Experiments Games with Mixed Strategy Equilibrium (混合策略均衡實驗)

Joseph Tao-yi Wang (王道一)

Lecture 5, EE-BGT

Games with MSE 有混合策略均衡的賽局

- Zero-Sum Games (零和賽局)
  - Rock-Scissor-Paper (剪刀石頭布)
  - Sports (PK, tennis serves, basketball drives, etc.)
    - 足球罰踢、網球發球、籃球切入或投籃
  - Military attack (軍事行動如登陸諾曼地或加萊)
- Deter Undesired Behavior (嚇阻「投機/不希望發生」的行為)
  - Searches of passengers after 9/11 (機場安檢、海關抓走私)
  - Randomizing across exam questions (老師隨機出題)
- But, there are interesting folk theories about these games... (但總有一些有趣的「理論」)

- 媒體報導，大多數人都知道，在猜拳遊戲中，石頭贏剪刀，剪刀贏布，布勝拳頭，但很少有人知道，如何贏得這個相當普遍的遊戲。現在死忠玩家透露了必殺秘技：先出剪刀。

  L0

- 英國「每日郵報」報導，研究顯示在這種快速擺出手部姿勢的猜拳遊戲中，石頭是三種猜拳手勢中玩家最喜歡出的一種。如果你的對手預期你會出石頭，他們就會選擇出布來贏過你，因此你要出剪刀才能贏，因為剪刀贏布。

  L1

  L2

- 報導說，這套剪刀策略讓拍賣商佳士得前年成功贏得一千萬英鎊的生意。一名有錢的日本藝術品收藏家，無法決定要讓哪家拍賣公司來拍賣自己收藏的印象派畫作，於是他要求佳士得與蘇富比兩家公司猜拳決定。

- 佳士得向員工討教猜拳策略，最後在一名主管十一歲的女兒的建議下決定出剪刀。這名女孩現在還在讀書，經常玩猜拳，她推論「所有人都以為你會出石頭」。這代表蘇富比會出布，想要打敗石頭，因此佳士得應該選擇出剪刀。

- 一如預期，蘇富比最後出布，輸給了佳士得的剪刀，拱手將生意讓給對方。

Mixed-Strategy Equilibrium in RPS

- How do you play Rock-paper-scissors (RPS)?
  - 如果你來玩剪刀石頭布，你會出什麼？

- What is the MSE here? (剪刀石頭布賽局的均衡為何？)

- Mix with probabilities (1/3, 1/3, 1/3) (三者隨機)

- Would you really play this MSE in RPS?
  - News article suggests a level-k model...
  - (你真的會按均衡策略來玩嗎？新聞故事所反映的多層次思考模型預測為何？想知道更多請看課本第五章)

- Janken/RPS Robot with 100% winning rate:
  - http://www.youtube.com/watch?v=3nxjjztQKtY

Advantages of Games with MSE (此種賽局的優點)

- Typically have unique equilibrium (有唯一均衡)
  - All games discussed have unique equilibrium
- Constant sum: No room for social preference
  - Not possible to help others without hurting self
    (總報酬為常數下通常無社會偏好，因為幫助別人一定傷到自己)
- Maximin leads to Nash in zero sum (避凶就是均衡)
  - Maximin is a <u>simple</u> rule: (對方就是要害我如何趨吉避凶)
  - "I want to maximize the worse case scenario..."
- A good place to test theory! (這是驗證理論的好地方)

Maximin in Matching Pennies (黑白猜下避兇)

|	H	T
H	1	-1
T	-1	1

- **Rowena thinks**: (列子認為)
- Play H: Worse case -1
- Play T: Worse case -1
- (1/2, 1/2): Worse case 0*

  (出正面最慘是對手選反面，出反面最慘是對手選正面，一半一半至少不賺不賠)

- Same for Colin (行家所見略同)

  – This is the MSE!

