

## Homework 5

(Experimental Economics: Spring 2019)

Consider the voter participation model. There are two alternatives,  $a$ ,  $b$ . There are two groups  $L$ ,  $M$ ; the number of group  $L$  members is  $l$  and that of group  $M$  members is  $m$ , where  $0 < l < m$ . All those in group  $L$  strictly prefer  $a$  to  $b$  and all those in group  $M$  strictly prefer  $b$  to  $a$ . In particular, for all  $i \in L$ ,  $u_i(a) = H$ ,  $u_i(b) = L$ , and for all  $j \in M$ ,  $u_j(a) = L$ ,  $u_j(b) = H$ , where  $H > L > 0$ . The cost of voting is now a uniformly distributed random variable on the support  $[0, 1]$ . One's voting cost is a private information, so one only knows his own (realized) voting cost, but not other members'. Individuals choose to vote either  $a$  or  $b$ , or abstain. The election is decided by plurality rule with ties broken by a fair coin toss.

- (a) A quasi-symmetric equilibrium is characterized by a pair of threshold costs  $(c_L^*, c_M^*)$ , hence members in group  $k$  participate in voting (and vote for their preferred alternative) if and only if their voting cost  $c$  is less than  $c_k^*$ , for  $k \in \{L, M\}$ . The threshold costs then decide the equilibrium participation rates  $(p_L^*, p_M^*)$ , and actually,  $(c_L^*, c_M^*) = (p_L^*, p_M^*)$  with the uniformly distributed cost on  $[0, 1]$ . Using these threshold costs/participation rates and binomial formula, write down the expressions for the pivot probabilities for  $a$ ,  $b$ ,  $\Pr[\text{Piv}_a|l, m]$  and  $\Pr[\text{Piv}_b|l, m]$ .
- (b) Using the expressions for the pivot probabilities, write down the two indifference conditions, one for each group  $k \in \{L, M\}$ , implying that the threshold cost type  $c_k^*$  is indifferent between voting and abstaining.
- (c) The two indifference conditions in part (b) give us two equations in two unknowns which we now write  $p_k^*(l, m)$ ,  $k \in \{L, M\}$  to indicate the dependence of our solutions on group sizes. Assume  $H = 105$ ,  $L = 5$ . Show by computation that  $p_L^*(2, 3) > p_L^*(4, 5)$  and  $p_M^*(2, 3) > p_M^*(4, 5)$ . That is, the participation rates are decreasing in total group size while the election remains "close" (size effect).
- (d) Also show by computation that  $p_L^*(4, 5) > p_L^*(3, 6)$  and  $p_M^*(4, 5) > p_M^*(3, 6)$ . That is, the participation rates are higher when the election is "close" than when it is a "landslide" (competition effect).
- (e) Finally, verify from the results in parts (c), (d) that  $p_L^*(2, 3) > p_M^*(2, 3)$ ,  $p_L^*(4, 5) > p_M^*(4, 5)$  and  $p_L^*(3, 6) > p_M^*(3, 6)$ . That is, the participation rates for the minority group  $L$  are higher than those for the majority group  $M$  (underdog effect).