

Coordination

協調賽局

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Lecture 10, EE-BGT

Outline

- ▶ Why is coordination important?
- ▶ Pure Coordination Game
 - ▶ GAMES magazine (1989)
 - ▶ Mehta et al. (AER 1994)
 - ▶ Mehta et al. (T&D 1994)
 - ▶ Bacharach and Bernasconi (GEB 1997)
- ▶ Battle of Sexes
 - Cooper et al. (AER 1990)
 - Blume et al. (AER98/GEB01)
- ▶ Market Entry Games
 - Kahneman (like magic...)
- ▶ Stag Hunt
 - Cooper et al. (AER 1990)
- ▶ Weak-Link Game
 - Van Huyck et al. (AER1990)
- ▶ Apply to Market Adoption:
 - Clemons & Weber (InfoSysR96)
- ▶ Apply to Culture:
 - Camerer & Weber (MS 2003)

Why is Coordination Important?

- ▶ Which Equilibrium to Select Among Many?
 - ▶ This requires Coordination!
- ▶ Examples of Coordination in Daily Life:
 - ▶ Language
 - ▶ Trading in Markets (Liquidity)
 - ▶ Industry Concentration

Why is Coordination Important?

- ▶ Equilibrium Selection in Game Theory
 1. **Desirable Features Approach:**
 - ▶ Payoff-Dominance, Risk Dominance, etc.
 2. **Convergence via Adaptation/Learning**
 - ▶ Weibull (1995), Fudenberg and Levine (1998)
 3. **Empirical Approach:** Infer Principles by
 - ▶ Putting people in experiments and observe actual behavior/outcome

Why is Coordination Important?

- ▶ Possible "Selection Principles":
 - ▶ Precedent, focal, culture understanding, etc.
- ▶ Why are observations useful?
- ▶ Schelling (1960, p.164):
 - ▶ "One cannot, without empirical evidence, deduce what understandings can be perceived in a nonzero-sum game of maneuver
 - ▶ any more than one can prove,
 - ▶ by purely formal deduction, that a particular joke is bound to be funny."

Why is Coordination Important?

- ▶ Can't Communication Solve This?
 - ▶ Not always... (See Battle of Sexes below)
- ▶ Sometimes communication is not feasible:
 - ▶ Avoiding Traffic Jams
 - ▶ Speed Limits (useful because they reduce speed "variance", and hence, enhance coordination!)
- ▶ Miscommunication can have big inefficiency!

Examples of Coordination Impact

- ▶ US railroad tracks is 4 feet and 8.5 inch
 - ▶ Because English wagons were about 5 feet (width of two horses), and lead to
- ▶ Space Shuttle Rockets smaller than ideal
 - ▶ since they need to be shipped back by train...
- ▶ Industries are concentrated in small areas
 - ▶ Silicon Valley, Hollywood, Hsinchu Science Park
- ▶ Urban Gentrification
 - ▶ I want to live where others (like me) live

Examples of Coordination Impact:

Drive on **Left**/**Right** side of the Road

- ▶ **Right**: Asia, Europe (Same continent!)
- ▶ **Left**: Japan, UK, Hong Kong (Islands!) 
- ▶ **Sweden** switched to **Right** (on Sunday morning)
- ▶ What about **America**? **Right**, to avoid
 - ▶ Hitting others with the whip on your right hand!
 - ▶ Bolivians switch to **Left** in mountainous area
 - ▶ To see outer cliffside from (left) driver seat
 - ▶ **Pittsburgh left**: 1st **left**-turner goes 1st at green
 - ▶ on two-lane streets to avoid blocking traffic

3 Types of Coordination Games

- ▶ Matching Games
 - ▶ Pure Coordination Game; Assignment Game
- ▶ Games with Asymmetric Payoffs
 - ▶ Battle of Sexes, Market Entry Game
- ▶ Games with Asymmetric Equilibria
 - ▶ Stag Hunt, Weak-Link Game
- ▶ Applications: Market Adoption and Culture

Examples of Coordination Impact

- ▶ Categorizing Products
 - ▶ Where should you find MCU? Disney or Action?
 - ▶ Find your favorite item at a new Costco store
- ▶ Common Language:
 - ▶ Internet promotes English
 - ▶ Some Koreans even get surgery to loosen their tongues, hoping to improve their pronunciation
- ▶ Key: Agreeing on something is better than not; but some coordinated choices are better

Matching Game: GAMES magazine (1989)

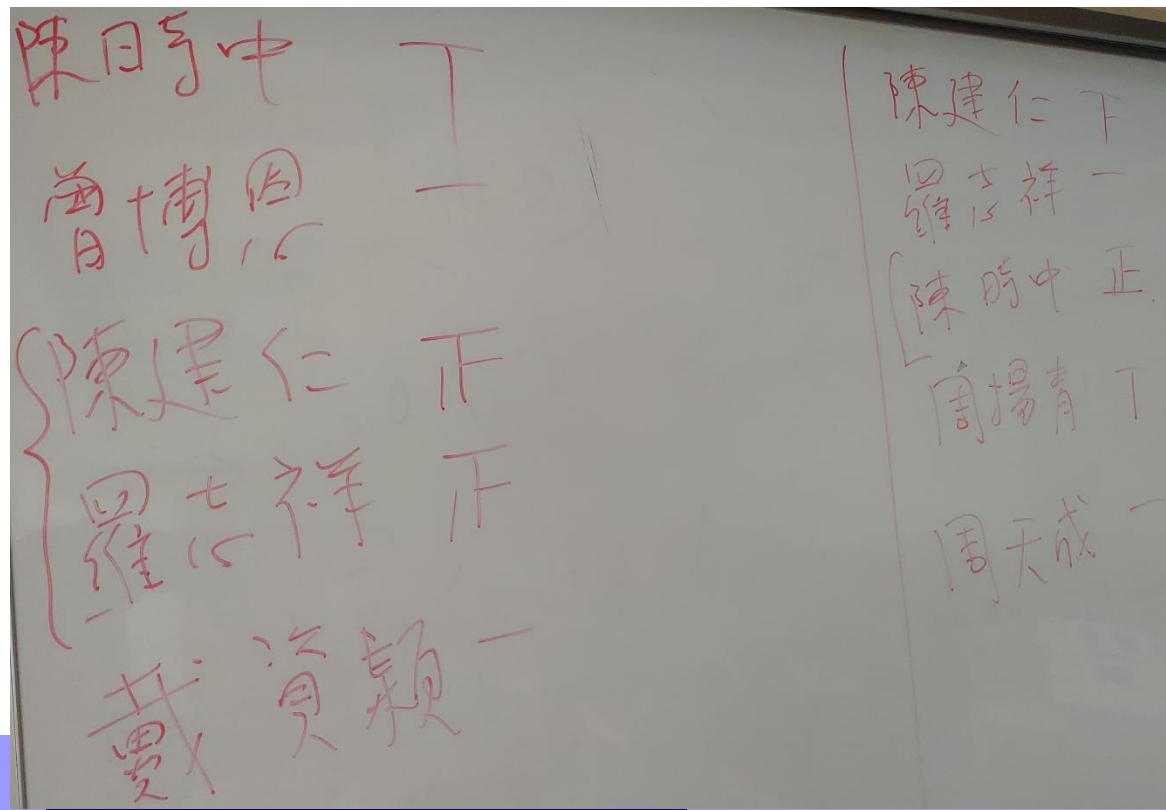
- ▶ Pick one celebrity (out of 9) for President, another for Vice-President:
 - ▶ Oprah Winfrey, Pete Rose,
 - ▶ Bruce Springsteen, Lee Iaccoca,
 - ▶ Ann Landers, Bill Cosby,
 - ▶ Sly Stallone, Pee-Wee Herman,
 - ▶ Shirley MacLaine
- ▶ One person is randomly awarded prize among those **who picked most popular one**

Matching Game: GAMES magazine (1989)

- ▶ Taiwanese Version:
 - ▶ 戴資穎、張育成、福原愛、瑞莎、趙婷、
陳時中、潘忠政、詹順貴、黃士修、趙介佑
- ▶ Prize?
- ▶ Results...

Matching Game: GAMES magazine (1989)

- ▶ Taiwanese example:
 - ▶ 戴資穎、周天成、羅志祥、周揚青、劉樂妍、曾博恩、陳時中、黃秋生、陳建仁、黃安
- ▶ Prize?
- ▶ Results...
(Last Year)



Matching Game: GAMES magazine (1989)

- ▶ Taiwanese example:
 - ▶ 戴資穎、陳偉殷、黃國昌、朱敬一、陳建仁、林立青、李來希、舒淇、林志玲、林奕含
- ▶ Prize?
- ▶ Results... (of 2019)

A photograph of a piece of lined paper with handwritten text in Chinese characters. The text is arranged in two columns. The left column lists names followed by numbers: 朱敬一 6, 黃國昌 2, 林奕含, 林志玲 1, 林立青, 陳建仁, 李來希. The right column lists names followed by numbers: 陳建仁 4, 舒淇 3, 李 2, 朱敬一, 林奕含, 林志玲 1, 黃國昌.

朱敬一 6	陳建仁 4
黃國昌 2	舒淇 3
林奕含	李 2
林志玲 1	朱敬一
林立青	林奕含
陳建仁	林志玲 1
李來希	黃國昌

Matching Game: GAMES magazine (1989)

▶ US Results:

1. Bill Cosby (1489): successful TV show
2. Lee Iacocca (1155): possible US candidate
3. Pee-Wee Herman (656): successful TV show
4. Oprah Winfrey (437): successful TV show
- ...
9. Shirley MacLaine (196): self-proclaimed reincarnate

Pure Coordination Game

	A	B
A	1, 1	0, 0
B	0, 0	1, 1

- ▶ Both get 1 if pick the same;
- ▶ Both get 0 if not
- ▶ Two pure NE,
- ▶ One mixed NE
- ▶ Which one will be played empirically?

