Midterm Exam for Experimental Economics I (Spring 2018)

Note: You have 180 minutes (1:20-4:20pm) and there are 138 points; allocate your time wisely.

Part A: Ultimatum Games (29 pts)

Paul the Proposer and Rachael the Respondent divide $10. Paul proposes how to split the money between the two of them, and Rachael decides to accept or reject. If Rachael accepts, the money is divided accordingly; if Rachael rejects, both earn zero. Find the SPE when the set of possible offers is: (P stands for the Proposer, R stands for the Respondent)

a. (10 pts) $A_p = \{$(P, R): (9.99, 0.01), (9.98, 0.02), (9.97, 0.03), … , (0.01, 9.99)$\}$.

b. (10 pts) $A_p = \{$(P, R): (10, 0), (9, 1), (8, 2), … , (0, 10)$\}$.

c. (9 pts) What do you think would happen when real people play this game?

Part B: Public Goods Game (31 pts)

There are N players, and each choose to invest c_i from their personal endowment e_i. Total investment, c_{all} = sum of c_i, is then multiplied by m and divided among all players. In other words, payoffs are $M = e_i - c_i + m * c_{all} / N$. What is the Nash Equilibrium of this game?

Part C: Dirty Face Game (37 pts)

Two agents each has a probability of 0.8 to be type X; 0.2 to be type O. They can only see the other person's type and are commonly told that at least one of them is type X. Both agents simultaneously choose Up (don't know) or Down (I am type X). If anyone chooses Down, the game ends. If nobody chooses Down, they will observe the other's choice and play again. Consider the cases below:

a. (13 pts) One is type X, and the other is type O. What would the SPE outcome be?

b. (13 pts) Both players are type X. What would the SPE outcome be?

c. (11 pts) What do you think would happen when real people play the games described in (a) and (b)?

Type		X	O
Probability		0.8	0.2
Action	Up	$0	$0
	Down	$1	-$5

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Part D: Legislative Bargaining (41 pts)

Consider the Baron-Ferejohn bargaining game with N=3 players. Three players bargain over how to divide a dollar and we assume closed-rule bargaining. In each round, a player is randomly selected to make a proposal and then majority voting among the players decides whether to accept the proposal or not. If accepted, the players receive their shares according to the proposal and the game ends. If rejected, discounting occurs according to a common discount factor δ and the process repeats itself with a newly selected proposer in the new round.

a. (8 pts) Explain the concept of stationary equilibrium in this game. You can take an example of (non)-stationary equilibrium.

b. (10 pts) Suppose now that every player is equally likely to be a proposer in each round. Write an equation for continuation value and solve for it. What is proposer share?

c. (13 pts) Suppose now that there is a player who is less likely to be a proposer than the other two players (in every round). Find an (stationary) equilibrium in which the player with smaller probability of being selected as a proposer has a smaller continuation value, by writing equations for continuation values. What is the condition on the parameter for this equilibrium?

d. (10 pts) In the asymmetric proposal power setting in part (c), find an equilibrium in which every player has the same continuation value.

Suggested Answer for the Midterm (Spring 2018).

Part A: See Past Exam of 2015

Part B: Let $C_{all} = \underbrace{C_i}_{\text{a person's investment}} + \underbrace{\sum_{j \neq i} C_j}_{\text{the sum of others' investment}}$

then
$$M(C_i) = e_i - C_i + \frac{m}{N}\left(C_i + \sum_{j \neq i} C_j\right)$$
$$= e_i + \left(\frac{m}{N} - 1\right) C_i + \frac{N}{N}\left(\sum_{j \neq i} C_j\right)$$

$$\Rightarrow \frac{\partial M(C_i)}{\partial C_i} = \left(\frac{m}{N} - 1\right)$$

There are three cases:

$\left(\frac{m}{N} - 1\right)$

(1) $> 0 \Rightarrow m > N$

(10) $\Rightarrow$ $M(C_i)$ is increasing in C_i

the more a person invests, the more he gains. Finally, he invests all his money $e_i \Rightarrow$ $C_i = e_i$ is N.E.

(2) $< 0 \Rightarrow m < N \Rightarrow$ $C_i = 0$ is N.E.,

(10 pts)

(3) $= 0 \Rightarrow m = N$

(11 pts) $\Rightarrow$ No matter how much a person invests, the payoff won't change.

Therefore, $C_i \in [0, e_i]$ can support an Equilibrium.

Part C

a. In the first round, Type X sees the other's type, knowing he is the only X. Since \$1 (Action Down) > \$0 (Action Up). Type X chooses Down.

In terms of Type O, he sees the other's type is X. he could be X with prob. 0.8, O w/p 0.2.

$$E[\text{Down}] = 0.8 \times 1 + 0.2 \times (-5) = -0.2$$
$$\wedge \quad (6\text{pts})$$
$$E[\text{Up}] = 0.8 \times 0 + 0.2 \times (0) = 0$$

Type O's best choice is Up.

Since Type X chooses down, the game ends at the first round.

SPE : (Down, Up) (7pts)
type X — type O

b. Since both players are type X, similar to a., they see the other's type is X, he could be X (p=0.8). O (p=0.2). Both of them choose Up in the first round. (6pts)

In the second round, knowing that the result of the first round is (Up. Up), they realize they both are type X through the previous action of his opponent. Therefore, they both chouse Down in the second.

