Midterm Exam for Experimental Economics II (Spring 2019)

Note: You have 180 minutes (1:20-4:20pm) and there are 137 points; allocate your time wisely.

PART A: The Sleeping Game (35 pts)

Read the (abridged) article below:

美客機「睡」過頭 怪ＦＡＡ不人道 2009/10/26 中國時報【劉屏／華盛頓廿五日電】

兩位客機駕駛員都睡著了。誰的錯？美國聯邦民航總署（ＦＡＡ）成為眾矢之的。專家說，ＦＡＡ不把駕駛員當人，不准他們輪流小睡，才導致這種緊急事件。

美國西北航空班機日前在明尼亞波利斯市上空過門不入，多飛了二百四十餘公里才降落。其間地面航管呼叫，但得不到回應，以為發生劫機，遂請軍方派遣戰鬥機升空。最後是空服員從客艙內打機內電話給駕駛員，才結束這場烏龍。兩位駕駛說是因為討論公司的政策太專心，以致忘了降落。…(中略)…專家普遍認為，最可能的解釋是「兩位駕駛員都睡著了」。…(中略)…迄今為止，沒有任何專家指責兩位駕駛，反倒頗多同情之詞，把矛頭指向ＦＡＡ。飛安專家、退休客機駕駛約翰.南斯接受ＡＢＣ主播吉布遜訪問時說，發生這種事，是因為「多年來，ＦＡＡ不承認駕駛員是人，不承認他們會睏，不准許駕駛在駕駛艙裡睡覺」。

由於ＦＡＡ不准睡，有的駕駛心想，「既然同伴不會睡，我小睡一下應無妨」，說不定兩人都這麼想，就都睡著了。專家說，很多國家准許駕駛員在嚴格的前提小睡一會兒，這些前提包括啟動自動駕駛儀、不能離開駕駛座、告知空服員等。這種作法的立論基礎是：一位擺明了要睡，另一位就絕不敢睡，反而比較安全。就像據傳台灣曾有客機駕駛告訴同僚，「我要在座位上打坐一會兒」，另一位於是格外專心，倒也平安。

Consider the following game played between the two sleepy pilots: Each pilot chooses to either sleep or stay awake. Falling asleep gives the sleepy pilot some rest, which is worth NT$2,000 to each pilot. The plane flies safely if at least one pilot to stay awake, which is worth NT$10,000 to each pilot. If both pilots fall asleep, the plane would be in danger, which would cost the pilot NT$100,000 each.

Now consider the case where the two pilots are inequality averse in the sense of Fehr and Schmidt (1999) and have the same utility function. Assume the pilots dislike earning less than the other player by a factor of α, but feel guilty about earn more by a factor of β.

Hint: Consider the following utility function:

$$U_i(X) = x_i - \frac{\alpha}{n-1}\sum_{k\neq i}\max(x_k - x_i, 0) - \frac{\beta}{n-1}\sum_{k\neq i}\max(x_i - x_k, 0)$$

a. (13 pts) Write down a utility function to represent the pilots' inequality-averse preferences and draw the new payoff matrix.

b. (12 pts) Solve for all of the pure and mixed Nash equilibrium of this game.

c. (10 pts) For what parameter values can this explain the intended outcome of the FAA (where both pilots always stay awake) and the outcome in Taiwan (where one pilot asks the other to cover him when he is taking a nap)? Why or why not?

PART B: Ultimatum Games (32 pts)

Paul the Proposer and Rachael the Respondent divide \$10. Paul proposes how to split the money between the two of them, and Rachael decides to accept or reject. If Rachael accepts, the money is divided accordingly; if Rachael rejects, both earn zero. Find the SPE when the set of possible offers is:

a. (10 pts) $A_p = \{$(P, R): (9.99, 0.01), (9.98, 0.02), (9.97, 0.03), ..., (0.01, 9.99)$\}$.
b. (10 pts) $A_p = \{$(P, R): (10, 0), (9, 1), (8, 2), ..., (0, 10)$\}$.
c. (12 pts) What do you think would happen when real people play this game?