Pure Coordination Game

- ▶ Mehta, Starmer and Sugden (AER 1994)
- ▶ **Picking Condition (P)**: Just pick a strategy
- ▶ **Coordinating Condition (C)**:
 - ▶ Win \$1 if your partner picks the same as you
- ▶ Difference between P and C = **How focal**
- ▶ Choices: Years, Flowers, Dates, Numbers, Colors, Boy's name, Gender, etc.

Pure Coordination Game

Category	Group P (n=88)		Group C (n=90)	
	Response	%	Response	%
Years	1971	8.0	1990	61.1
Flowers	Rose	35.2	Rose	66.7
Dates	Dec. 25	5.7	Dec. 25	44.4
Numbers	7	11.4	1	40.0
Colors	Blue	38.6	Red	58.9
Boy's Name	John	9.1	John	50.0
Gender	Him	53.4	Him	84.4

Pure Coordination Game: Follow-up 1

- ▶ Bardsley, Mehta, Starmer, Sugden (EJ 2010)
 - ▶ Incorporate (Replace?) Bardsley, et al. (wp 2001)
- ▶ 14 Games: One in choice set is **distinctive**
 - ▶ EX: {Bern, Barbodos, Honolulu, Florida}
- ▶ Add additional condition besides P and C:
 - ▶ **Guess Condition (G)**: Guess partner's pick
- ▶ **Design question**: How do you avoid **focality of physical location** (first/last/top-left)?
 - ▶ Have things swim around the computer screen...

Pure Coordination Game: Follow-up 1

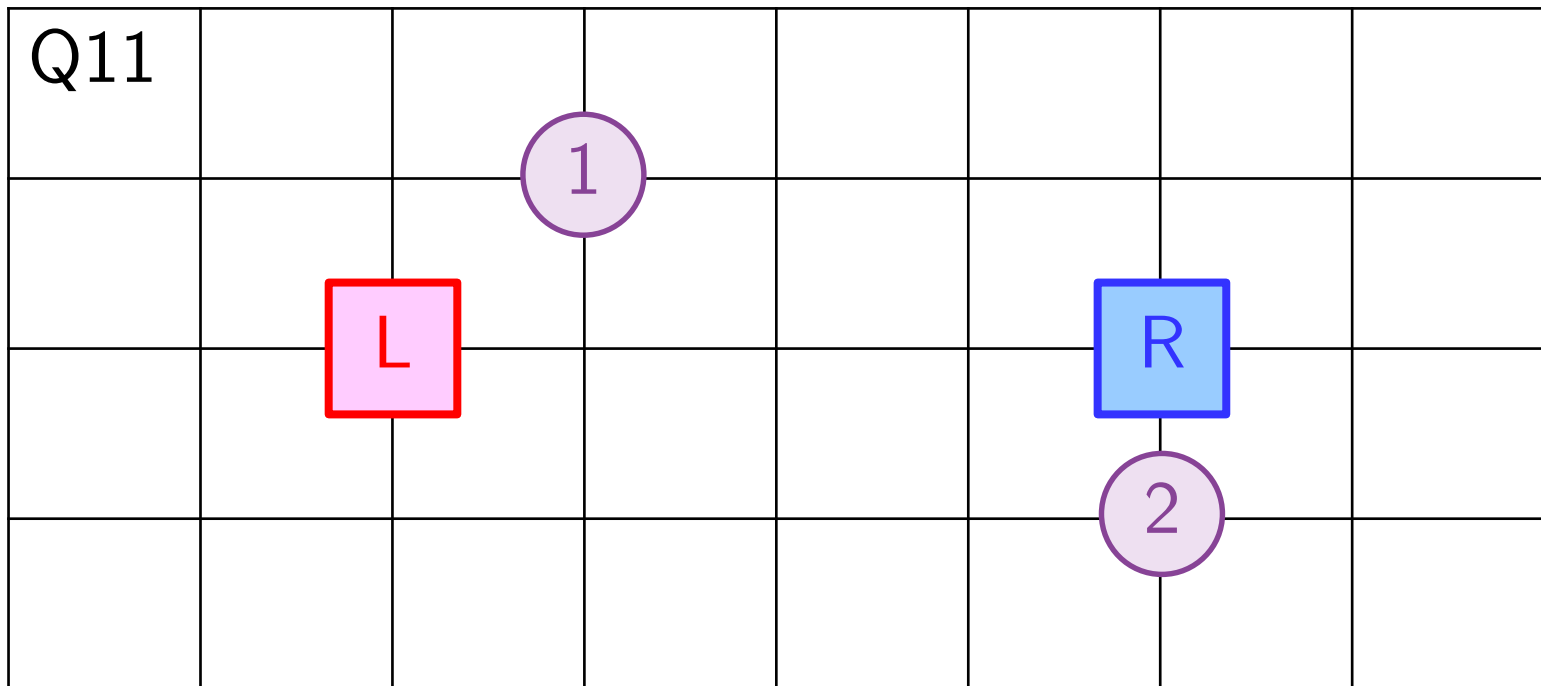
- ▶ EX: {Bern, Barbodos, Honolulu, Florida}
- ▶ **Derivative Saliience:** $P=G=C$
 - ▶ (See how paper use) Cognitive Hierarchy Model
 - ▶ Ex: Choose Bern in C since Bern in P and G
- ▶ **Schelling Saliience:** $P=G \neq C$
 - ▶ Team Reasoning: Pick distinctive choice **only** in C
 - ▶ Ex: Choose Bern in C, **but** Florida in P and G
- ▶ Coordinate on this: Even though I would not pick this and I know you would not pick this!

Pure Coordination Game: Follow-up 1

- ▶ Derivative Salience: $P=G=C$
- ▶ Schelling Salience: $P=G \neq C$
- ▶ Schelling Salience wins here!
 - ▶ In 12 games (out of 14):
- ▶ Chose distinctive choice 60% in C (modal!)
 - ▶ But less often in P and G
- ▶ EJ 2010: Follow-up w/ Nottingham subjects
 - ▶ Both saliences rejected with subtle design differences (used to coordinate)

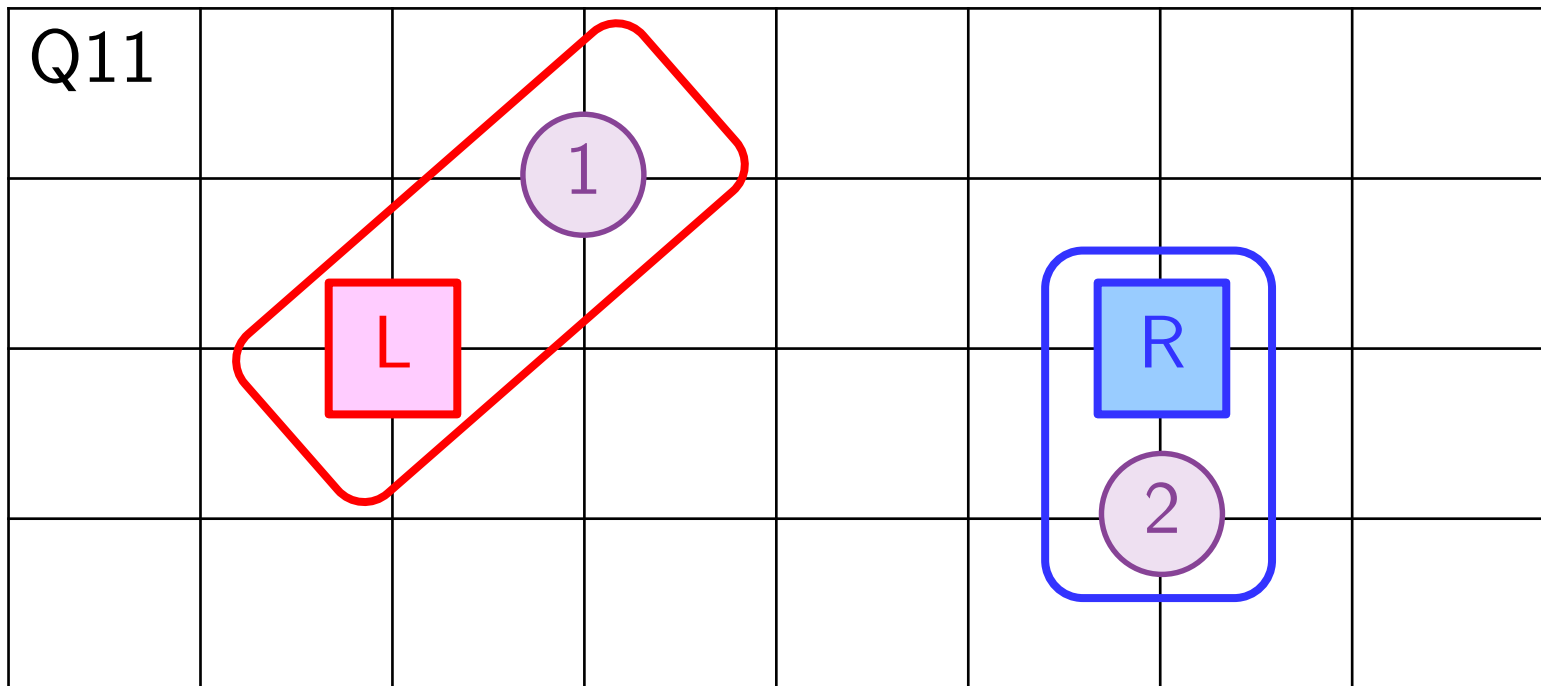
Assignment Game (Follow-up 2)