SPE : (Up. Up) → (Down. Down) (7pts)

c. (11pts) You should discuss the possibility that people in the real life can use the reasoning skills in (a) and (b), including calculation of expected value and level-k thinking

(a) Stationary equ'm in BF bargaining implies:

① A proposer proposes the same division every time s/he is selected as a proposer regardless of the history of the game; and

② Voters vote only on the basis of the current propos[al] and expectations about future proposals, ~~why~~ which by ①, have the same distribution of outcomes in each period.

* Hence if you assign zero to someone who previously assigned you zero as a proposer, this strategy woul[d] not constitute a stationary equilibrium.

(b) Assume symmetric equ'm: hence everyone has the same continuation value $v_i = v \ \forall i$.

Any voter i accepts a proposal that gives as much as $x_i \geq \delta v$, and by majority rule, a proposer gives a positive share to only one of the remaining members: hence a proposer share $z = 1 - \delta v$.

Now the continuation value v is the expected value of the game starting next period:

$$v = \frac{z}{3} + \frac{2}{3}\cdot\frac{1}{2}\delta v = \frac{z}{3} + \frac{\delta v}{3} = \frac{1-\delta v}{3} + \frac{\delta v}{3} = \frac{1}{3}$$

$$\Rightarrow \quad z = 1 - \frac{\delta}{3} \quad \text{proposer share}$$

(c) Assume two members are selected as a proposer with prob. $p > \frac{1}{3}$, and the remaining member, with prob $q < \frac{1}{3}$.

Assume two continuation values, v_A for those w/ p and v_B for those w/ q; and conjecture $v_A > v_B$

The immediate consequence of this conjecture is that the one w/ v_B 大 has priority to be included in any coalition, hence proposer shares

$$z_A = 1 - \delta v_B \qquad z_B = 1 - \delta v_A$$

~~p=~~ $q = 1 - 2p$

$\Rightarrow p = \frac{1}{2} - \frac{q}{2}$

$$\Rightarrow \quad v_A = p z_A + q \cdot \frac{1}{2}\delta v_A = \left(\frac{1}{2} - \frac{q}{2}\right)(1 - \delta v_B) + \frac{q}{2}\delta v_A$$

$$v_B = q z_B + (1-q)\delta v_B = q(1 - \delta v_A) + (1-q)\delta v_B$$

$$\Rightarrow \quad 2v_A = (1-q) - (1-q)\delta v_B + q\delta v_A$$

$$v_B = q - q\delta v_A + (1-q)\delta v_B$$

Adding the two equtions, we obtain

$$2v_A + v_B = 1 - q + q = 1 \quad \text{or} \quad v_B = 1 - 2v_A$$

Substituting for v_B, $\begin{cases} v_A = \dfrac{(1-q)(1-\delta)}{2 + q\delta - 2\delta} \\ v_B = \dfrac{q(2-\delta)}{2 + q\delta - 2\delta} \end{cases}$ ✓

We finally verify the condition for $v_A > v_B$, which is equivalent to: $q < \dfrac{1-\delta}{3 - 2\delta}$. ✓

(d) We also consider $\dfrac{1-\delta}{3-2\delta} \le q < \dfrac{1}{3}$ and now conjecture $v_A = v_B$. Being ~~indiff~~ indifference, the proposer w/ v_A chooses the member w/ v_A w/ prob x and the member w/ v_B w/ prob $1-x$ (a mixed strategy). Hence the proposer share $z_A = 1 - \delta v_A$ w/ prob x and $z_A = 1 - \delta v_B$ w/ prob $1-x$; and $z_B = 1 - \delta v_A$.

$$v_A = p[x(1 - \delta v_A) + (1-x)(1 - \delta v_B)] + p x \delta v_A + \frac{q}{2}\delta v_A$$

$$v_B = q(1 - \delta v_A) + 2p(1-x)\delta v_B$$

$p = \frac{1}{2} - \frac{q}{2}$

Letting $v_A = v_B = v$ and solve for v, x:

$$v = p(1-\delta v) + px\delta v + \frac{q}{2}\delta v$$
$$v = q(1-\delta v) + 2p(1-x)\delta v$$

$$\Rightarrow v = \tfrac{1}{2}(1-q)(1-\delta v) + \tfrac{1}{2}(1-q)x\delta v + \frac{q}{2}\delta v$$
$$v = q(1-\delta v) + (1-q)(1-x)\delta v$$

$$\Rightarrow 2v = (1-q)(1-\delta v) + (1-q)x\delta v + q\delta v$$
$$+\ v = q(1-\delta v) + (1-q)(1-x)\delta v$$

$$3v = 1 - \delta v + (1-q)\delta v + q\delta v$$
$$= 1 - \delta v + \delta v = 1$$

$$\therefore v = \frac{1}{3}, \quad x = \frac{q(3-2\delta) - (1-\delta)}{(1-q)\delta}$$

Here, $q < \frac{1}{3}$ implies $q(3-2\delta) - (1-\delta) < (1-q)\delta$.