PART C: 2-period Bargaining Game (30 pts)

Player 1 offers how to split a pie of \$100 with player 2; player 2 can accept the offer (and split accordingly), or reject it. If player 2 rejects, the pie shrinks to \$25 and player 2 gets to offer how to split it with player 1. Player 1 can accept the offer (split accordingly), or reject it (both earn zero).

a. (14 pts) What is the Nash equilibrium of the subgame after player 2 rejects?
b. (16 pts) What is the subgame perfect Nash equilibrium of this game?

PART D: Voter Participation Model (40 pts)

Consider the voter participation model in class and assume now the number of voters in the minority group L is $l = 1$ and that in the majority group M is $m = 2$. (Suppose that the cost of voting $\kappa < \frac{1}{2}$ is the same for all individuals).

a. (20 pts) Find out a quasi-symmetric mixed-strategy equilibrium in which all individuals in group L use the same mixed strategy, i.e., for all $i \in L$, $v_i = \hat{v} \in (0,1)$, and the number of participants from group M is exactly $m^v = l = 1$. Do you think there also exists a similar equilibrium with $m^v = 2$? Show why or why not.
b. (20 pts) Also find out a totally mixed quasi-symmetric equilibrium in which all $i \in L$ use a mixed strategy $\hat{v}$ and all $j \in M$ use a mixed strategy $\overline{v}$ (there can be many such equilibria). Among the equilibria you found (if there were many), can you identify any equilibrium of the form $v = (\hat{v}, 1 - \hat{v})$—that is, all $i \in L$ play $\hat{v}$ and all $j \in M$ play $\overline{v} = 1 - \hat{v}$?

[Experimental Economics II] Midterm Solution

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1 The Sleeping Game (35 pts)

a. (13 pts)

(5 pts) Utility functions to represent pilots' inequality-averse preferences are

$$\begin{cases} U_1(X) = x_1 - \alpha \max(x_2 - x_1, 0) - \beta \max(x_1 - x_2, 0) \\ U_2(X) = x_2 - \alpha \max(x_1 - x_2, 0) - \beta \max(x_2 - x_1, 0) \end{cases}.$$

(8 pts) The original payoff matrix, where $U_i(X) = x_i$, was:

Pilot 1 \ Pilot 2	Sleep	Awake
Sleep	(-$98000, -$98000)	($12000, $10000)
Awake	($10000, $12000)	($10000, $10000)

Applying the utility functions above, the new payoff matrix would be:

Pilot 1 \ Pilot 2	Sleep	Awake
Sleep	(-$98000, -$98000)	($12000-$\beta$$2000, $10000-$\alpha$$2000)
Awake	($10000-$\alpha$$2000, $12000-$\beta$$2000)	($10000, $10000)

b. (12 pts)

(9 pts) For simplicity, we assign characters to each candidate of Nash equilibrium:

Pilot 1 \ Pilot 2	Sleep	Awake
Sleep	$\mathbb{A}$	$\mathbb{B}$
Awake	$\mathbb{C}$	$\mathbb{D}$

The pure Nash equilibrium depend on the values of α and β. After some calculation, we obtain:

α \ β	$\beta < 1$	$\beta = 1$	$\beta > 1$
$\alpha < 54$	$\mathbb{B}$, $\mathbb{C}$	$\mathbb{B}$, $\mathbb{C}$, $\mathbb{D}$	$\mathbb{D}$
$\alpha = 54$	$\mathbb{A}$, $\mathbb{B}$, $\mathbb{C}$	$\mathbb{A}$, $\mathbb{B}$, $\mathbb{C}$, $\mathbb{D}$	$\mathbb{A}$, $\mathbb{D}$
$\alpha > 54$	$\mathbb{A}$	$\mathbb{A}$, $\mathbb{D}$	$\mathbb{A}$, $\mathbb{D}$

