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Eyetracking Spatial Beauty Contests: Replication and Beyond

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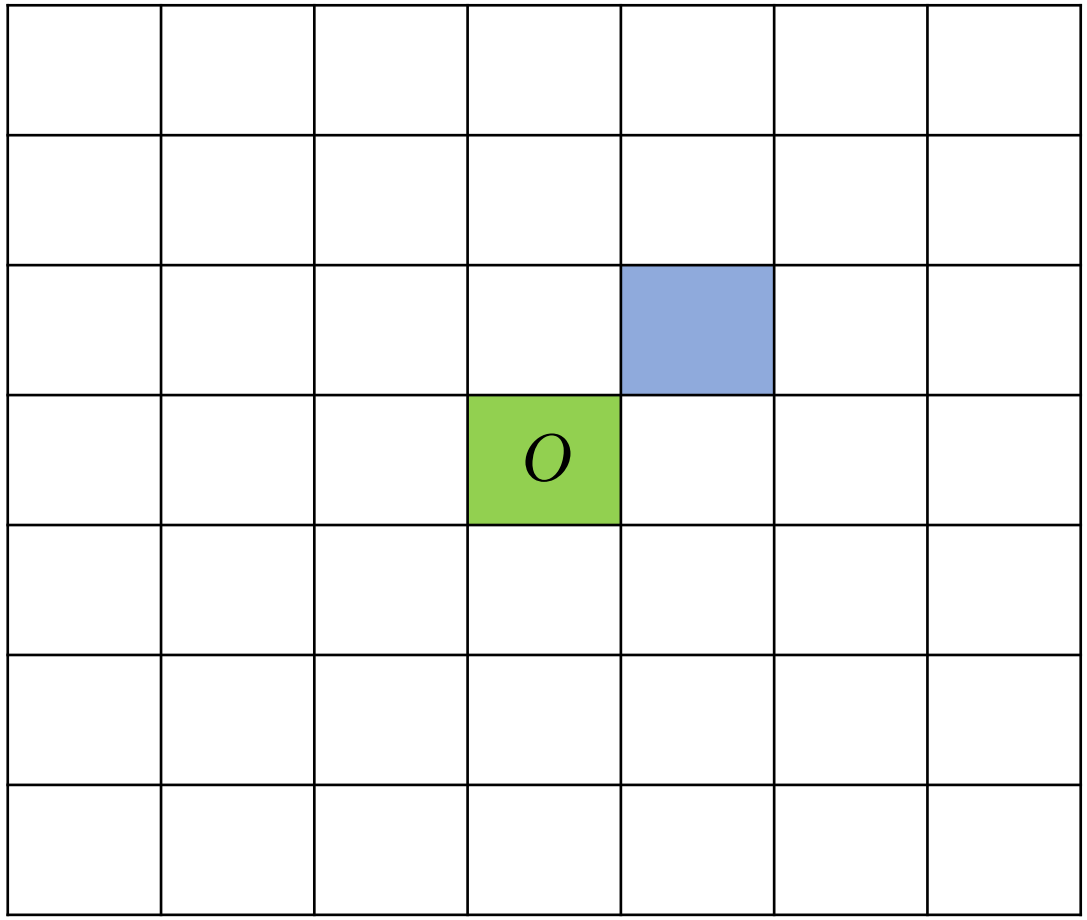
Spatial Beauty Contest Game

