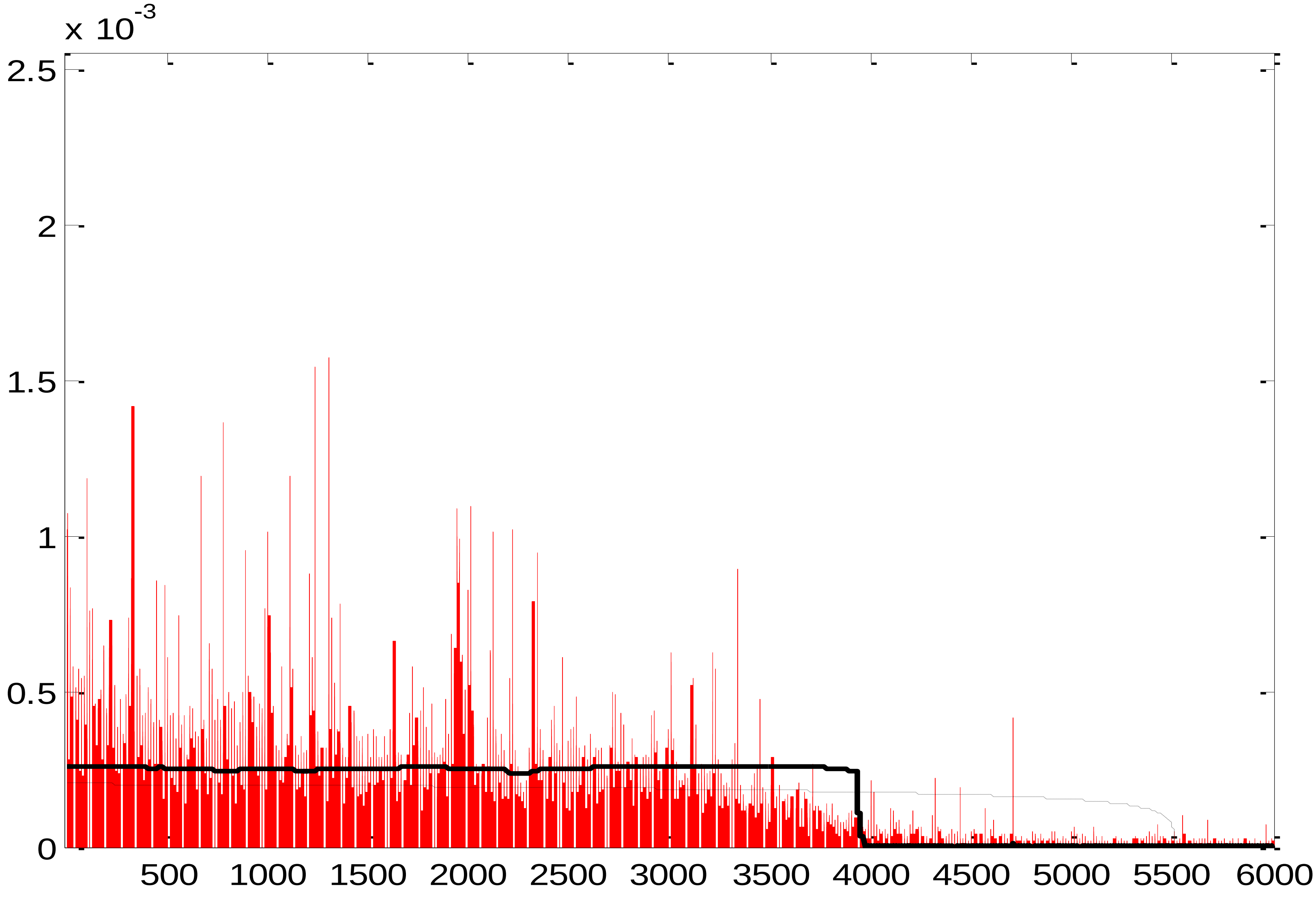
Learning in the Field (Week 3)