  (這正好是此賽局的混合策略均衡！)

*We assume preferences satisfy axioms for EU... (假設偏好滿足期望效用公理)

Challenges of Games with MSE (對理論的挑戰)

- Epistemic Foundation (認知基礎: 須清楚知道對手的策略)
    - Requires precise knowledge of opponent strategy
- Learning Dynamics may not work (動態學習不見得好)
    - Gradient processes spiral away (梯度逼近會螺旋脫離均衡)
    - No incentive to mix properly at MSE (均衡時亂做沒差)
- Randomization can be unnatural
    - Especially in repeated play (重複做的話，隨機亂選不太自然)
- Purification (純化: 個體可做不同單純策略，整體看起來「混合」即可)
    - MSE can occur at population level, not individually

Chart Title

Actual Data (實驗資料)

MSE predictions(均衡預測)
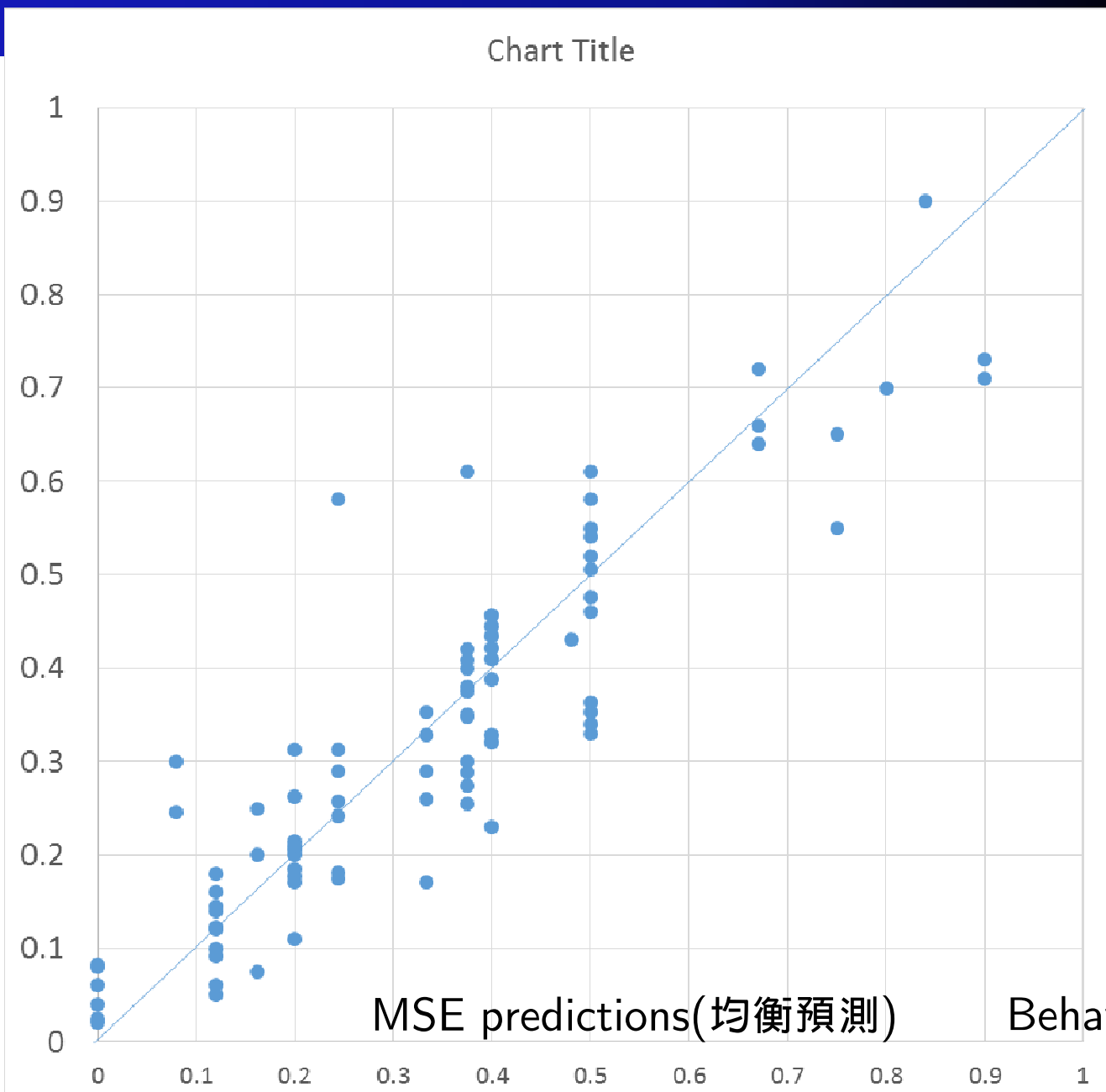
Joker Game: O'Neill (1987) (出鬼牌賽局)