- ▶ Hume (1978/1740) - Ownership conventions: spatial/temporal proximity, cultural, etc.
- ▶ Mehta, Starmer and Sugden (T&D 1994)



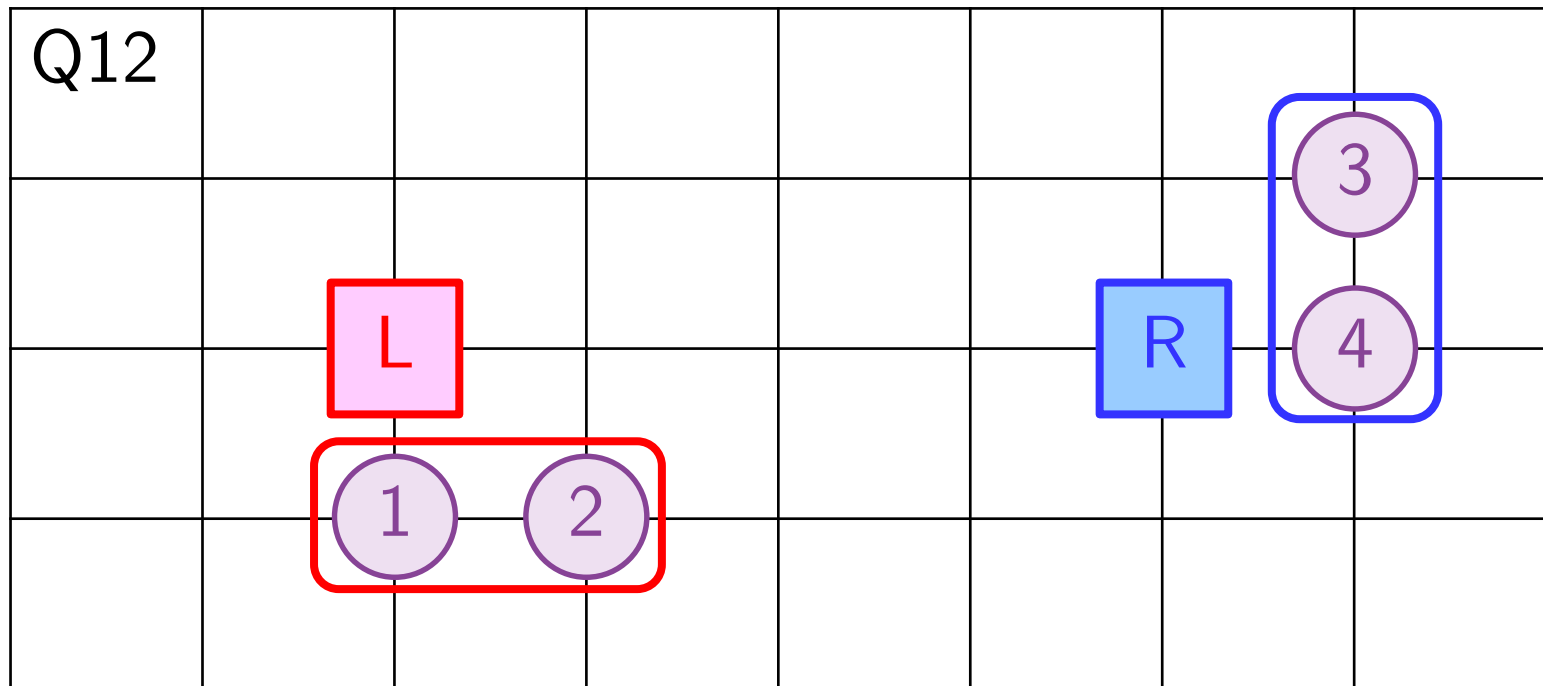
Assignment Game and Visual Selection

- ▶ Assign circles to L or R
- ▶ Earn \$\$ if all circles match partner assignment
- ▶ Focal Principle 1: Closeness (C)



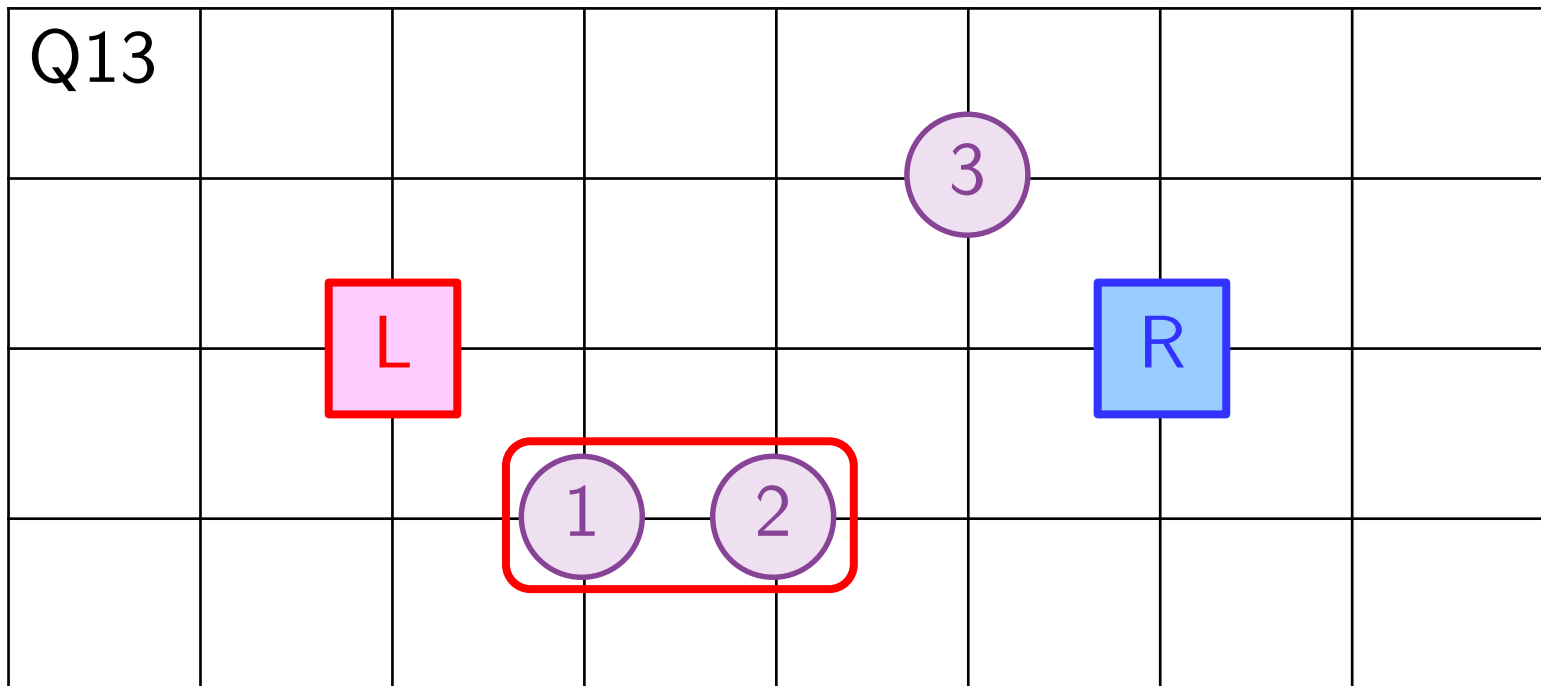
Assignment Game and Visual Selection

- ▶ Assign circles to L or R
- ▶ Earn \$\$ if all circles match partner assignment
- ▶ Focal Principle 2: Equality (E)