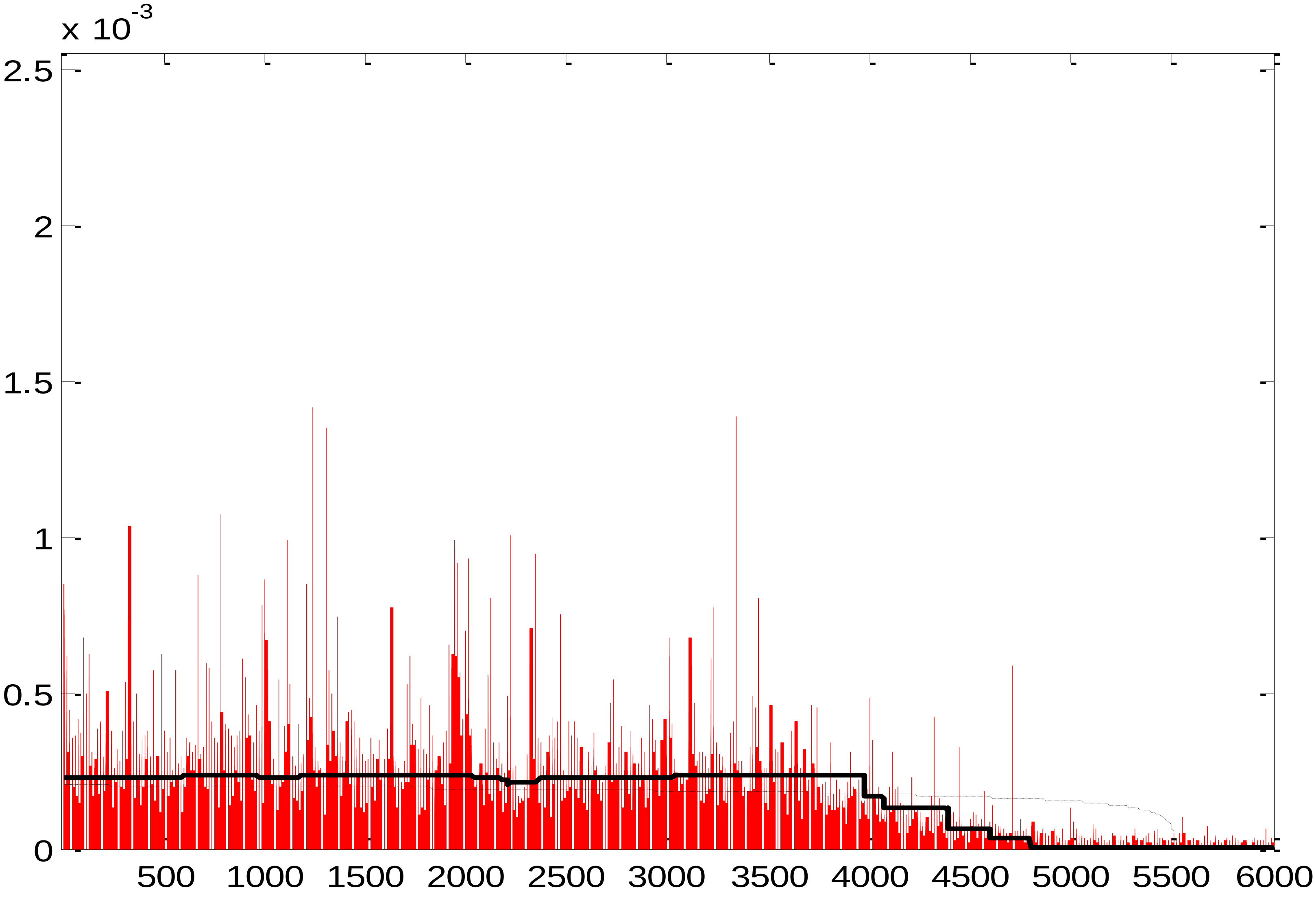
Learning in the Field (Week 4)