- Earlier studies: Play between MSE & random
  - – But had computerized opponents and/or low incentives, so hard to interpret the results... (早期實驗結果介於MSE和亂選之間，但通常對手是電腦且不見得有誘因)
- First "Modern" Studies: O'Neill (1987)
- Good Design Trick: (很棒的實驗設計技巧！)
  - – Risk aversion plays no role when there are only two possible outcomes
  - – (當實驗結果只有兩種可能時，風險偏好不會影響受試者的決定)

Joker Game: O'Neill (1987) (出鬼牌賽局)

	1	2	3	J	MSE	Actual	QRE
1	-5	5	5	-5	0.2	0.221	0.213
2	5	-5	5	-5	0.2	0.215	0.213
3	5	5	-5	-5	0.2	0.203	0.213
J	-5	-5	-5	5	0.4	0.362	0.360
MSE	0.2	0.2	0.2	0.4			
Actual	0.226	0.179	0.169	0.426			
QRE	0.191	0.191	0.191	0.427			

實際的出牌頻率跟MSE預測很接近

QRE的預測更接近，但無法解釋「不平均」

- Actual frequencies are quite close to MSE
- QRE better, but cannot get "imbalances"

Quantal Response Equilibrium (手滑反應均衡)

- QRE – McKelvey and Palfrey (1995)

- Better response, not best response (更適/非最適)

- Logit payoff response function: (常用logit報酬反應函數)

$$P(s_i) = \frac{e^{\lambda \cdot \left[\sum\limits_{s_{-i}} P(s_{-i})u_i(s_i,s_{-i})\right]}}{\sum\limits_{s_k} e^{\lambda \cdot \left[\sum\limits_{s_{-i}} P(s_{-i})u_i(s_k,s_{-i})\right]}}$$

Quantal Response Equilibrium (QRE)

- $\lambda = 0$: Noise (do not respond to payoffs)
  (對報酬無反應)

- $\lambda = \infty$: Nash (perfectly respond to payoffs)
  (完全反應)

$$P(s_i) = \frac{e^{\lambda \cdot \left[\sum\limits_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i}) \right]}}{\sum\limits_{s_k} e^{\lambda \cdot \left[\sum\limits_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i}) \right]}}$$

Response to O'Neill (1987)

- Brown and Rosenthal (1990) criticize O'Neill:
  - Overly support MSE (太過支持混合策略均衡)
  - Aggregate tests not good enough (只有總體檢定不夠)
- They run (temporal dependence): (應該檢定跨期相關性)

$$J_{t+1} = a_0 + a_1 J_t + a_2 J_{t-1}$$

$$b_0 J_{t+1}^* + b_1 J_t^* + b_2 J_{t-1}^*$$

$$c_1 J_t J_t^* + c_2 J_{t-1} J_{t-1}^* + \epsilon$$

$$J_t = \text{Own Choice}; J_t^* = \text{Other's Choice};$$

- MSE implies only a_0 is not zero (均衡: 只有 a_0 不是0)

Brown & Rosenthal (1990) Results

Effect	Coefficient	% Players s.t. $p<0.05$
Guessing	b_0	8%
Previous opp. choices	b_1, b_2	30%
Previous outcomes	c_1, c_2	38%
Previous choices & outcome	b_1, b_2, c_1, c_2	44%
Previous own choices	a_1, a_2	48%
All effects		62%

Source: Table 3.4, BGT.

Response to O'Neill (1987) (後續討論)

- Run: 2 JJJJ 1 2 33 (連發太短)
- Too Short runs: play J twice too rarely (鮮有連續J)
- Subjects react to what they see/do (對歷史有反應)
  - But most cannot use temporal dependence to guess opponent current action (無法用跨期相關性猜中對方這次行動)
- Equilibrium-in-beliefs somewhat supported (信念)
  - Each player may deviate from MSE (每人各自可能偏離)
  - But these deviations cannot be detected (卻沒有被破解)
- Purification interpretation of MSE (純化的MSE)
  - Equilibrium in beliefs, not in mixtures (信念非策略)

Response to O'Neill (1987) (後續討論)

- ## Other similar studies (相關延伸研究)
  - Rapoport and Boebel (1992) [BGT, Table 3.5]
  - Mookerjhee and Sopher (1997) [BGT, Table 3.6-3.7]
  - Tang (1996abc, 2001) [BGT, Table 3.8]
  - Binmore, Swierzbinski, and Proulx (2001) [BGT, Table 3.9]
- ## Stylized Facts: (整體實驗結果)
  - Actual frequencies not far from MSE (出牌頻衡很接近MSE)
  - Deviations small but significant (跟MSE差距小但統計上顯著)
  - Temporal dependence at individual level (個人有跨期相關性)
- ## Can a theory explain these? (有何理論可以解釋這些實驗結果?)