Your goal
Left 4, Below 2

Other's goal
Right 2, Above 4



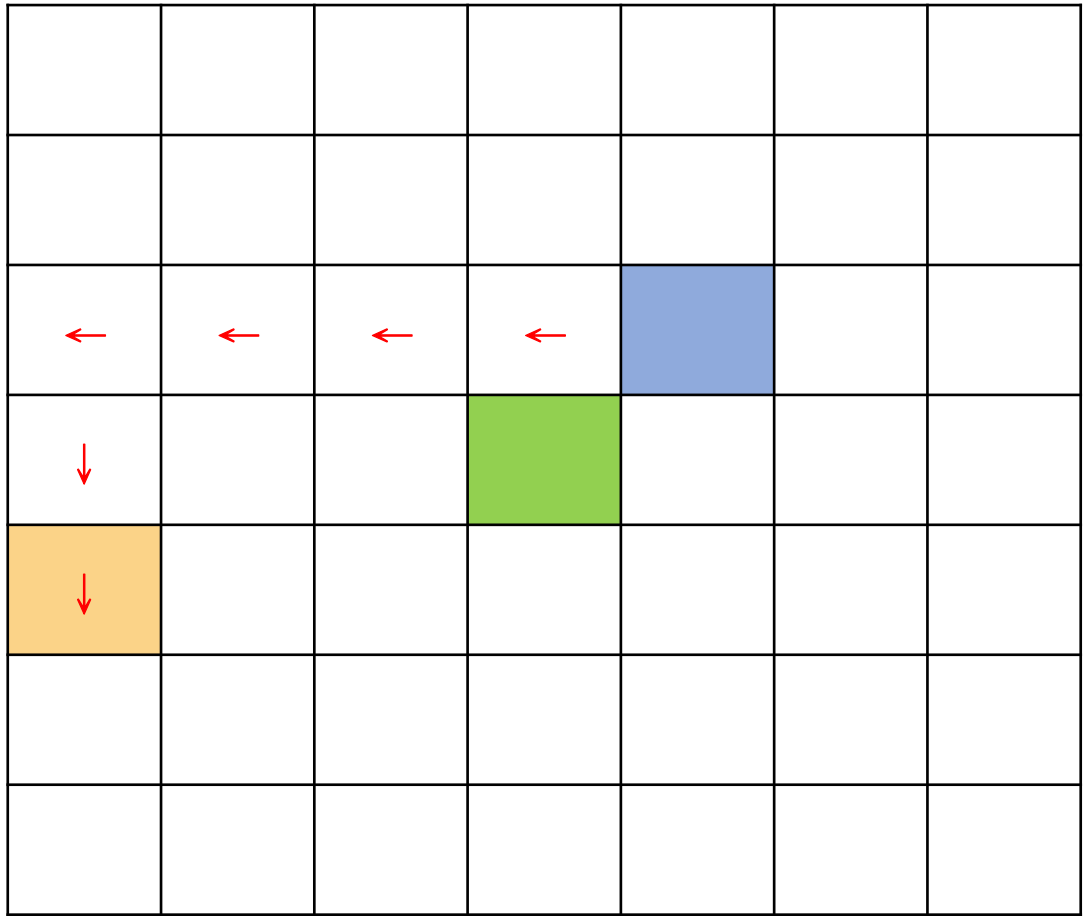
Your choice
Other's choice

Game 15

Spatial Beauty Contest Game

Your goal
Left 4, Below 2

Other's goal
Right 2, Above 4



Your choice