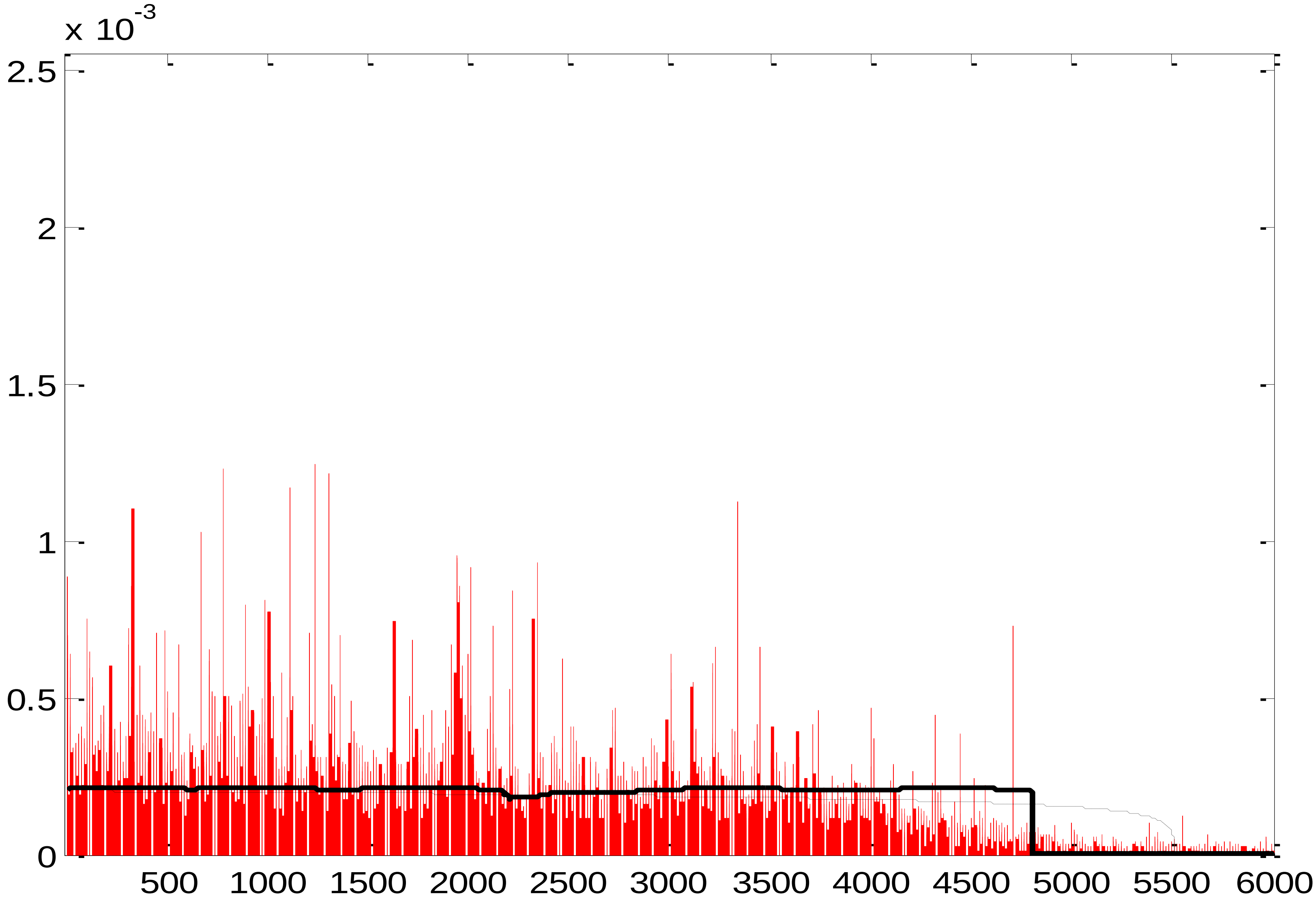
Learning in the Field (Week 5)

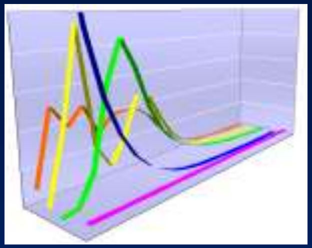


Learning in the Field (Week 6)



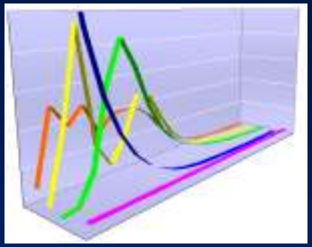
Learning in the Field (Week 7)





How did this START?

