Learning: Reinforcement, Fictitious Play and EWA
學習理論：制約、計牌與EWA

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Lecture 9, EE-BGT

Outline: Learning

1. Learning: What you do after you see "results"...
2. What we know now: (various learning rules)
    1. Reinforcement
    2. Belief learning
    3. EWA: a hybrid of reinforcement and belief learning
    4. Others: Evolutionary, anticipatory learning, imitation, learning direction theory, rule learning,...
3. Further research:
    1. Beyond: New direction for research in learning
    2. Application: How can we use these tools?

What you do after you see…

- Suppose you are playing Stag Hunt
- (B, L) happened last time
- What would you do now?

- Change strategy?
- Stick to it?

	L	R
T	3, 3	0, 1
B	1, 0	1, 1

What you do after you see...

- A robot (pre-programmed) would stick to it
  - Evolutionary approach
- But humans think twice
- How would you switch?
- Reinforcement:
  - Choices "reinforced" by previous payoffs
  - "Very bad" reasoning

	L	R
T	3, 3	0, 1
B	1, 0	1, 1

Reinforcement Learning

- Update attractions (tendency to play a certain strategy) after given history
- Reinforcement:
  - Choices "reinforced" by previous payoffs
  - Allow spillovers to "neighboring strategies" ϵ
- Example: (cumulative reinforcement)

$$\underline{\underline{A^B(t)}} = \varphi \underline{\underline{A^B(t-1)}} + (1-\epsilon)\cdot \boxed{1}$$

$$\underline{\underline{A^T(t)}} = \varphi \underline{\underline{A^T(t-1)}} + \epsilon \cdot \boxed{1}$$

Reinforcement Learning

- (More General) Cumulative Reinforcement:

$$\underline{\underline{A^B(t)}} = \varphi \underline{\underline{A^B(t-1)}} + (1-\epsilon) \cdot 1 \boxed{\cdot [1 - \rho(t-1)]}$$
$$\underline{\underline{A^T(t)}} = \varphi \underline{\underline{A^T(t-1)}} + \epsilon \cdot 1 \boxed{\cdot [1 - \rho(t-1)]}$$

- Alternatively,
- Weighted Average Reinforcement

$$\underline{\underline{A^B(t)}} = \varphi \underline{\underline{A^B(t-1)}} + \boxed{(1-\varphi)} \cdot (1-\epsilon) \cdot 1$$
$$\underline{\underline{A^T(t)}} = \varphi \underline{\underline{A^T(t-1)}} + \boxed{(1-\varphi)} \cdot \epsilon \cdot 1$$

What "else" could you do...

- Would you update your beliefs about what others do"?
  - Belief learning models
- Fictitious Play
  - Keep track of frequency
  - Ex: rock-paper-scissors
- Cournot Best-Response
  - What you did last time is what you'll do now

	L	R
T	3, 3	0, 1
B	1, 0	1, 1

Weighted Fictitious Play

- Other weights? Weighted fictitious play
  - Fictitious play: weigh all history equally ($\rho = 1$)
  - Cournot: focus only on the last period ($\rho = 0$)
- Prior:
  - $P_{t-1}(L) = 3/5$, $P_{t-1}(R) = 2/5$
- Posterior:
  - $P_t(L) = (3\rho + 1) / (5\rho + 1)$
  - $P_t(R) = (2\rho + 0) / (5\rho + 1)$
  - ρ = decay factor

Weighted Fictitious Play

- Posterior:
  - $P_t(L) = (3\rho + 1) / (5\rho + 1)$
  - $P_t(R) = (2\rho + 0) / (5\rho + 1)$
- Use this belief to compute payoffs and use them as attractions:
  - $A^T(t) = [\, 3\,(3\rho + 1) + 0\,(2\rho + 0)\,] / (5\rho + 1)$
  - $A^B(t) = [\, 1\,(3\rho + 1) + 1\,(2\rho + 0)\,] / (5\rho + 1)$
- Note: Actually payoff received play no role

Could you being doing both?

