# Experimental Economics l – Midterm Quiz Fall 2015

Exam Time: 10/26 2:20pm – 5:20pm. You have 3 hours; allocate your time wisely.

**Part A (15 points): The Professor-Student Problem**

Consider the relationship between Professor Joseph and Student Yu. Professor Joseph has expected utility function satisfying $u'(x) = x^{-R}$ where $R > 0$, while Student Yu has expected utility function satisfying $v'(x) = x^{-r}$ with $r < R$. Consider the ten lottery choices of Holt and Laury (2002) listed below:

You will roll a ten-sided die and get paid according to your decision (choice A or B):

| **Decision** | Lottery A | Lottery B | Your choice (A or B) |
|---|---|---|---|
| Question 1 | 1：Gain NT$200<br>2～10：Gain NT$160 | 1：Gain NT$385<br>2～10：Gain NT$10 | |
| Question 2 | 1～2 ：Gain NT$200<br>3～10：Gain NT$160 | 1～2 ：Gain NT$385<br>3～10：Gain NT$10 | |
| Question 3 | 1～3 ：Gain NT$200<br>4～10：Gain NT$160 | 1～3 ：Gain NT$385<br>4～10：Gain NT$10 | |
| Question 4 | 1～4 ：Gain NT$200<br>5～10：Gain NT$160 | 1～4 ：Gain NT$385<br>5～10：Gain NT$10 | |
| Question 5 | 1～5 ：Gain NT$200<br>6～10：Gain NT$160 | 1～5 ：Gain NT$385<br>6～10：Gain NT$10 | |
| Question 6 | 1～6 ：Gain NT$200<br>7～10：Gain NT$160 | 1～6 ：Gain NT$385<br>7～10：Gain NT$10 | |
| Question 7 | 1～7 ：Gain NT$200<br>8～10：Gain NT$160 | 1～7 ：Gain NT$385<br>8～10：Gain NT$10 | |
| Question 8 | 1～8 ：Gain NT$200<br>9～10：Gain NT$160 | 1～8 ：Gain NT$385<br>9～10：Gain NT$10 | |
| Question 9 | 1～9 ：Gain NT$200<br>10：Gain NT$160 | 1～9 ：Gain NT$385<br>10：Gain NT$ 10 | |
| Question 10 | 1～10：Gain NT$200 | 1～10：Gain NT$385 | |

1. (5 pt) Show that both Professor Joseph and Student Yu exhibit constant relative risk aversion. Hence or otherwise, solve for their Von Neumann-Morgenstern utility functions $u(.)$, $v(.)$, and corresponding degree of relative risk aversion $R(x)$.

2. (5 pt) Show that a risk neutral person would choose lottery A for Questions 1~4 and lottery B otherwise.
3. (5 pt) Would Professor Joseph choose more or less lottery A's than a risk neutral person? Why or why not? What about Student Yu (compared to a risk neutral person and/or to Professor Joseph)?

**Part B (15 points): Ultimatum Games**

Paul the Proposer and Rachael the Respondent divide $10. Paul proposes how to split the money between the two of them, and Rachael decides to accept or reject. If Rachael accepts, the money is divided accordingly; if Rachael rejects, both earn zero. Find the SPE when the set of possible offers is:

a. (5 pt) $A_P$ = {(P, R): (9.99, 0.01), (9.98, 0.02), (9.97, 0.03), …, (0.01, 9.99)}.
b. (5 pt) $A_P$ = {(P, R): (10, 0), (9, 1), (8, 2), …, (0, 10)}.
c. (5 pt) What do you think would happen when real people play this game?