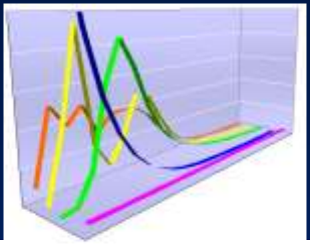
- Cognitive hierarchy (Camerer et al, 2004): Players have incorrect & heterogeneous beliefs.
  - Zero-step thinkers randomize uniformly
  - Higher-step thinkers best respond given the belief that other players are a mixture of lower-step thinkers
- The type distribution is Poisson; players' beliefs are a truncated Poisson distribution

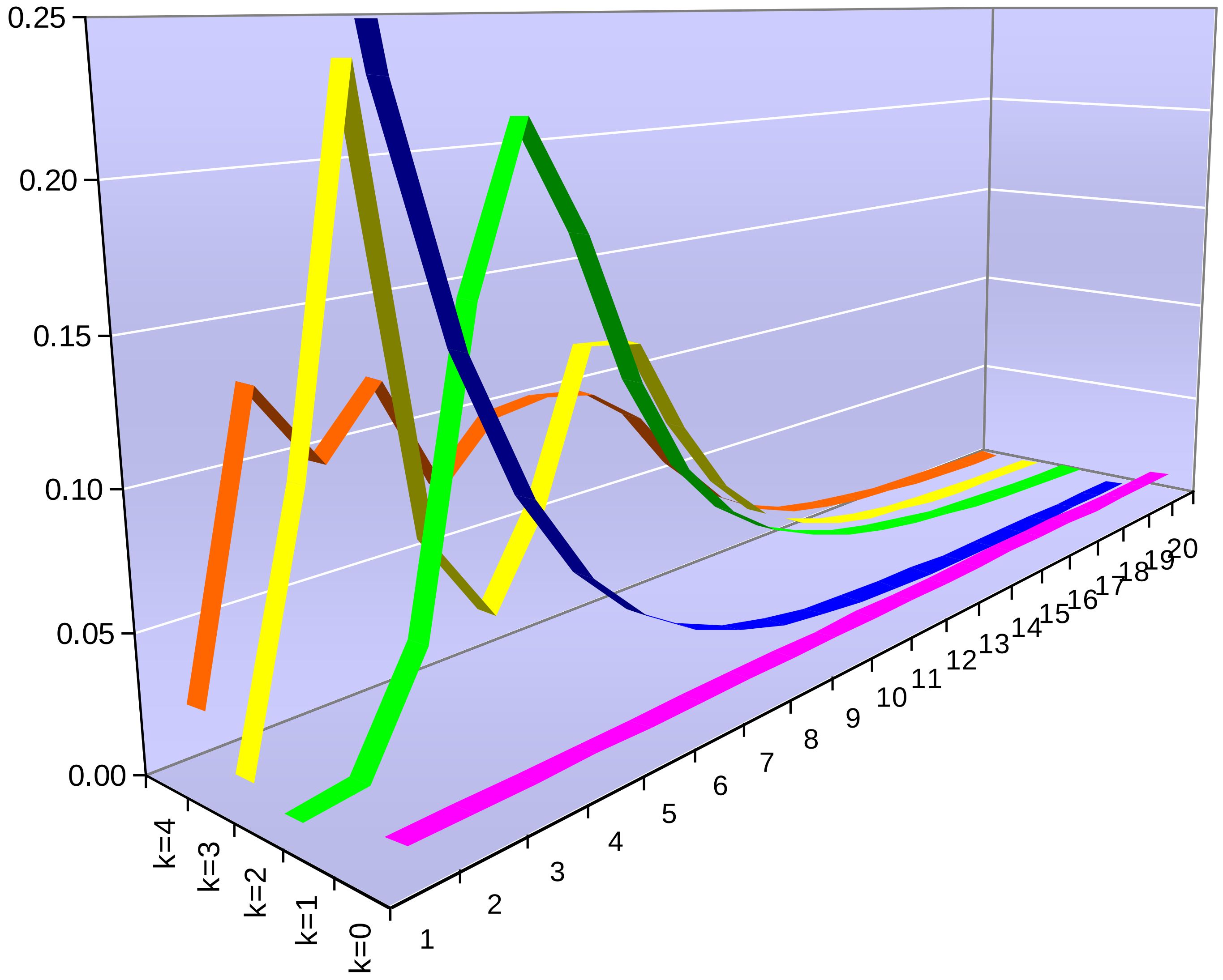


How did this START?