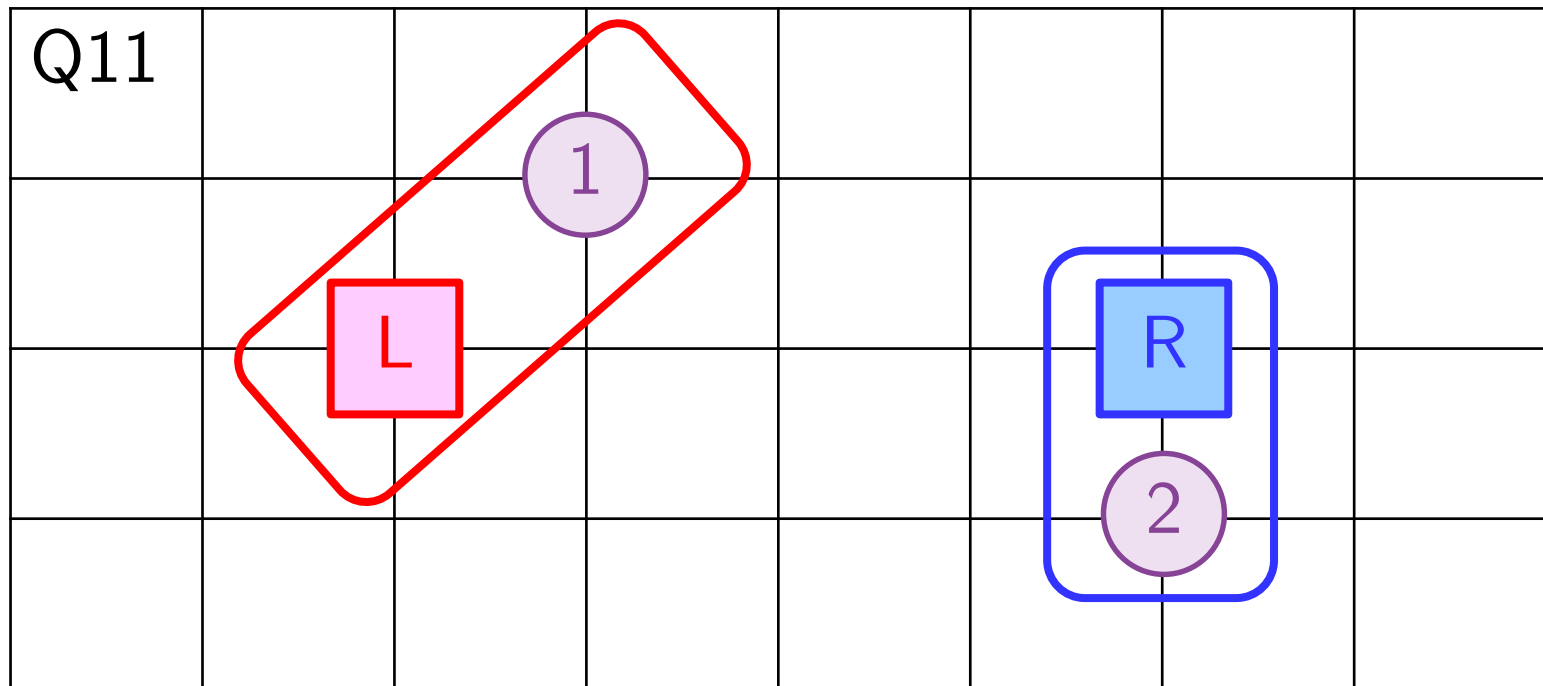
Assignment Game and Visual Selection

- ▶ Assign circles to L or R
- ▶ Earn \$\$ if all circles match partner assignment
- ▶ Focal Principle 3: Accession (A)



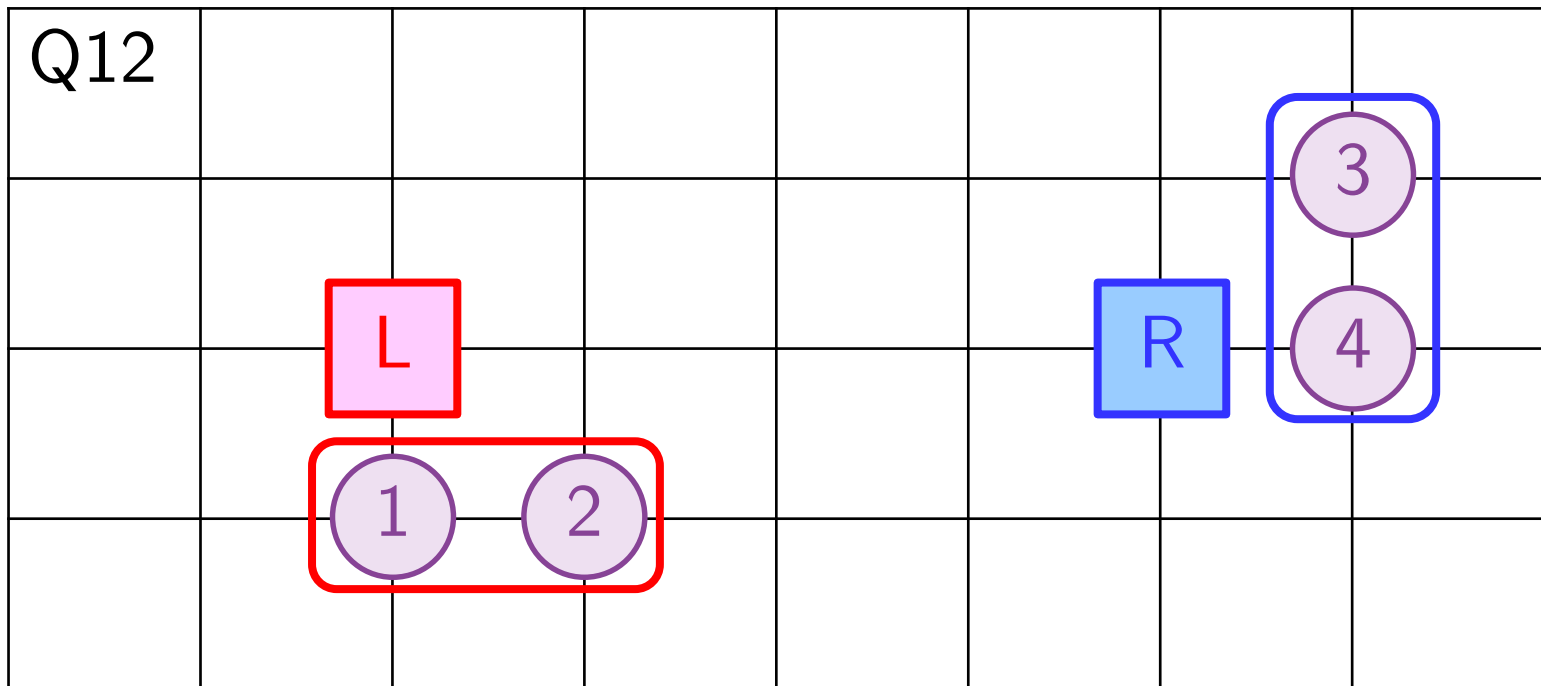
Assignment Game and Visual Selection

- ▶ How would you assign the circles?
- ▶ What about this? ($C = A = E$)
- ▶ In fact, 74% chose this!



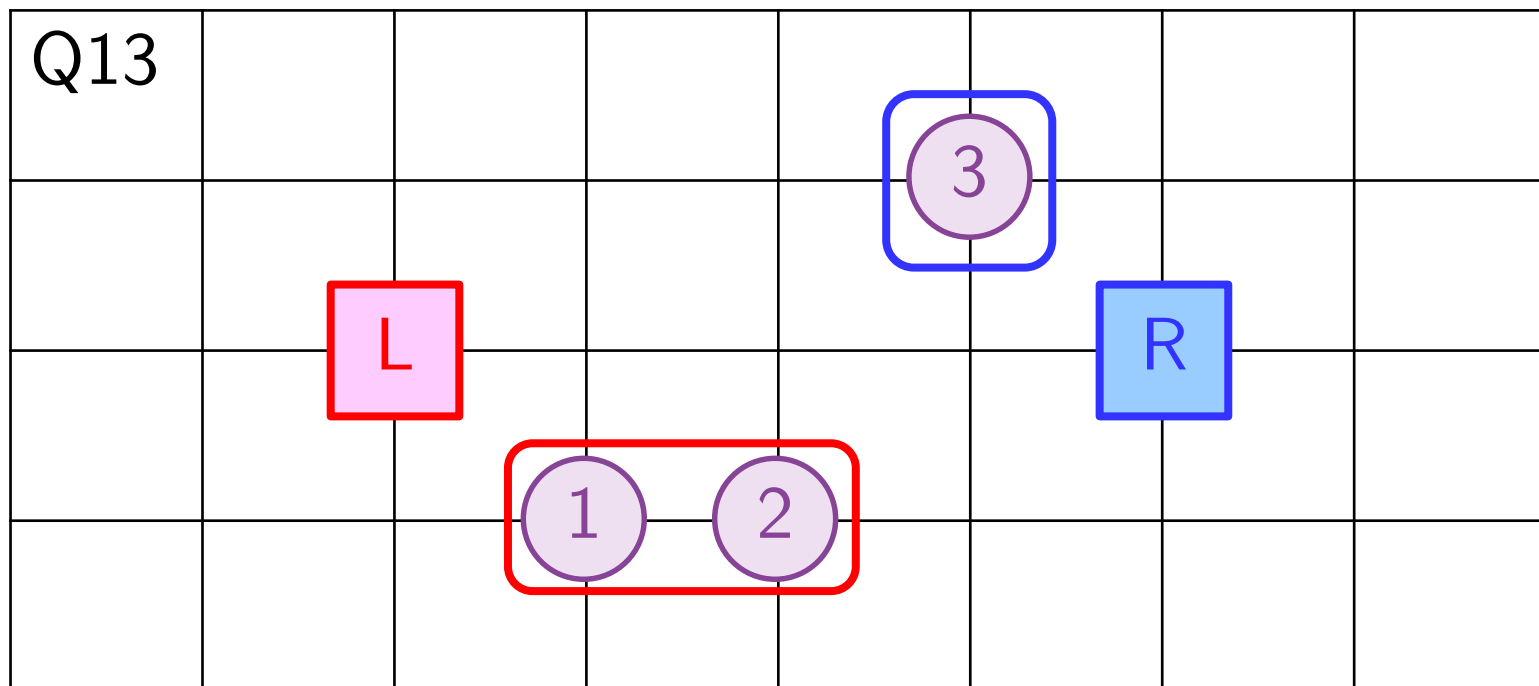
Assignment Game and Visual Selection

- ▶ How would you assign the circles?
- ▶ What about this? ($C = A = E$)
- ▶ In fact, 68% chose this!