**Part C (15 points): Matching Pennies Games**

Find all Nash equilibria in Ochs (1995b)'s three matching pennies games:

| | H | T |
|---|---|---|
| H | **1**, 0 | 0, 1 |
| T | 0, 1 | 1, 0 |

| | H | T |
|---|---|---|
| H | **9**, 0 | 0, 1 |
| T | 0, 1 | 1, 0 |

| | H | T |
|---|---|---|
| H | **4**, 0 | 0, 1 |
| T | 0, 1 | 1, 0 |

**Part D (20 points): The Sleeping Game**

Read the (abridged) article below and answer the following questions:

1. (5 pt) Consider the following game played between the two sleepy pilots: Each pilot chooses to either sleep or stay awake. Falling asleep gives the sleepy pilot some rest, which is worth NT$2,000 to each pilot. The plane flies safely if at least one pilot to stay awake, which is worth NT$10,000 to each pilot. If both pilots fall asleep, the plane would be in danger, which would cost the pilot NT$100,000 each. Draw the game matrix (assuming each pilot only cares about the sum of their own payoffs).
2. (5 pt) Is it consistent with equilibrium for both pilots to stay awake? Why or why not?
3. (5 pt) Solve for all of the pure and mixed Nash equilibrium of this game.
4. (5 pt) Which equilibrium could result in the case described in the news below where both pilots fall asleep despite FAA forbidding pilots sleeping? Which equilibrium corresponds to the case where one pilot tells the other s/he is going to rest for a while? Which one has a higher monetary payoff for the pilots?

## Both pilots "slept" for an HOUR as packed airliner flew on for 600 miles

2009/10/23 by Paul Thompson for "MailOnline"

An airliner with 144 passengers onboard flew 600 miles as its pilots slept at the controls for more than an hour, it was claimed yesterday. The pilot and co-pilot are believed to have nodded off as the plane flew on autopilot at 37,000ft, causing it to overshoot its destination by 150 miles. In the meantime, F-16 fighter jets were readied to escort or even shoot down the jet over fears it had been hijacked.

After a tense hour and 18 minutes of radio silence contact was finally made - and the Airbus A320 from San Diego in the U.S. landed safely in Minneapolis, with its passengers blissfully unaware of the drama. The pilots insisted they had been engaged in a "heated" argument about airline policy. By the time their "discussion" had ended - or they had woken up - the plane was 150 miles off course and flying over the neighbouring state of Wisconsin. Instead of being above Minneapolis, population 380,000, it was instead over the town of Eau Claire, population 61,000. When it eventually landed at Minneapolis police and FBI agents stormed the plane, ordering passengers to remain in their seats while they went to the cockpit.

Two pilots are being investigated after they overshot the airport by 150 miles. Officials said controllers had tried repeatedly to make contact with Northwest Airlines Flight 188. Airline officials had also tried to raise them through a secure link to the cockpit, but had failed. The U.S. National Transportation Safety Board has admitted it is looking into pilot fatigue as the reason for the overshoot on Wednesday.

… … (omitted) … …

Since the FAA does not allow pilots to sleep, some of them may think, "Since my companion will not sleep, it is okay for me to take a nap." However, if both of them think so, they would both fall asleep. Experts said that many countries allow pilots to take a nap on strict premises such as starting autopilot, not leaving the seats, informing the flight attendants, and so on. The rationale of this policy is: "Knowing the other is sleeping, one definitely dares not sleep making it safer." It is said that a Taiwanese pilot had told his co-pilot that he wanted to take a nap for a while, and the other stayed extremely concentrated to have a safe flight.

(This paragraph is translated from "美客機「睡」過頭 怪ＦＡＡ不人道", *China Post* 2009/10/26)

# Midterm Quiz Suggested Answer

## Part A

1. ① Professor Joseph : $u'(x) = x^{-R}$

Student Yu : $v'(x) = x^{-r}$

$\Rightarrow u''(x) = -Rx^{-R-1}$

$R(x) = -x\frac{u''(x)}{u'(x)} = -x\cdot\frac{(-Rx^{-R-1})}{x^{-R}} = R$ (1 pt)

$v''(x) = -rx^{-r-1}$

$r(x) = -x\frac{v''(x)}{v'(x)} = -x\cdot\frac{(-rx^{-r-1})}{x^{-r}} = r$ (1 pt)

Hence, both of them are constant relative risk aversion.