Psychology: Production Task (心理學: 產生數列)

- Ask subjects generate random sequences (產生數列)
- Sequences resemble the underlying statistical process <u>more closely than</u> what short random sequences actually do (產生的比真正隨機數列還要<u>更隨機</u>)
  - Too balanced (太平衡)
  - Too few runs (連發太少)
  - Longest run is too short (最長的連發太短)
- Children do not learn this misconception until after 5th grade (小孩子在五年級之前沒有這個問題)
  - A learned mistake (這是一個<u>後天學會</u>的錯誤)

Game Play (賽局實驗) vs. Production (產生數列)

- ## Rapoport and Budescu (1992, 1994, 1997)
  - Compare sequences from a production task to strategies in a constant-sum game (R&B, 1992)
  - 比較產生的數列和零和賽局實驗中的數列

Condition D: Matching pennies 150 times 1-by-1
  - 150次逐次黑白猜

Condition S: Give sequence of 150 plays at once
  - 一次給150回合黑白猜的決定

Condition R: Produce the outcome of tossing an unbiased coin 150 times
  - 產生數列——丟銅板150次的結果

Game Play (賽局實驗) vs. Production (產生數列)

- iid rejected for 40% (D), 65% (S), 80% (R) of the subjects in the three conditions
  - 三種分別有40%, 65% 和80%的受試者拒絕 iid 假設
  - Game play reduces deviations from randomness
  - 真的去玩會讓受試者比較隨機(降低偏離情形)
- Are subjects better motivated?
- Or, are their working memory interfered and randomize "memory-lessly"?
  - 這是因為受試者有更好的誘因，還是因為他們的腦部運作(工作記憶)受到干擾，以致於「忘記過去，努力面前」？

3-action Matching Pennies

	1	2	3		MSE
1	2	-1	-1		1/3
2	-1	2	-1		1/3
3	-1	-1	2		1/3

MSE	1/3	1/3	1/3

- Rapoport and Budescu (1994)

Runs in 3-action Matching Pennies: R&B '94

Pattern	Game Freq.	Production Freq.	iid Freq.
xx	0.269	0.272	0.333
xxx	0.073	0.063	0.111
xxy	0.196	0.209	0.222
xyy	0.196	0.210	0.222
xxxx	0.020	0.018	0.037
xxxy	0.053	0.045	0.074
yxxx	0.054	0.045	0.074
xyxx	0.056	0.035	0.074
xxyx	0.058	0.037	0.074

Other Play in 3-action Matching Pennies

Pattern	Game Freq.	Production Freq.	iid Freq.
xy	0.731	0.728	0.667
xyx	0.237	0.160	0.222
xyz	0.297	0.359	0.222
yxzx	0.096	0.078	0.074
xyxz	0.099	0.079	0.074
xyzx	0.121	0.173	0.074

Source: Table 3.10, BGT.

A Limited Memory Model (有限記憶模型)

- Subjects only remember last m elements (記得最後 m 回合)
- Chose the $(m+1)$st to balance the number of H and T choices in the last $(m+1)$ flips
  - 受試者第$(m+1)$回合做決定來平衡正反面在$(m+1)$ 次中出現的次數
- If m is small, alternate choices too frequently
  - 如果 m 很小，就會正反變換太頻繁
- Experimental Data: (Should all be 0.5 if iid)
  - P(H|H)=0.42　　　　　(實驗結果：如果iid的話應該都是0.5)
  - P(H|HH)=0.32
  - P(H|HHH)=0.21
- Requires m=7 to generate this (Magic 7?) (才符合實驗結果)

Explicit Randomization (使用亂數產生器)