(3 pts) For mixed Nash equilibrium, we assume that each pilot chooses "Sleep" with probability p and chooses "Awake" with probability $1-p$. At Nash equilibrium, the two strategies should be indifferent, and thus:

$$p \cdot (-98) + (1-p) \cdot (12 - 2\beta) = p \cdot (10 - 2\alpha) + (1-p) \cdot 10 \Rightarrow \; p = \frac{1-\beta}{55 - \alpha - \beta}$$

Note that $0 \leq p \leq 1$ should be satisfied, or else mixed Nash equilibrium doesn't exist.

c. (10 pts)

(4 pts) The intended outcome of the FAA (where both pilots always stay awake) is when the only Nash equilibrium is $\mathbb{D}$. Therefore, it could be explained by parameter values $\alpha < 54$ and $\beta > 1$.

(4 pts) On the other hand, the outcome in Taiwan (where one pilot asks the other to cover him when he is taking a nap) is when the Nash equilibrium is $\mathbb{B}$ or $\mathbb{C}$. Therefore, it could be explained by parameter values $\alpha < 54$ and $\beta < 1$.

(2 pts) The two outcomes differ because of the value of β. To put it short, FAA is actually asking pilots to be very averse to advantageous inequality, which may not apply to the real world.

2 Ultimatum Games (32 pts)

a. (10 pts)

(5 pts) For any offer in A_p, Rachael should choose to accept since it's better than earning nothing.

(5 pts) Therefore, Paul should propose $(P, R) = (9.99, 0.01)$. This is the SPE.

b. (10 pts)

(3 pts) For the offer $(P, R) = (10, 0)$, both "accept" and "reject" are Rachael's best response.

(3 pts) If Rachael accepts, Paul should propose $(P, R) = (10, 0)$. If Racheal rejects, Paul should propose $(P, R) = (9, 1)$.

(4 pts) Hence, there are two SPEs, $(P, R) = (10, 0)$ and $(P, R) = (9, 1)$.

c. **(12 pts)** I think when real people play this game, they will consider about fairness. Therefore, it is unlikely that such an unequal split would be accepted.

3 2-Period Bargaining Game (30 pts)

a. (14 pts)

(6 pts) In this subgame, player 1 is choosing between accepting player 2's offer and earning zero. Thus, player 1 would accept the offer $(\$\epsilon, \$25 - \$\epsilon)$, where $\epsilon > 0$ can be infinitely close to zero.

(8 pts) Hence, $(0, \$25)$ is the Nash equilibrium of this subgame.

b. (16 pts)

(6 pts) From the results above, we know that in this whole game player 2 is choosing between accepting player 1's offer and earning $\$25 - \$\epsilon$, where $\epsilon > 0$ can be infinitely close to zero.

(10 pts) Hence, to make player 2 accept the offer, player 1 should propose $(\$75, \$25)$. This is the subgame perfect Nash equilibrium.

4 Voter Participation Model (40 pts)

a. (20 pts)

(5 pts) When $m^v = 1$, the voter in group L will earn $\frac{1}{2} - \kappa$ if he choose to vote, or earn 0 if he choose not to vote. Since $\kappa < \frac{1}{2}$, the voter in group L should choose to vote. Hence, $\hat{v} = 1$.

(5 pts) With the voter in group L voting for sure, the voter who abstains in group M is actually preferring a tie over a win. However, he earns $\frac{1}{2}$ in a tie and $1 - \kappa$ in a win, where the former payoff should be smaller than the latter given $\kappa < \frac{1}{2}$.

(3 pts) Therefore, the quasi-symmetric mixed-strategy equilibrium doesn't exist for the case $m^v = 1$.

(2 pts) Next, we consider the case $m^v = 2$. Under this condition, the voter in group L chooses to abstain for sure, since there's no way he could change the result.