② VNM utility function:

$$u(x) = \begin{cases} \frac{x^{1-R}}{1-R} & \text{if } R \neq 1 \\ \ln x & \text{if } R = 1 \end{cases}$$

$(\because R=1,\ u'(x) = \frac{1}{x} \Rightarrow \int \frac{1}{x}dx = \ln x)$ (1.5 pt)

$$v(x) = \begin{cases} \frac{x^{1-r}}{1-r} & \text{if } r \neq 1 \\ \ln x & \text{if } r = 1 \end{cases}$$

(1.5 pt)

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2. Suppose a risk neutral person chooses A at question k:

$\frac{k}{10}\times 200 + \frac{10-k}{10}\times 160 > \frac{k}{10}\times 385 + \frac{10-k}{10}\times 10$

$\Leftrightarrow -\frac{k}{10}\times 185 + (1-\frac{k}{10})\times 150 > 0$

$\Leftrightarrow -k\times 185 + (10-k)\times 150 > 0 \Leftrightarrow 335k < 1500 \therefore k < 4.478$

∴ He will choose A at Question 1-4. # (5 pt)

3. ① Joseph vs. neutral

$\because R(x) = R > 0 \therefore$ He is risk averse

$\Rightarrow \because$ A is safer, ∴ He will choose "more" A than neutral person. (2 pt)

② Yu vs. neutral

$R(x) = r$, <1> If $r>0$, then he will choose "more" A than neutral person. (1 pt)

<2> If $r<0$, then he will choose "less" A (He is now risk-loving. (1 pt)

③ Studuent Yu vs. Joseph

∵ $r < R$

∴ Yu has smaller degree of risk averse than Joseph.

⇒ Yu will choose "less" A. (1pt)

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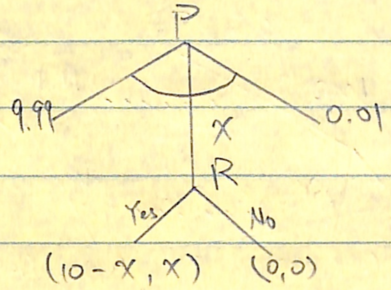
## Part B

(a) $A_p = \{(P, R) = (9.99, 0.01), \cdots (0.01, 9.99)\}$




$(x = 9.99, 9.98, \cdots, 0.01)$

$\forall p$, Rachael should accept it!  (5 pt)

⇒ Given R will accept it, P should choose (9.99, 0.01) ⇒ This is SPE!
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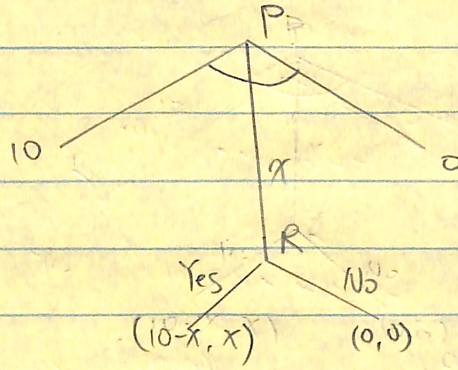
(b) $A_p = \{(P, R) = (10, 0), (9, 1), \cdots, (0, 10)\}$




$(x = 10, 9, \cdots 1, 0)$

Two SPE: In the subgame of (10, 0), both "accept" or "reject" are best reponse.
(2.5 pt) ① If R chooses "accept", Paul should choose (10, 0) ⇒ {(10, 0), accept} is a SPE.
(2.5 pt) ② If R chooses "reject", Paul should choose (9, 1) and R will accept. ⇒ {(9, 1), accept} is also a SPE
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(c) I think this is "not" a good prediction, since this prediction does not consider any social preference. Hence, in the real world, I think such a very unbalanced distribution will not be accepted in the real world.
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(If your answer is reasonable, you can get 5.)