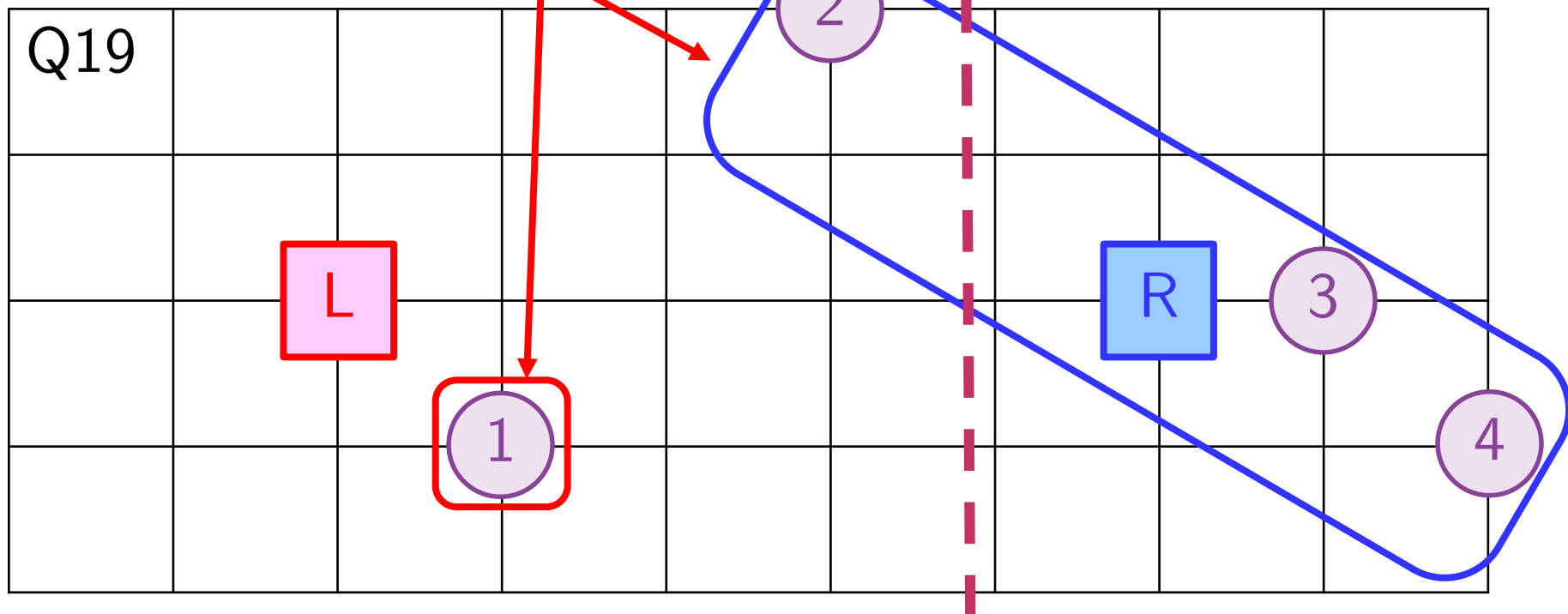
Assignment Game and Visual Selection

- ▶ How would you assign the circles?
- ▶ What about this? (Accession!)
- ▶ In fact, 70% chose this! (What does C/E say?)



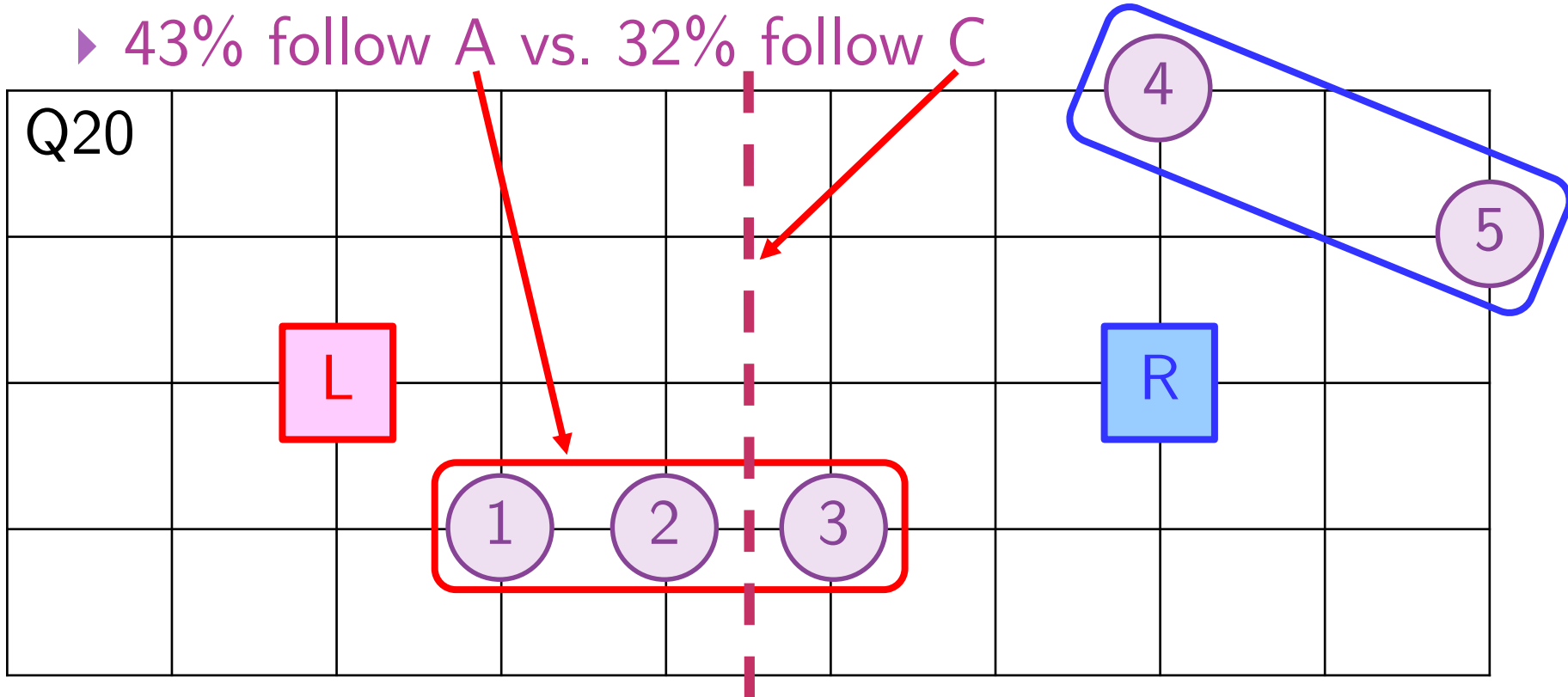
Assignment Game: C & A vs. Equality

- ▶ What does Closeness/Accession say?
- ▶ What does Equality say about this? 😊
- ▶ 29% follow C & A vs. 45% follow E



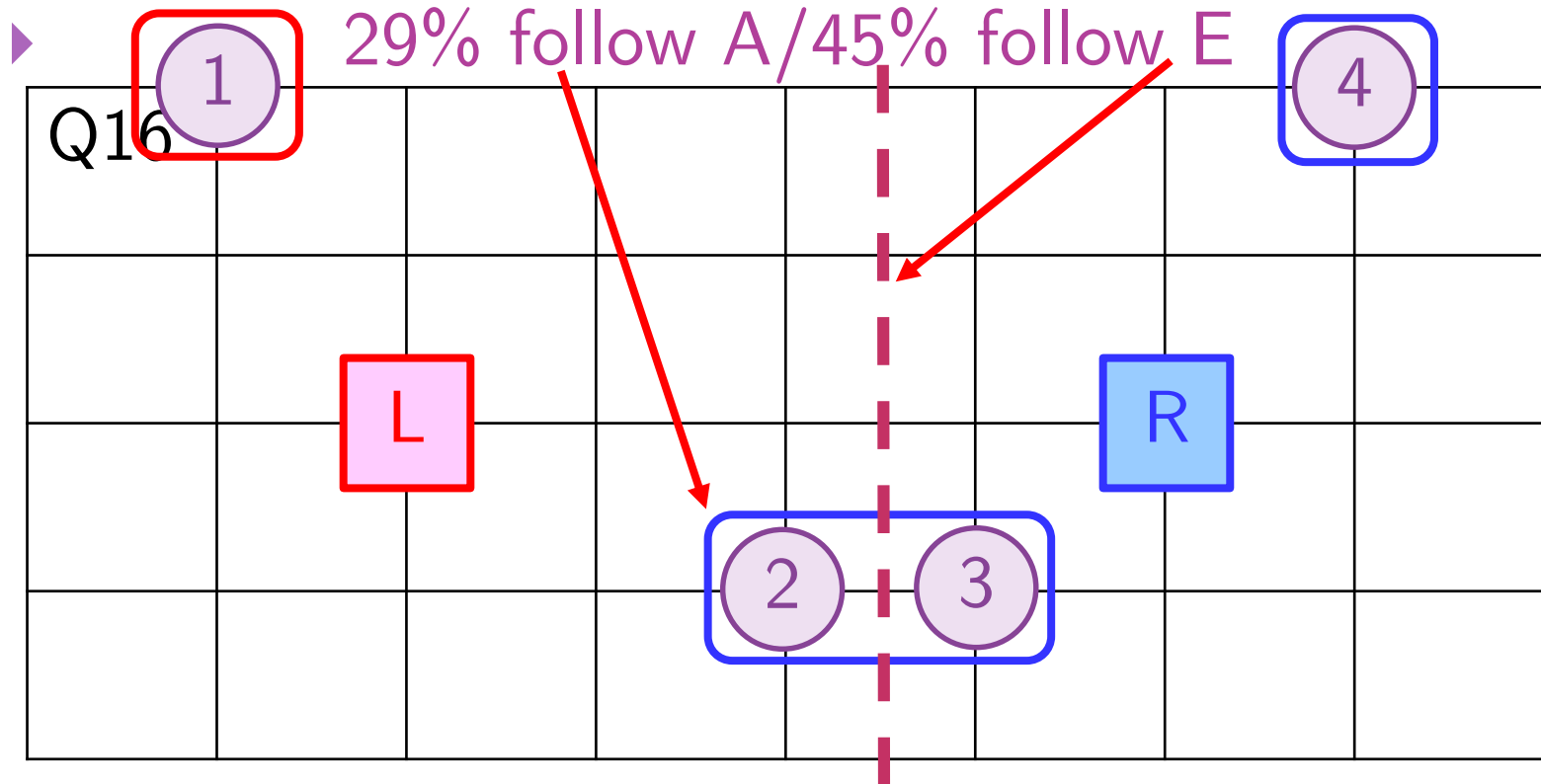
Assignment Game: Accession vs. Closeness