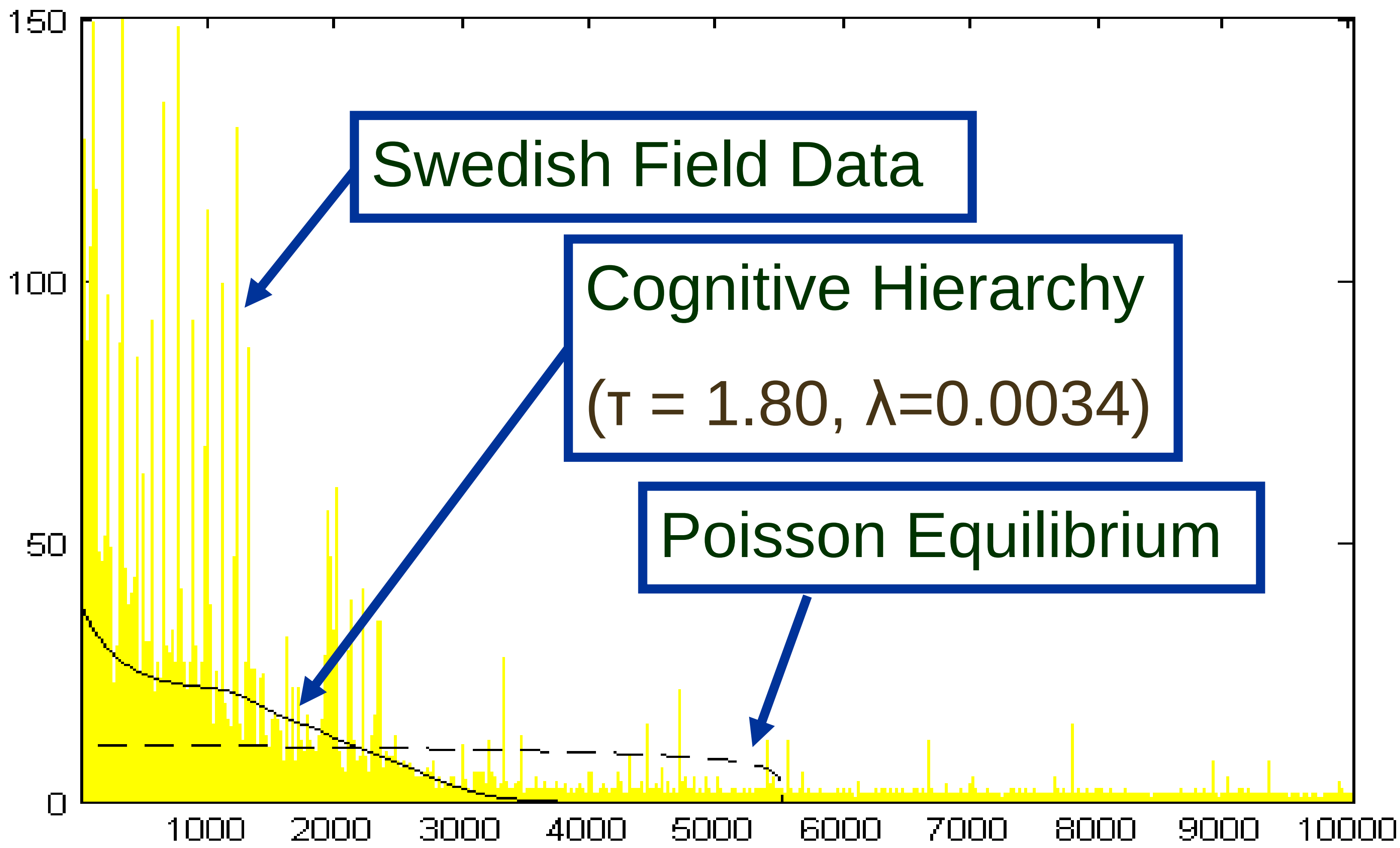
- We extend the standard model in two respects:
  - Number of players is random (Poisson); allows computation of expected payoffs
  - Players best-respond noisily using a power function
- τ: Average number of thinking steps
- λ: Degree of precision in best responses

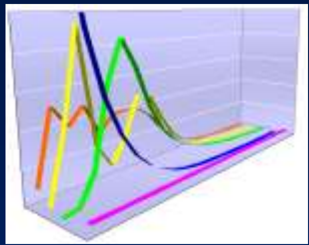
Cognitive Hierarchy





Initial Response in the Field (Week 1)