- **Observe the randomization subjects want to play**
  - 觀察人們會為亂數產生器設定何種機率來做
  - Bloomfield (1994), Ochs (1995b), Shachat (2002)
- Explicit Randomization: (使用亂數產生器)
1. Allocate 100 choices to either strategies
2. Choices are shuffled and computer selects one
   - 總共100張牌/選擇，決定兩邊各放幾張讓電腦隨機打一張出來…
- **Deviations cannot be due to cognitive limit!**
  - 如果還偏離均衡，就不是因為不能產生亂數！
- Result: Deviations from MSE small but significant
- About 10% purists (偏離MSE很小但顯著。10%「單純的人」)

Explicit Randomization (使用亂數產生器)

- Ex: Ochs (1995b) - Matching Pennies (黑白猜)
  - Row player payoff of (H, H): 1→9→4 (改變列子報酬)
- MSE: Column MSE changes; row is same...
  - 行家的MSE會改變；列子的反而不會變
- Allocate 10 plays of H or T (分配十個選擇給正或反)
  - Becomes a 10-play sequence (變成「做十次的數列」)
- Note: Random draw without replacement
  - This is not exactly randomization of MSE...
  - 註：這是隨機抽取不放回，不是真的MSE...

Matching Pennies (Baseline)

	H	T
H	1,0	0,1
T	0,1	1,0

- MSE:
  - R: (0.500, 0.500)
  - C: (0.500, 0.500)

- Actual Frequency: (實際頻率)
  - R: (0.500, 0.500)
  - C: (0.480, 0.520)

- QRE:
  - R: (0.500, 0.500)
  - C: (0.500, 0.500)

Matching Pennies (Game 2)

	H	T
H	9,0	0,1
T	0,1	1,0

- MSE:
  - R: (0.500, 0.500)
  - C: (0.100, 0.900)

- Actual Frequency: (實際頻率)
  - R: (0.600, 0.400)
  - C: (0.300, 0.700)

- QRE:
  - R: (0.649, 0.351)
  - C: (0.254, 0.746)

Matching Pennies (Game 3)

	H	T
H	4,0	0,1
T	0,1	1,0

Source: Table 3.12, BGT.

- MSE:
  - R: (0.500, 0.500)
  - C: (0.200, 0.800)

- Actual Frequency:(實際頻率)
  - R: (0.540, 0.460)
  - C: (0.340, 0.660)

- QRE:
  - R: (0.619, 0.381)
  - C: (0.331, 0.669)

MSE in Field Context (實際現場的MSE)

- Rapoport and Almadoss (2000)
- Patent races games (競相專利賽局)
  - Two firms with endowment e (兩家廠商，各有財產)
  - Invest $1, 2, ..., e$ (integer)
  - Win r if invest most
- Unique MSE: Invest e with prob. $1\text{-}e/r$, invest others with prob. $1/r$ (not obvious)

Patent Race Results (競相專利賽局實驗結果)

(Table 3.14)	Game L: e=5,r=8		Game H: e=5,r=20	
Investment	MSE	Actual	MSE	Actual
0	0.125	0.169	0.050	0.141
1	0.125	0.116	0.050	0.055
2	0.125	0.088	0.050	0.053
3	0.125	0.118	0.050	0.053
4	0.125	0.090	0.050	0.069
5	0.375	0.418	0.750	0.628

MSE in Field Context

- 3 Firm Hotelling: Collins and Sherstyuk (2000)
  - 2-Firm: Brown-Kruse, Cronshaw & Schenk (1993)
  - 4-Firm: Huck, Muller and Vreiend (2002)

- Location Games (3 Firm Hotelling Model)
  - Three firms simultaneously choose [0,100]
  - Consumers go to nearest firm
  - Profits proportional to units sold

- Unique MSE: Randomize uniformly [25,75]