Other's choice
Your target

Game 15

Spatial Beauty Contest Game

Your goal
Left 4, Below 2

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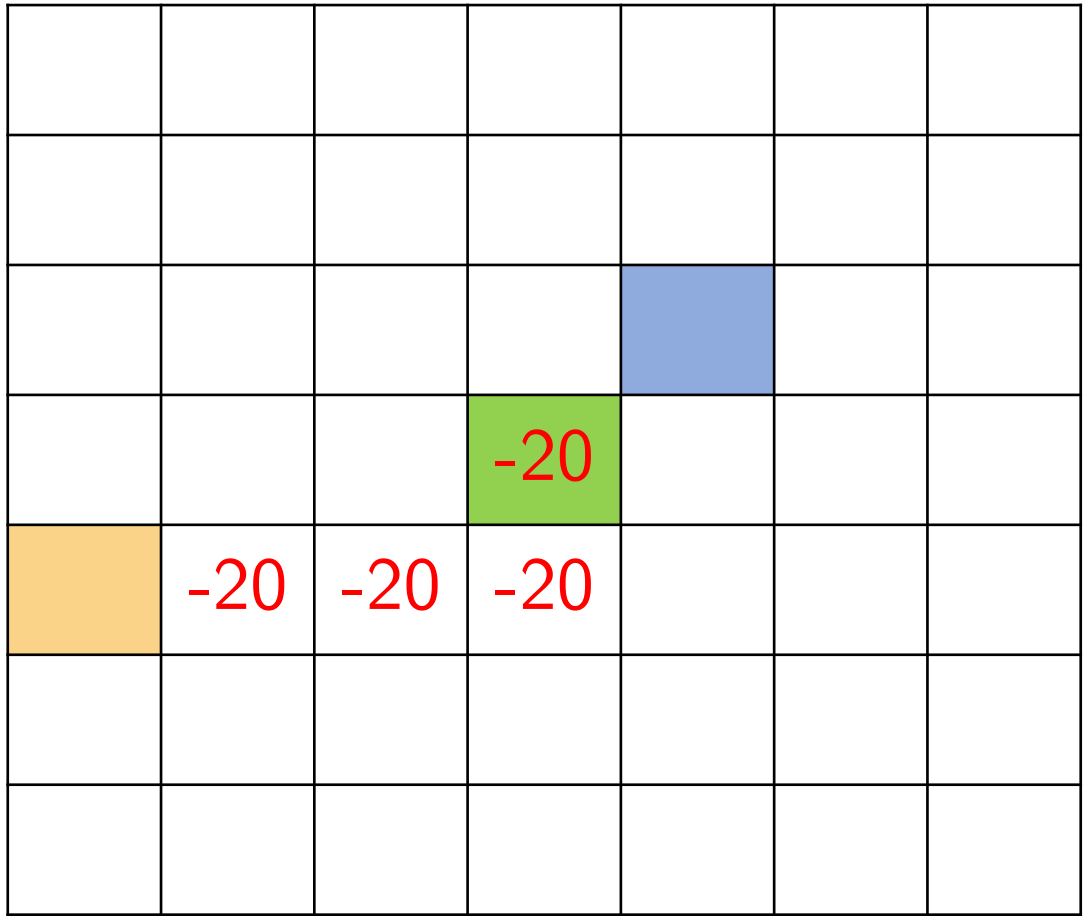
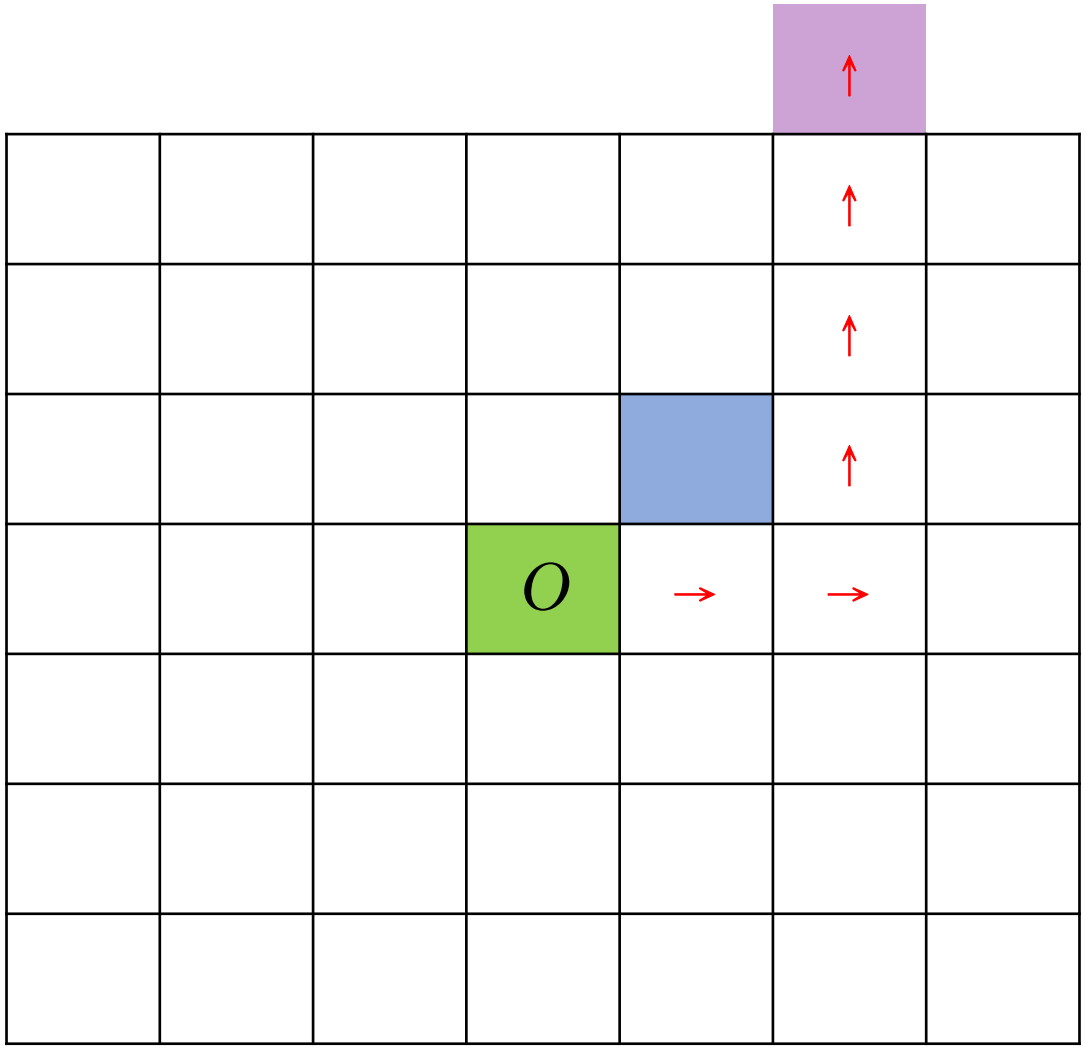