(2 pts) And with the voter in group L abstaining for sure, one of the voters in group M has no point to vote, as he gets 1 if voting and $1 - \kappa$ if abstaining.

(3 pts) Thus, the quasi-symmetric mixed-strategy equilibrium doesn't exist for the case $m^v = 2$ either.

b. (20 pts)

(8 pts) Using mixed strategy means that the voters are indifferent between voting and abstaining. First we consider from the viewpoint of the voter in group L:

Probability / Decision	$\bar{v}^2$ (2 votes for M)	$2\bar{v}(1-\bar{v})$ (1 vote for M)	$(1-\bar{v})^2$ (0 vote for M)
Vote	$-\kappa$	$\frac{1}{2} - \kappa$	$1 - \kappa$
Abstain	0	0	$\frac{1}{2}$

Since the two options are indifferent,

$$\begin{aligned}&\bar{v}^2(-\kappa)+2\bar{v}(1-\bar{v})(\frac{1}{2}-\kappa)+(1-\bar{v})^2(1-\kappa)=\frac{1}{2}(1-\bar{v})^2\\ \Rightarrow\ &\bar{v}(1-\bar{v})+(1-\bar{v})^2-\kappa=\frac{1}{2}(1-\bar{v})^2\\ \Rightarrow\ &\frac{1}{2}\bar{v}^2=\frac{1}{2}-\kappa\Rightarrow\ \bar{v}=\sqrt{1-2\kappa}.\end{aligned}$$

(8 pts) Next, we consider from the viewpoint of a voter in group M:

Prob. / Decision	$\hat{v}\cdot\bar{v}$ (1 vote for each)	$\hat{v}\cdot(1-\bar{v})$ (1 vote for L)	$(1-\hat{v})\cdot\bar{v}$ (1 vote for M)	$(1-\hat{v})\cdot(1-\bar{v})$ (0 vote for both)
Vote	$1-\kappa$	$\frac{1}{2}-\kappa$	$1-\kappa$	$1-\kappa$
Abstain	$\frac{1}{2}$	0	1	$\frac{1}{2}$

Since the two options are indifferent as well,

$$\begin{aligned}&\hat{v}\cdot\bar{v}(1-\kappa)+\hat{v}\cdot(1-\bar{v})(\frac{1}{2}-\kappa)+(1-\hat{v})\cdot\bar{v}(1-\kappa)+(1-\hat{v})\cdot(1-\bar{v})(1-\kappa)\\ &\quad=\frac{1}{2}\hat{v}\cdot\bar{v}+(1-\hat{v})\cdot\bar{v}+\frac{1}{2}(1-\hat{v})\cdot(1-\bar{v})\\ \Rightarrow\ &1-\hat{v}+\hat{v}\cdot\bar{v}+\frac{1}{2}\hat{v}-\frac{1}{2}\hat{v}\cdot\bar{v}-\kappa=\frac{1}{2}\hat{v}\cdot\bar{v}+\bar{v}-\hat{v}\cdot\bar{v}+\frac{1}{2}-\frac{1}{2}\hat{v}-\frac{1}{2}\bar{v}+\frac{1}{2}\hat{v}\cdot\bar{v}\\ \Rightarrow\ &\frac{1}{2}-\kappa+\frac{1}{2}\hat{v}\cdot\bar{v}=\frac{1}{2}\bar{v}\Rightarrow\ \sqrt{1-2\kappa}\cdot\hat{v}=\sqrt{1-2\kappa}-(1-2\kappa)\\ \Rightarrow\ &\hat{v}=1-\sqrt{1-2\kappa}=1-\bar{v}.\end{aligned}$$

(4 pts) From the above, we can see that there exists an equilibrium of the form

$$v=(\hat{v},1-\hat{v})=(\sqrt{1-2\kappa},1-\sqrt{1-2\kappa}).$$