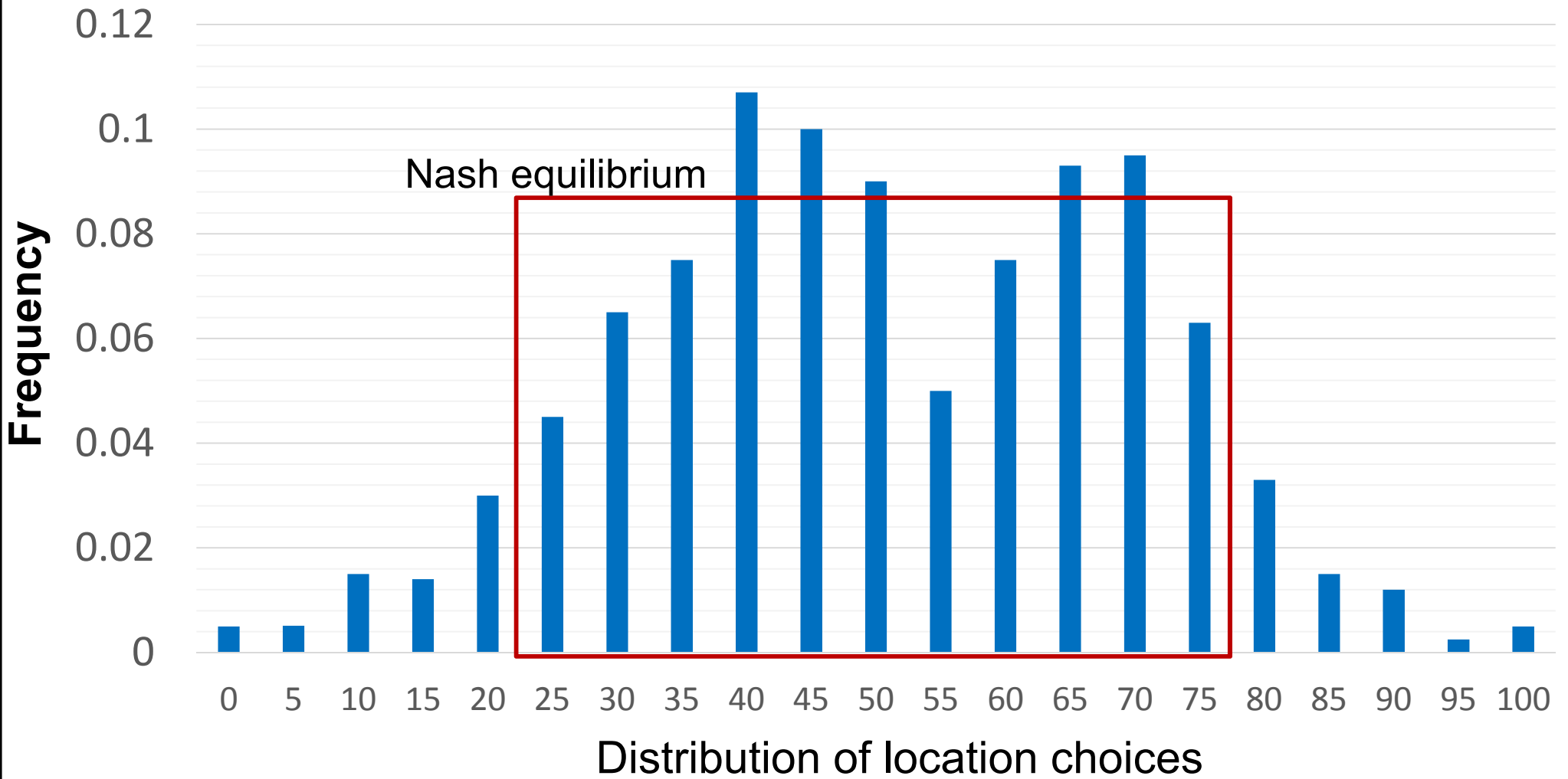Figure 3.2 Behavioral Game Theory
Source: Based on Collins and Sherstyuk (2000)

Two Field Studies

- **Walker and Wooders (2001)**
  - serve decisions (L or R) of tennis players in 10 Grand Slam matches
- Result:
  - Win rates across two different directions are not statistically different (p<0.10 for only $2/40$)
  - Players still exhibit some over-alteration in serve choices through temporal dependence (p<0.10 for $8/40$) [weaker than lab subjects]

# Two Field Studies

- Palacios-Huerta (2001): soccer penalty kicks
  - Code both kicker and goalie's choices
  - No selection bias (look at all games)
- Win rates are equal; no serial dependence
  - Not surprising since penalty kicks are few and are often done by different players
- Recent: Huang, Hsu, and Tang (AER 2007)
  - Chen-Ying Huang (here at NTU)

Conclusion

- **Take-home Message:**
- Aggregate frequencies of play are close to MSE but the deviations are statistically significant.