- ▶ What does Accession say about this? 😊
- ▶ What does Closeness say about this?
 - ▶ 43% follow A vs. 32% follow C



Assignment Game: Accession vs. Equality

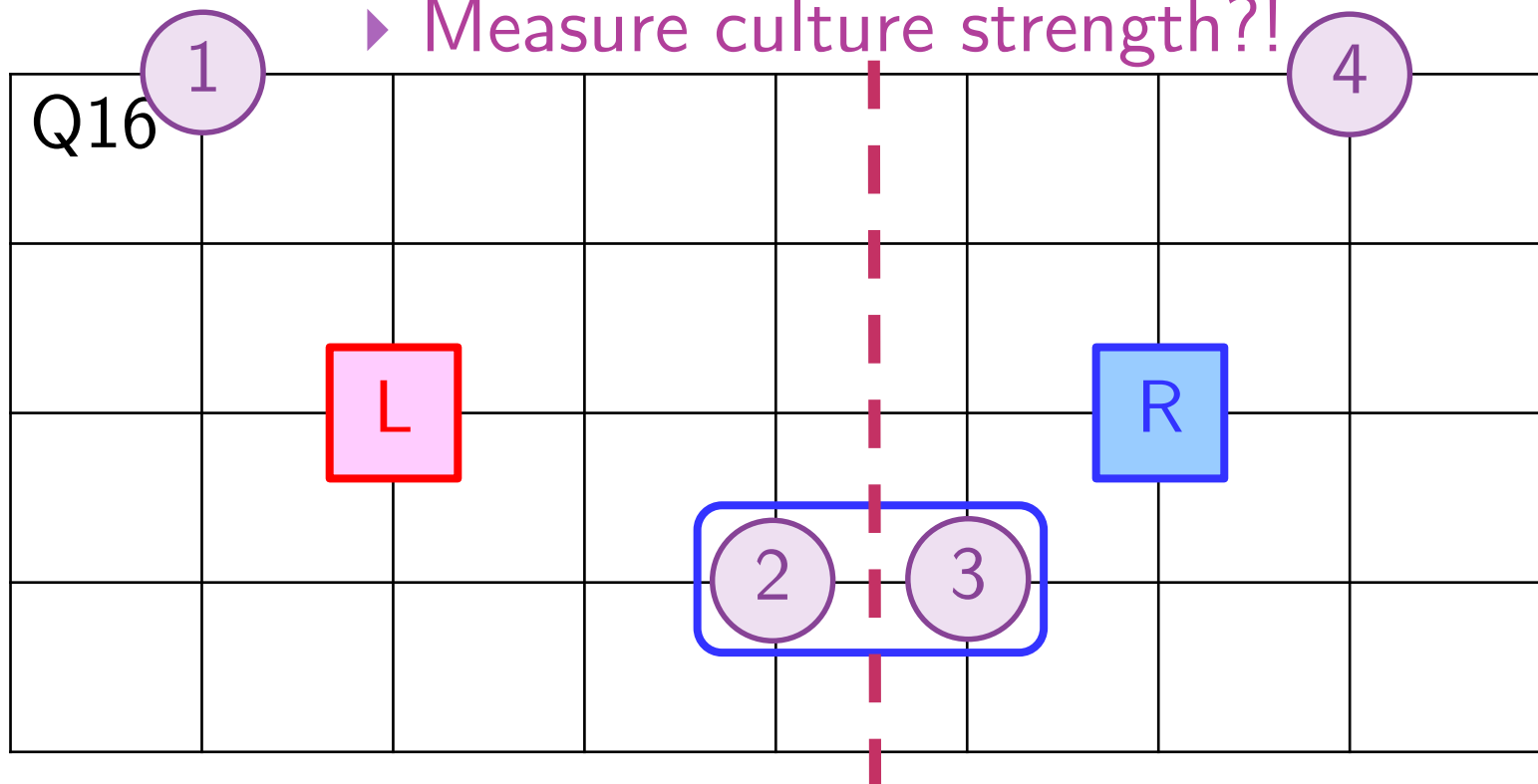
- ▶ What does Accession say about this?
- ▶ What does Equality say about this? 😊



Equality > Accession > Closeness

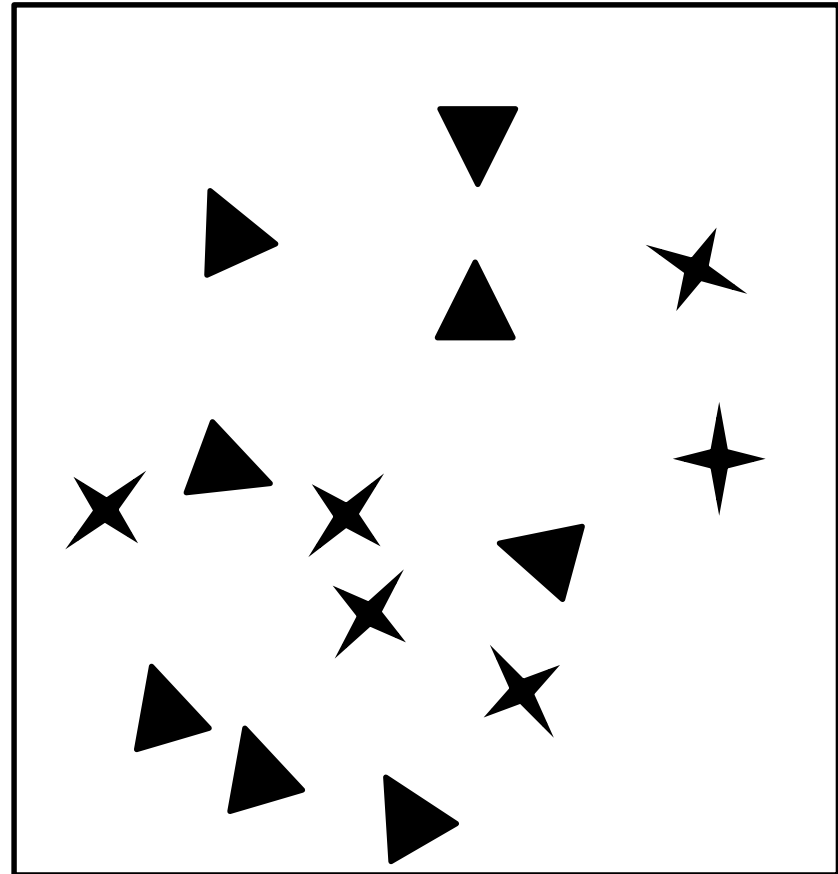
- ▶ First Focal Principle: **Equality** 😊
- ▶ Then **Accession** (if Equality satisfied/silent)

▶ Measure culture strength?!

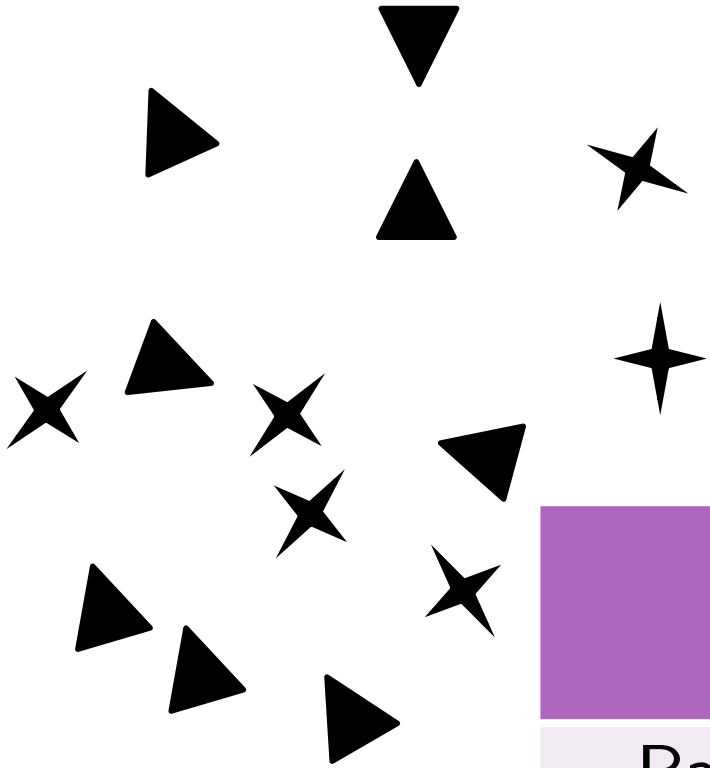


Unpacking Focality

- ▶ Bacharach and Bernasconi (GEB 1997)
- ▶ Visual matching game
 - ▶ Pick one from picture:
- ▶ **Test rarity preferences**
 - ▶ 6 vs. 8
- ▶ Are **Rare** item chosen more frequently
 - ▶ As **Rarity** increases?
 - ▶ 6/8, 2/3, 6/18, 1/15