- Reinforcement does not update beliefs
  - But people do update!
- Fictitious play doesn't react to actual payoffs
  - But people do respond
- EWA: a hybrid of two
  - Camerer and Ho (Econometrica, 1999)

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Experience-Weighted Attraction

- $N(t)$: Experience Weight (weakly increasing)

$$N(t) = \varphi(1-\kappa)N(t-1) + 1, N(t) \leq \frac{1}{1-\varphi(1-\kappa)}$$

- Attraction (for chosen action B)

$$A^B(t) = [\varphi N(t-1) A^B(t-1) + 1]/N(t)$$

- For unchosen action T, add δ:
  - Weight players give to foregone payoffs from unchosen strategies
  - Law of effect vs. Law of simulated effect

$$A^T(t) = [\varphi N(t-1) A^T(t-1) + 3\underline{\underline{\delta}}]/N(t)$$

EWA Special Case: Reinforcement

- $A^B(t) = [\varphi N(t-1) A^B(t-1) + \pi(B, L)]/N(t)$
- $A^T(t) = [\varphi N(t-1) A^T(t-1) + \pi(T, L)\underline{\underline{\delta}}]/N(t)$

  where $N(t) = \varphi(1-\kappa) N(t-1) + 1$
- Becomes Reinforcement if $\delta = 0, N(0) = 1$
- (Simple) cumulative reinforcement: $\kappa = 1$
  - $N(t) = 1$ for all t
- (Weighted) average reinforcement: $\kappa = 0$
  - Weights are $\frac{\varphi}{\varphi+1}$ and $\frac{1}{\varphi+1}$

EWA Special Case: W. Fictitious Play

- $A^B(t) = [\varphi N(t-1) A^B(t-1) + \pi(B, L)] / N(t)$
- $A^T(t) = [\varphi N(t-1) A^T(t-1) + \pi(T, L)\underline{\underline{\delta}}] / N(t)$

  where $N(t) = \varphi(1-\kappa) N(t-1) + 1$
- Weighted Fictitious Play if $\delta = 1, \kappa = 0$
  - Good Homework exercise...
  - Hint: Since $N(t) = 1 + \varphi + \varphi^2 + \cdots + \varphi^{t-1}$
  - Posterior is

$$P_t(L) = \frac{I(L, h(t)) + (\varphi + \cdots \varphi^{t-1}) \cdot P_{t-1}(L)}{1 + \varphi + \cdots \varphi^{t-1}}$$

EWA Special Case: W. Fictitious Play

- $A^B(t) = [\varphi N(t-1) A^B(t-1) + \pi(B, L)]/N(t)$
- $A^T(t) = [\varphi N(t-1) A^T(t-1) + \pi(T, L)\underline{\underline{\delta}}]/N(t)$

  where $N(t) = \varphi(1-\kappa) N(t-1) + 1$
- Weighted Fictitious Play if $\delta = 1, \kappa = 0$
  - Fictitious Play: $\varphi = 1$
  - Cournot best-response: $\varphi = 0$

EWA Cube: Camerer, Wang, Ho (EJ 08) vs. Wang, Knoepfle, Camerer (JEEA 09)

