Homework 7: optional

(Political Economy: Spring 2018)

1. Verify the following two (approximate) equalities.

   (1)
$$\frac{\Pr[Piv_A|\alpha] + \Pr[Piv_B|\alpha]}{\Pr[Piv_A|\beta] + \Pr[Piv_B|\beta]} \approx \frac{e^{2n\sqrt{r(1-r)}}}{e^{2n\sqrt{s(1-s)}}} \times K(r,s)$$

   (2)
$$\frac{\Pr[Piv_A|\alpha]}{\Pr[Piv_A|\beta]} = \frac{e^{-n(1-r)p_b}}{e^{-nsp_b}} \times \frac{1+n(1-r)p_b}{1+nsp_b}$$

2. Consider a jury model with $n = 3$ voters and equal prior $\pi = 1/2$. Also assume asymmetric signal precision;

$$\Pr[\theta_i = 1|\omega = G] = p, \quad \Pr[\theta_i = 0|\omega = I] = q$$

   where $1 > p > q > 1/2$. We have utility $u(c|G) = u(a|I) = 1$ and $u(c|I) = u(a|G) = 0$. Each voter has a voting cost drawn (independently) from the uniform distribution on $[0,1]$, hence $F(x) = x$. You are asked to find a sincere voting equilibrium with voluntary participation in this environment.

   (1) Write $x = p_1$, $y = p_0$ for the participation rates of those with signal $i = 0, 1$. Assuming sincere voting, derive expressions for $\Pr[c|\omega]$, $\Pr[a|\omega]$ and also for $\Pr[T|\omega]$, $\Pr[T_{-1}|\omega]$, $\Pr[T_{+1}|\omega]$, $\omega = G, I$, where the events T, $T_{\pm 1}$ are as defined in class. *Hint.* For a fixed number of voters, the probability of the event $(k,l)|\omega$, meaning k votes for "c" and l votes for "a" in state ω, is given by the multinomial probability

$$\binom{n}{k+l}\binom{k+l}{k} \Pr[c|\omega]^k \Pr[a|\omega]^l (1 - \Pr[c|\omega] - \Pr[a|\omega])^{n-k-l}$$

   where $n - k - l = \#$ abstainers.

   (2) Combine the probabilities in part (1) to write the pivot probabilities $\Pr[Piv_c|\omega]$, $\Pr[Piv_a|\omega]$, $\omega = G, I$, and state the individual rationality conditions (IR_1), (IR_0) for the signals $i = 0, 1$ in terms of these pivot probabilities.

   (3) Assume $p = 0.9$, $q = 0.6$ and solve the two equations (IR_1), (IR_0) for the equilibrium participation rates x^*, y^* (by using some mathematics package such as Mathematica or Matlab). Are these participation rates strictly positive? Which one is greater between x^* and y^*?

   (4) Finally, state the incentive compatibility conditions (IC_1), (IC_0) for signals $i = 0, 1$ and verify that the above equilibrium participation rates x^*, y^* in part (3) indeed satisfy these incentive compatibility conditions.