Unpacking Focality: Test Rarity

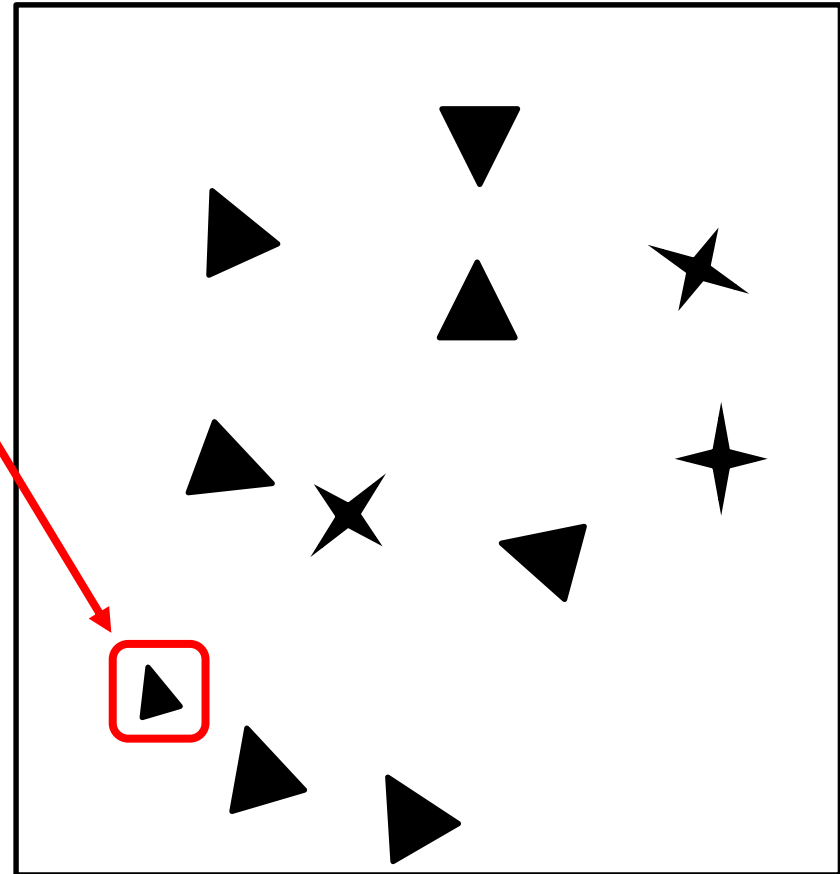


- ▶ Yes!
- ▶ As **Rarity** increases,
 - ▶ Frequency of rare choice increases!

	# of Rare/Frequent Items			
	6/8	2/3	6/18	1/15
Rare	65%	76%	77%	94%
Frequent	35%	24%	23%	6%

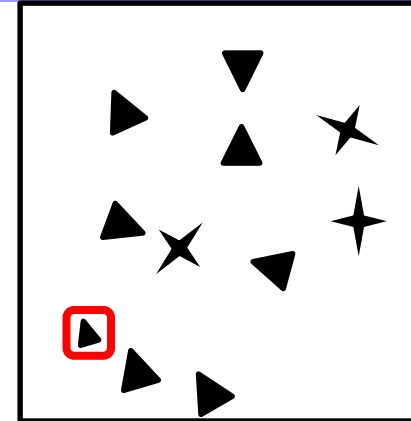
Unpacking Focality: Test Trade-offs

- ▶ **Rarity** ($r=3$ vs. $n=8$)
 - ▶ against
- ▶ **Oddity** (size or color)
 - ▶ $p(F)$ = prob. of notice
 - ▶ Would you choose
 - ▶ **Oddity** if $p(F) > 1/r$?
- ▶ **Obvious** Treatments:
 - ▶ $p(F) = 0.94 \gg 1/3$
- ▶ **Subtle** Treatments:
 - ▶ $p(F) = 0.40 > 1/3$



Unpacking Focality: Test Trade-offs

- ▶ Violate $p(F) > 1/r$
 - ▶ Mostly chose **Obvious** Oddity
 - ▶ Less than half chose **Subtle** Oddity



$r = \#$ of Rare	Obvious Oddity (r)				Subtle Oddity (r)				
	2	3	4	5	2	3	4	5	6
Rare	14%	19%	9%	7%	77%	55%	45%	69%	55%
Oddity	83%	79%	91%	88%	23%	31%	45%	19%	20%
Other	2%	2%	0%	5%	0%	14%	10%	12%	25%
$p(F)$	0.95	0.91	0.95	0.93	0.55	0.40	0.62	0.25	0.25

Unpacking Focality

- ▶ Munro (wp 1999)
- ▶ Field study of coordination
- ▶ Narrow bike lanes in **Japan**
 - ▶ No center line
- ▶ Two bikes coming from opposite directions
 - ▶ Both ride close to middle
- ▶ How they avoid colliding?
 - ▶ Both move **Left!**

Asymmetric Players: Battle of Sexes

	1	2
1	0, 0	200, 600
2	600, 200	0, 0

- ▶ 100 lottery tickets =
 - ▶ 10% chance to win \$1/\$2
- ▶ Pure NE: (1,2) and (2,1)
 - ▶ Players prefer equilibrium where **they** play strategy 2
- ▶ Mixed NE:
 - ▶ (1/4, 3/4) each
- ▶ Which would you pick?

Asymmetric Players: Battle of Sexes

- ▶ Cooper, DeJong, Forsythe & Ross (AER 90')
- ▶ **BOS**: Baseline (MSE mismatch 62.5%)
- ▶ **BOS-300**: Row player has outside option 300
 - ▶ Forward induction predicts (2,1)
- ▶ **BOS-100**: Row player has outside option 100
 - ▶ Forward induction doesn't apply
- ▶ Compare BOS-100 and BOS-300 shows if "any outside option" works...

Battle of Sexes (Last 11 Periods)

Game	Outside	(1,2)	(2,1)	Other	# Obs
BOS	-	37(22%)	31(19%)	97(59%)	165
BOS-300	33	0(0%)	119(90%)	13(10%)	165
BOS-100	3	5(3%)	102(63%)	55(34%)	165
BOS-1W					165
BOS-2W					165
BOS-SEQ					165

Asymmetric Players: Battle of Sexes

- ▶ Cooper, DeJong, Forsythe & Ross (AER 90')
- ▶ BOS-1W: 1 way communication by Row
- ▶ BOS-2W: 2 way communication by both
- ▶ BOS-SEQ: Both know that Row went first, but Column doesn't know what Row did
 - ▶ Information set same as simultaneous move
 - ▶ Would a sequential move act as an coordination device?

Battle of Sexes (Last 11 Periods)

Game	Outside	(1,2)	(2,1)	Other	# Obs
BOS	-	37(22%)	31(19%)	97(59%)	165
BOS-300	33	0(0%)	119(90%)	13(10%)	165
BOS-100	3	5(3%)	102(63%)	55(34%)	165
BOS-1W	-	1(1%)	158(96%)	6(4%)	165
BOS-2W	-	49(30%)	47(28%)	69(42%)	165
BOS-SEQ	-	6(4%)	103(62%)	56(34%)	165

Where Does Meaning Come From?

- ▶ Communication can help us coordinate
- ▶ But how did the **common language for communication** emerge in the first place?
- ▶ Put people in a situation of **no meaning** and see how they create it!
- ▶ Blume, DeJong, Kim & Sprinkle (AER 1998)
 - ▶ See also BDKS (GEB 2001) which is better!

Evolution of Meaning: Game 1 (Baseline)

	A	B
T1	0, 0	7, 7
T2	7, 7	0, 0

- ▶ Blume et al. (AER 1998)
- ▶ Sender has private type T1 or T2
- ▶ Sends message "*" or "#" to receiver
- ▶ Receiver chooses A or B (to coordinate type)

Evolution of Meaning

- ▶ Blume et al. (AER 1998)
- ▶ **Game 1:** Baseline as above
- ▶ **Game 1NH:** See only history of own match
- ▶ **Game 2:** Receiver can choose C (safe action) that gives (4,4) regardless of T1/T2
 - ▶ Theory: Pooling or Separating Equilibrium

Evolution of Meaning: Game 2

	A	B	C
T1	0, 0	7, 7	4, 4
T2	7, 7	0, 0	4, 4

- ▶ **Game 2:** Receiver can choose C (safe action) that gives (4,4) regardless of T1/T2

Percentage Consistent with Separating

Game \ Period	1	5	10	15	20
1st Session					
Game 1	48	65	74	89	95
2nd Session					
Game 1	49	72	61	89	100
Game 1NH	55	55	28	55	72
Game 2					
Separating	44	88	88	88	94
Pooling	39	05	00	05	05

Evolution of Meaning (Blume et al. AER 1998)

- ▶ **Game 1:** Baseline as above
- ▶ **Game 1NH:** See only history of own match
- ▶ **Game 2:** Receiver can choose C (safe action) that gives (4,4) regardless of T1/T2
 - ▶ Theory: Pooling or Separating Equilibrium
- ▶ **Game 3:** Coordinate payoffs become (2,7)
 - ▶ So sender wants to disguise types to force receiver to choose C (safe action)
 - ▶ Allowed to send 2 or 3 messages...

Evolution of Meaning: Game 3

	A	B	C
T1	0, 0	2, 7	4, 4
T2	2, 7	0, 0	4, 4

► **Game 3:** Coordinate payoffs become (2,7)

Results of Game 3: 2 vs. 3 messages

# of Messages	1-10	11-20	21-30	31-40	41-50	51-60
2-Separating	43	53	38	39		
2-Pooling	33	34	41	43	2 nd Session	
3-Separating	43	38	33	24		
3-Pooling	33	37	42	60		
2-Separating	39	27	23	24	24	23
2-Pooling	39	48	51	60	63	61
3-Separating	23	22	23	25	22	24
3-Pooling	55	61	58	56	57	61
					1 st Session	

Example of Asymmetric Payoffs

- ▶ Market Entry Game
 - ▶ n players decide to enter market with capacity c
 - ▶ Payoffs declines as number of entrants increase;
 < 0 if number $> c$ (= capacity)
- ▶ Kahneman (1988): Number close to equil.
 - ▶ "To a psychologist, it looks like magic."
- ▶ See BI-SAW paper by Chen et al. (2012)...