- QRE seems to fit behaviors well.
- Temporal dependence frequently observed

Actual
Data
實驗資料

QRE predictions（模型預測）

Source: BGT, Ch. 3
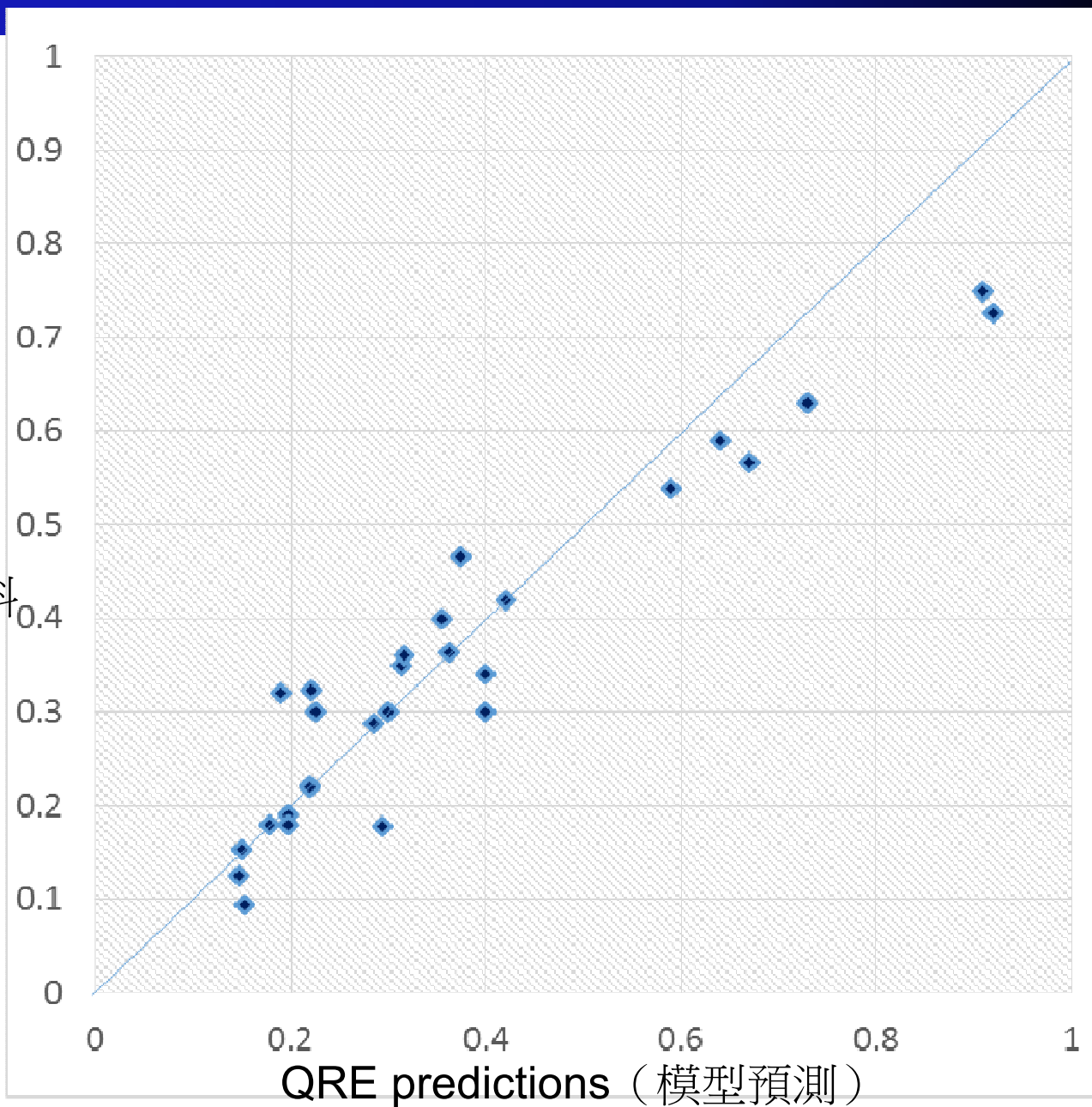
Conclusion

- With explicit randomization, the existence of purists hint on equilibrium in beliefs
  - Players cannot guess what opponents are doing
  - Beliefs about opponents are correct on average
  - But, they may not be randomizing themselves
- Field-Lab-Theory: Ostling, Wang, Chou & Camerer (2011), "*Testing Game Theory in the Field: Evidence from Swedish Poisson LUPI Lottery Games*," American Economic Journal: Microeconomics, 3(3), 1-33.