Part C

| | | H | T |
|---|---|---|---|
| p | H | 1, 0 | 0, 1 |
| 1-p | T | 0, 1 | 1, 0 |

Trivially, $\nexists$ pure NE (1 pt)

Suppose player 1 chooses H with prob. p, T w/ prob. 1-p

At NE, player 2 should be indifferent between H and T

$\Rightarrow \begin{cases} H: \text{ expected payoff } = 1-p \\ T: \text{ expected payoff } = p \end{cases}$

$\Rightarrow p = 1-p \quad \therefore p^* = \frac{1}{2}$ (2pt)

Similarly, for player 2, he should play H, T w/ prob. $\frac{1}{2}$. (2pt)

$\therefore$ NE: $(\frac{1}{2}H \oplus \frac{1}{2}T, \frac{1}{2}H \oplus \frac{1}{2}T)$ #

| | | q | 1-q |
|---|---|---|---|
| | | H | T |
| p | H | 9, 0 | 0, 1 |
| 1-p | T | 0, 1 | 1, 0 |

Trivially, $\nexists$ pure NE. (1 pt)

Suppose player 1 chooses H w/ prob. p, T w/ prob. 1-p

$\Rightarrow$ player 2 should be indifferent

$\therefore p = 1-p \Rightarrow p^* = \frac{1}{2}$ (2 pt)

Suppose player 2 plays H w/ prob. q, T w/ prob. 1-q

$\Rightarrow$ player 1 should be indifferent

$\therefore 9q = 1-q \Rightarrow q^* = \frac{1}{10}$ (2 pt)

$\therefore$ NE: $(\frac{1}{2}H \oplus \frac{1}{2}T, \frac{1}{10}H \oplus \frac{9}{10}T)$ #

| | | q | 1-q |
|---|---|---|---|
| | | H | T |
| p | H | 4, 0 | 0, 1 |
| 1-p | T | 0, 1 | 1, 0 |

Trivially, $\nexists$ pure NE (1 pt)

Suppose player 1 plays H w/ prob. p, T w/ prob. 1-p

$\Rightarrow$ player 2 should be indifferent

$\therefore p = 1-p \Rightarrow p^* = \frac{1}{2}$ (2 pt)

Suppose player 2 plays H w/ prob. q, T w/ prob. 1-q

$\Rightarrow$ player 1 should be indifferent

$\therefore 4q = 1-q \Rightarrow q^* = \frac{1}{5}$ (2 pt)

$\therefore$ NE: $(\frac{1}{2}H \oplus \frac{1}{2}T, \frac{1}{5}H \oplus \frac{4}{5}T)$ #

## Part D

1.

| | | sleep | awake |
|---|---|---|---|
| p | sleep | (−100,000 , −100,000) | (12,000 , 10,000) |
| 1−p | awake | (10,000 , 12,000) | (10,000 , 10,000) |

(5 pt)

#

2. No, given his co-pilot will stay awake, the other pilot should deviate to take a nap. # (5 pt)

3. ① pure : (sleep, awake) , (awake , sleep) (2 pt)

② mixed :

Suppose player 1 plays "sleep" w/ prob. p , plays "awake" w/ prob. 1−p.

⇒ player 2 should be indifferent when NE

∴ $\underbrace{-100{,}000\,p + 12{,}000(1-p)}_{\text{"sleep"}} = \underbrace{10{,}000}_{\text{"awake"}}$

⇒ $12{,}000 - 112{,}000\,p = 10{,}000$

⇒ $112{,}000\,p = 2{,}000$ ∴ $p^* = \frac{1}{56}$

By symmetry, player 2 should play the same strategy.

∴ mixed NE : $(\frac{1}{56}$ sleep ⊕ $\frac{55}{56}$ awake , $\frac{1}{56}$ sleep ⊕ $\frac{55}{56}$ awake$)$ (3 pt)

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4. ① The case in the news article is the "Mixed Equilibrium" that both pilots have positive probability to fall asleep. (2 pt)

② (sleep , awake) or (awake , sleep) (2 pt)

③ That a pilot sleeps and the other one stays awake will bring higher payoff to the pilots. (1 pt)

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