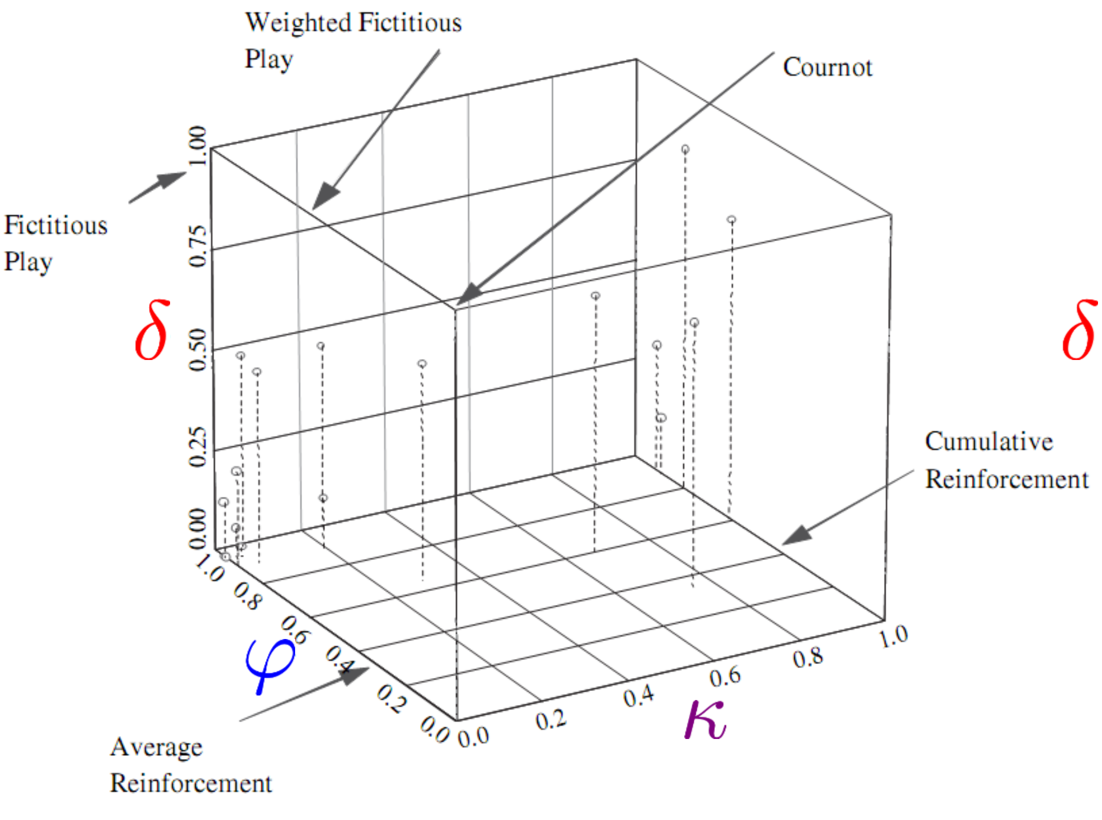
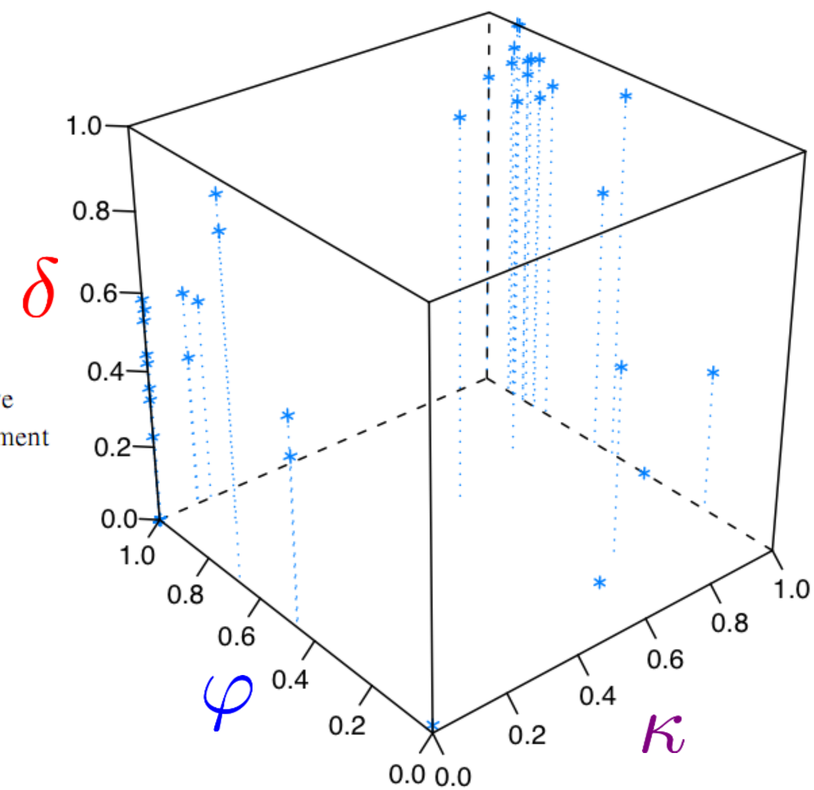