Market Entry Game Results

Market capacity	1	3	5	7	9	11	13	15	17	19
MSE	0	2.1	4.2	6.3	8.4	10.5	12.6	14.7	16.8	18.9
all data	1.0	3.7	5.1	7.4	8.7	11.2	12.1	14.1	16.5	18.2
1 st block	1.3	5.7	9.7	6.7	3.7	14.0	11.3	11.3	16.0	18.0

- ▶ Sundali, Rapoport and Seal (OBHDP 1995)

Games with Asymmetric Equilibria

	1	2
1	800, 800	800, 0
2	0, 800	1000, 1000

▶ Stag Hunt

- ▶ Cooper et al. (AER 1990)
- ▶ 100 lottery tickets =
 - ▶ 10% chance to win \$1/ \$2
- ▶ Pure NE:
 - ▶ (1,1) & (2,2)
- ▶ Which would you pick?

Games with Asymmetric Equilibria

- ▶ Cooper et al. (AER 1990)
- ▶ **CG**: Baseline Stag Hunt
- ▶ **CG-900**: Row has outside option 900 each
 - ▶ Forward induction predicts (2,2)
- ▶ **CG-700**: Row has outside option 700 each
 - ▶ Forward induction won't work
- ▶ **CG-1W**: 1 way communication by Row
- ▶ **CG-2W**: 2 way communication by both

Stage Hunt (Last 11 Periods)

Game	Outside	(1,1)	(2,2)	Other	# Obs
CG	-	160(97%)	0(0%)	5(3%)	165
CG-900	65	2(2%)	77(77%)	21(21%)	165
CG-700	20	119(82%)	0(0%)	26(18%)	165
CG-1W	-	26(16%)	88(53%)	51(31%)	165
CG-2W	-	0(0%)	150(91%)	15(9%)	165

Weak-link Game


- ▶ Van Huyck, Battalio and Beil (AER 1990)
- ▶ Each of you belong to a team of n players
- ▶ Each of you can choose effort $X_i = 1-7$
- ▶ Earnings depend on your own effort and the smallest effort $\min\{X_j\}$ of your team
 - ▶ Each person has to do his/her job for the whole team project to fly
- ▶ Payoff = $60 + 20 * \min\{X_j\} - 10 * X_i$


Team Project Payoff

Cost of Effort X_i

Weak-link Game: Van Huyck et al. (AER 1990)

▶ Payoff = 60 + 10 * $\min\{X_j\}$ - 10 * $(X_i - \min\{X_j\})$

Team Minimum 

Deviation from Min 

Your X_i	Smallest X_j in the team						
	7	6	5	4	3	2	1
7	130	110	90	70	50	30	10
6	-	120	100	80	60	40	20
5	-	-	110	90	70	50	30
4	-	-	-	100	80	60	40
3	-	-	-	-	90	70	50
2	-	-	-	-	-	80	60
1	-	-	-	-	-	-	70

Weak-link Game: Van Huyck et al. (AER 1990)

- ▶ What is your choice when...
 - ▶ Group size = 2?
 - ▶ Group size = 3?
 - ▶ Group size = 20?

- ▶ Can some kind of communication help coordinate everyone's effort?

Classroom Experiment: 害群之馬

最弱環節賽局
(Weak-Link Game)

Weak-Link Game (最弱環節賽局)

- ▶ Each DM chooses effort $X=1-4$
 - ▶ Spade = 4, Heart = 3, Diamond = 2, Club = 1
- ▶ DM (Decision Maker) = a team of two
 - ▶ 每組每回合都會有四張撲克牌，分別為黑桃(4)、紅心(3)、方塊(2)、梅花(1)
 - ▶ 主持人會跟每組收一張牌
 - ▶ 交出來的花色代表你們花多少時間排練
 - ▶ 你們的努力程度：黑桃 = 4小時、紅心 = 3小時、方塊 = 2小時、梅花 = 1小時
 - ▶ 各組要討論屆時交出哪一張牌...

Payoff Calculation (記分方式)

$$\text{Payoff} = 3 * \min\{X_j\} - 1 * X_i$$

Team Project Payoff

Cost of Effort X

- 「花最少時間排練那一組的排練時數」，每一小時的排練大家都會得到3分。各組自己每花一小時排練，就少1分。

Your X_i (本組時數)	$\min\{X_j\}$ (最低那組時數)			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

Payoff Calculation (記分方式)

1. How much would you earn if all DM choose $X=4$?

▶ 8!

▶ 如果所有各組都花四小時排練，這樣各組會拿幾分？8分！

Your X_i (本組時數)	$\min\{X_j\}$ (最低那組時數)			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

Payoff Calculation (記分方式)

2. How much would you earn if you choose $X=3$ while others choose $X=4$?

▶ 6 (< 8 , not worth it!)

▶ 如果別組都花四小時排練，但你們這組只花三小時排練，這樣你們會拿幾分？你們這麼做值得嗎？6分！小於8分所以不值得！

Your X_i (本組時數)	$\min\{X_j\}$ (最低那組時數)			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

Payoff Calculation (記分方式)

3. How much would you earn if you choose $X=2$ while some other DM choose $X=1$?
- ▶ 1 (< 2 , if you also choose $X=1$!)
 - ▶ 如果有某一組只花一小時排練，你們這組如果花兩小時排練，值得嗎？不值得，因為只得1分，但如果也花一小時就會跟他們一樣得到2分！

Your X_i (本組時數)	$\min\{X_j\}$ (最低那組時數)			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

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▶ Please decide now and we will see the results...

6. Are you satisfied with the results? How can you encourage cooperation next time?

▶ 你對結果滿意嗎？如果你希望大家都更好，該怎麼鼓勵大家合作？讓我們再來做一次...

Your X_i (本組時數)	$\min\{X_j\}$ (最低那組時數)			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

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- ▶ In reality, people would see each other's effort and increase effort gradually
- ▶ Let's try again by committing hour-by-hour!
 - ▶ 現實中你們彼此多半清楚大家的排練情況，而且時數可以逐步加碼。這次我們採一小時、一小時逐步加碼方式進行

本組排練時數	最低那組排練時數			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

Weak-link Game: Van Huyck et al. (AER 1990)

► Payoff = $60 + 10 * \min\{X_j\} - 10 * (X_i - \min\{X_j\})$

Your X_i	Smallest X_j in the team						
	7	6	5	4	3	2	1
7	130	110	90	70	50	30	10
6	-	120	100	80	60	40	20
5	-	-	110	90	70	50	30
4	-	-	-	100	80	60	40
3	-	-	-	-	90	70	50
2	-	-	-	-	-	80	60
1	-	-	-	-	-	-	70

Weak-link Game: Large Group ($n=14-16?$)

- ▶ Start at $X_i = 4-7$, but quickly drop to $X_i = 1-2!$

X_i	Period									
	1	2	3	4	5	6	7	8	9	10
7	33	13	9	4	4	4	6	3	3	8
6	10	11	7	0	1	2	0	0	0	0
5	34	24	10	12	2	2	24	1	0	1
4	17	23	24	18	15	5	3	3	2	2
3	5	18	25	25	17	9	8	3	4	2
2	5	13	17	23	31	35	39	27	26	17
1	2	5	15	25	37	50	47	70	72	77

Weak-link Game: Large Group (Extensions)

- ▶ No penalty above min: 83% choose 7 in 1st period
- ▶ Show effort distribution: Accelerate race to bottom

X_i	Period									
	1	2	3	4	5	6	7	8	9	10
7	33	13	9	4	4	4	6	3	3	8
6	10	11	7	0	1	2	0	0	0	0
5	34	24	10	12	2	2	24	1	0	1
4	17	23	24	18	15	5	3	3	2	2
3	5	18	25	25	17	9	8	3	4	2
2	5	13	17	23	31	35	39	27	26	17
1	2	5	15	25	37	50	47	70	72	77

Weak-link Game: Small Group ($n=2$)

- ▶ Start at $X_i = 1$ or 7, but quickly converge to $X_i = 7$!
- ▶ Wait a couple periods for partner if choose $X_i = 7$.

X_i	Period						
	1	2	3	4	5	6	7
7	9	13	13	17	19	19	21
6	0	1	4	2	1	1	0
5	4	1	1	1	0	0	0
4	0	1	2	0	1	1	0
3	1	2	1	1	0	0	0
2	1	2	0	0	0	0	1
1	8	4	3	3	3	3	2

Weak-link Game: Small Group (Extension)

- ▶ Random Matching:
 - ▶ Start high (4-7), but drop to 1!
 - ▶ Small group size not enough to induce good equilibrium!
 - ▶ Clark and Sefton (wp 1999)
 - ▶ Replicate random matching results in stag hunt
 - ▶ Still unpublished as of 2021...
- (See how difficult to publish replications...Orz)

Weak-link Game: Group Size (various papers)

- ▶ Table 7.27 (Camerer, BGT 2003)
- ▶ Group size ≥ 6 :
 - ▶ 1st period $\min\{X_j\} \leq 4$
 - ▶ 5th period $\min\{X_j\}$ mostly 1
- ▶ Group size 2-3:
 - ▶ 1st period $\min\{X_j\}$ is 5-7
 - ▶ 5th period $\min\{X_j\}$ mostly (86%) reaches 7 if $n=2$
- ▶ But 1st period median $X_i = 4-5$ for all n !
 - ▶ Why? Maybe subjects think they play against representative opponent (and clone for large n)