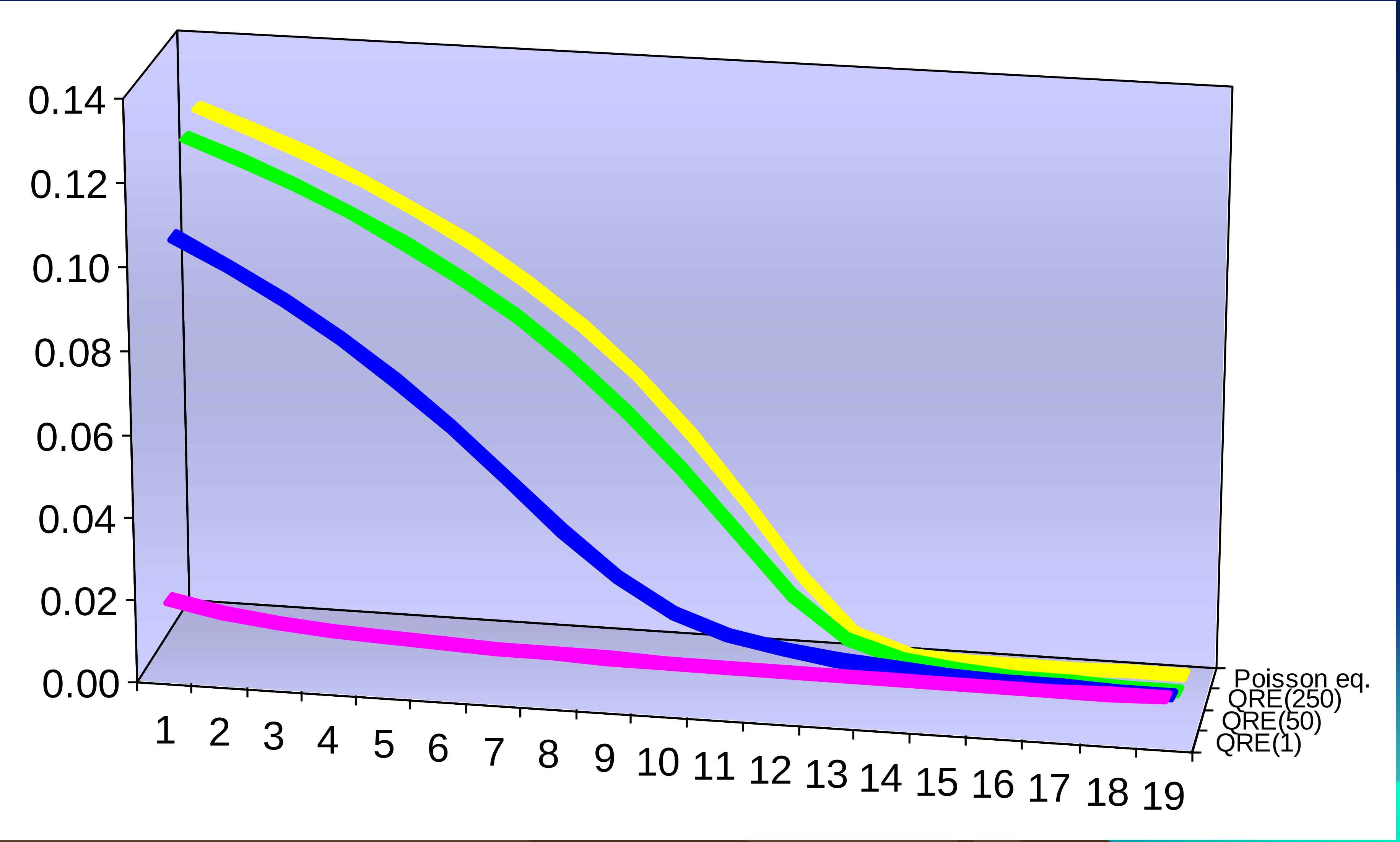




Quantal Respone Equilibrium

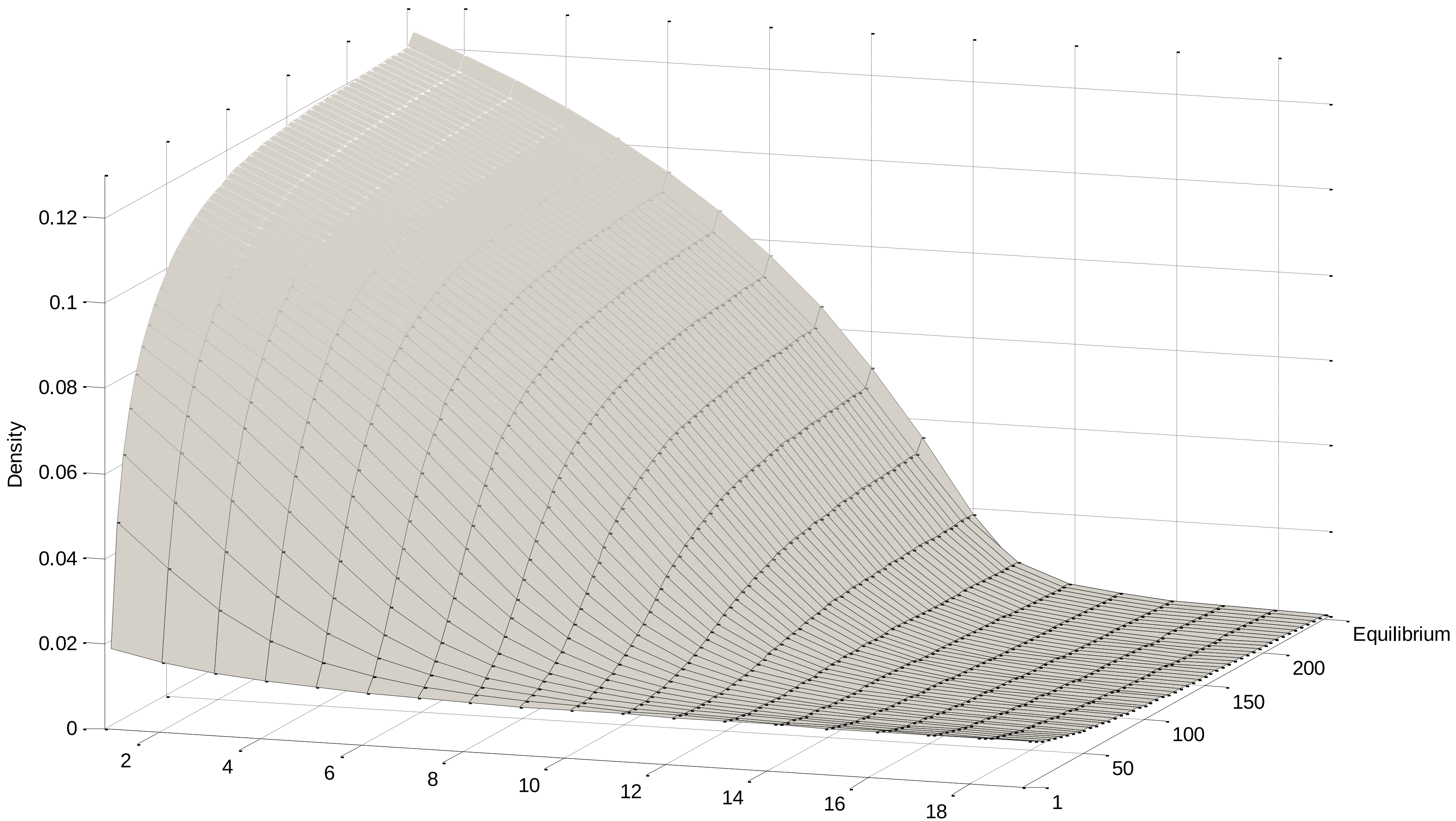
- We maintain the assumption that *N* ~ *Poisson (n)*.
- Replace best responses with noisy (quantal) responses.
- QRE: Players know both are doing quantal response (correct beliefs)
- Can’t explain overshooting
  - Converges “UP” to equilibrium

Logit QRE



0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
Poisson eq.
QRE(250)
QRE(50)
QRE(1)

Logit QRE Approx. from Below



Conclusion

- Observe a well-defined game (LUPI) played in the field
- Poisson equilibrium explains the data surprisingly well
- Imitation learning explains convergence
- CH (τ = 1.80) accounts for initial overshooting of low numbers
- Shouldn't we apply population uncertainty to other games?
- LUBA (Least Unique Bid Auctions)