Fig. 2. *EWA's Model Parametric Space*

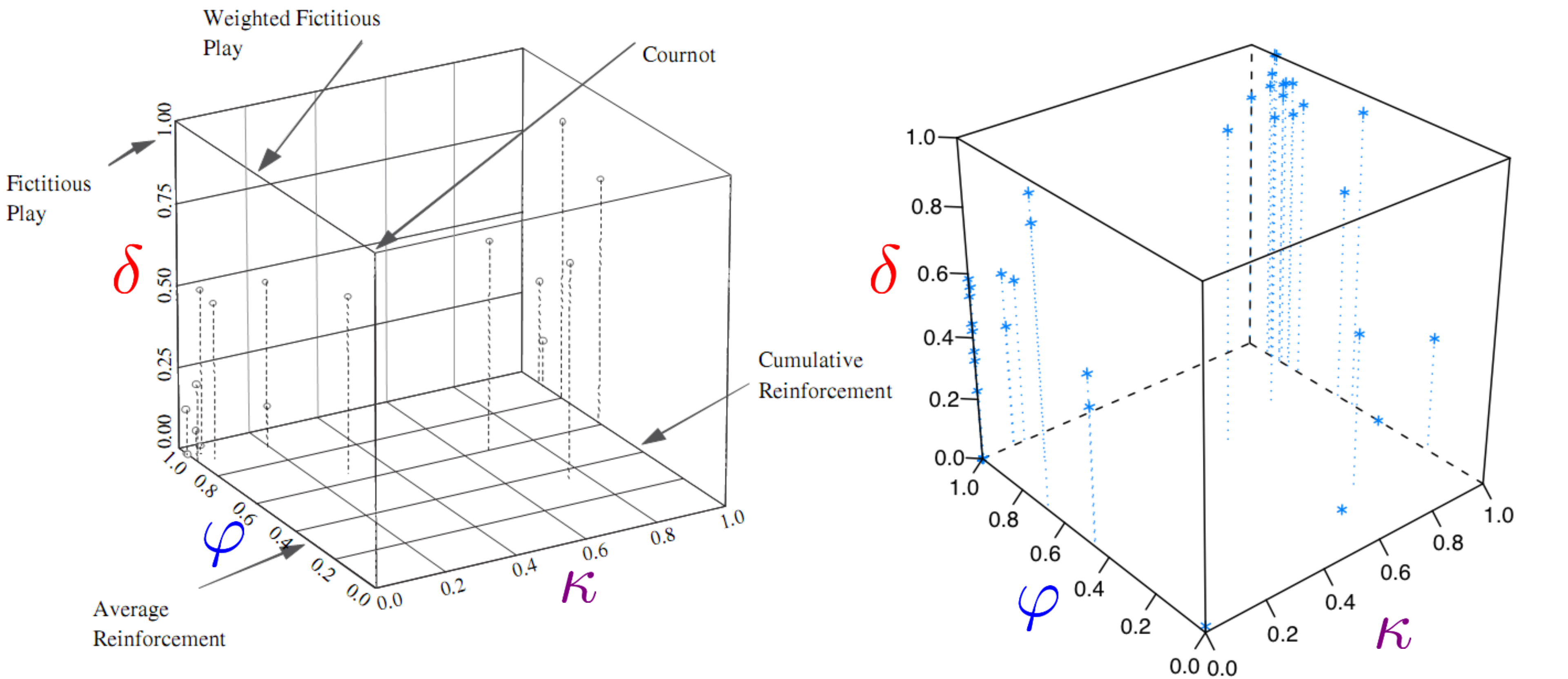


- δ: Attraction weight on foregone payoffs
- φ: Decay of previous attractions; κ: Growth rate of attractions

Interpretation of EWA Parameters

- δ: Attraction weight on foregone payoffs
  - Diff. between received vs. opportunity gains
- κ: The rate attractions grow
  - Cumulative vs. Average
- $N(t)$: The strength of initial attractions
  - (in units of "experience-equivalence")
- φ: Weight in $N(t)$
  - Decay of previous attractions

EWA Cube: Camerer, Wang, Ho (EJ 08) vs. Wang, Knoepfle, Camerer (JEEA 09)



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Prediction Power of EWA

- EWA generally improves accuracy in about 35 games (except for mixed ones)
  - See Camerer and Ho (book chapter, 1999)
  - "Long version" of the Econometrica paper?
- BGT, Ch. 6 provides two examples:
  - Continental Divide
  - p-Beauty Contest

Prediction Power of EWA

- Overfitting: Too many parameters?
- Can be tested:
  - LR test: Restricted fit vs. Unrestricted
- Better Out-of-sample Prediction Power:
  - Estimate parameters and predict "new data"
  - Not prone to overfitting (because of new data)
- 1-parameter self-tuned EWA works too:
  - This EWA-Lite does as good as reinforcement or fictitious play, even on data with new games

Other Learning Rules

- Other rules include:
- Anticipatory learning (Sophistication):
  - Sophisticated players are aware that others are learning – BR to Cournot, etc. (level-k)
  - Soph. EWA: Camerer, Ho, Chong (JET 2002)
- Imitation: Imitate average or "best" player
- Learning direction theory: Move toward BR
- Rule learning: Learn which "rule" to use
  - Stahl (GEB 2000)

Further Research

- Here is where we stand.
- Are there new direction for research in learning?
  - How does information acquisition help us study how people learn?
  - Learning direction theory and imitation are still loose ends to be filled

Holy Grail: How do people "actually" learn?

Further Research

- How can we use these tools?
- Econometric Properties of learning rules:
  - Salmon (Econometrica 2001): Simulate data via certain learning rules and estimate them
  - Identification is bad for mixed strategy equilibrium and games with few strategies
  - EWA estimation does well on δ; others okay only for 1000 periods (but not 30 periods)
- Can use this to test designs

Conclusion

- Learning: How people react to past history
- Reinforcement
- Belief Learning
  - Fictitious play, Cournot, etc.
- EWA: a Hybrid model
  - Performs better even out-of-sample
- Design tests: simulate and estimate