Your choice
Other's choice
Your target

Game 15

Spatial Beauty Contest Game

Your goal
Left 4, Below 2



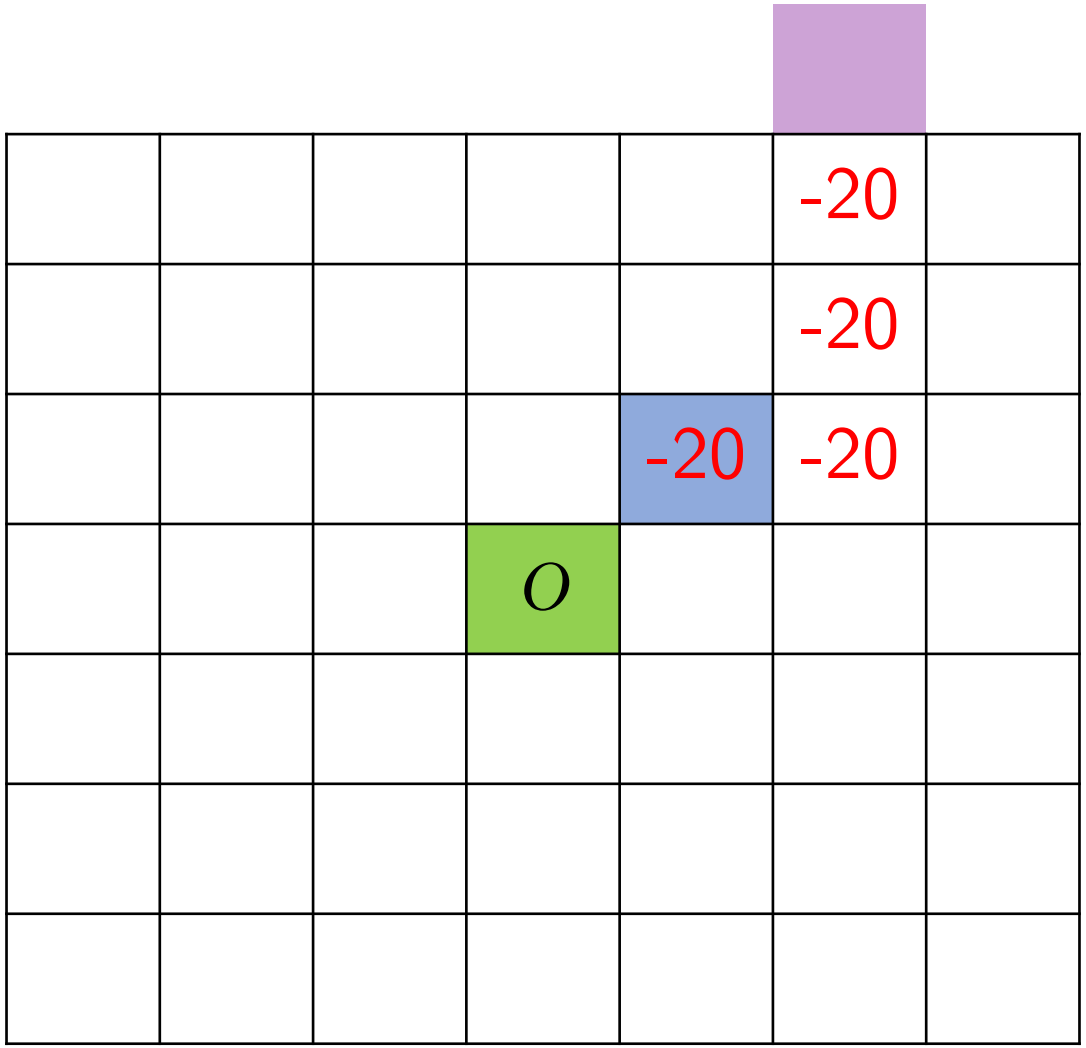
Other's goal
Right 2, Above 4

Your choice
Other's choice
Other's target

Game 15

Spatial Beauty Contest Game

Your goal
Left 4, Below 2



Other's goal
Right 2, Above 4

Your choice
Other's choice
Other's target

Game 15

Experimental Design

- Map size and goals vary across 36 trials.
- Each player choose simultaneously
- The payoff is the following :

$$\pi_i = 480 - 20 \times (|x_i - t_i^x| + |y_i - t_i^y|)$$

Experimental Design

- 200 subjects:
  - 88 eye-tracked subjects
  - 112 mouse-tracked subjects.

- Post experiment questionnaire

The Level-k Model

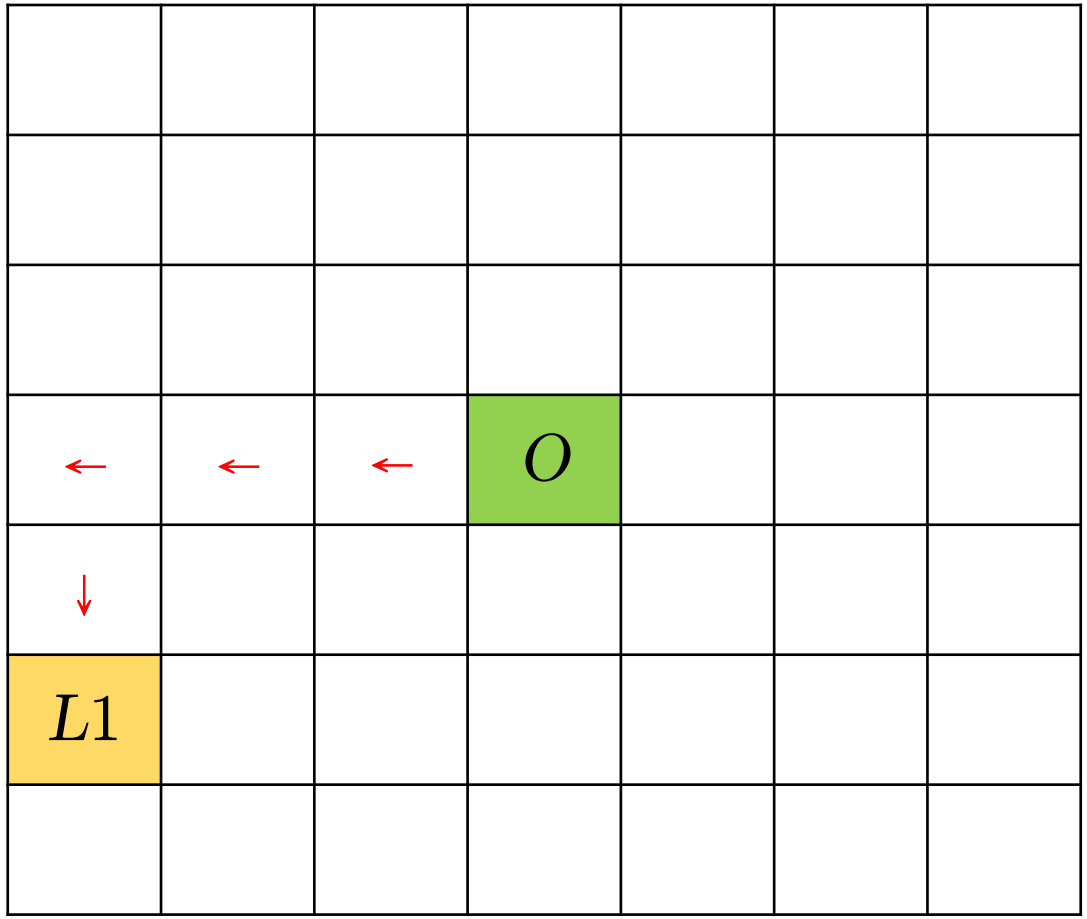
- $L0$ randomly choose on the map
  - On average the center

- BR to $L0$ = BR to $(0,0)$

The Level-k Model

Your goal
Left 4, Below 2

Other's goal
Right 2, Above 4



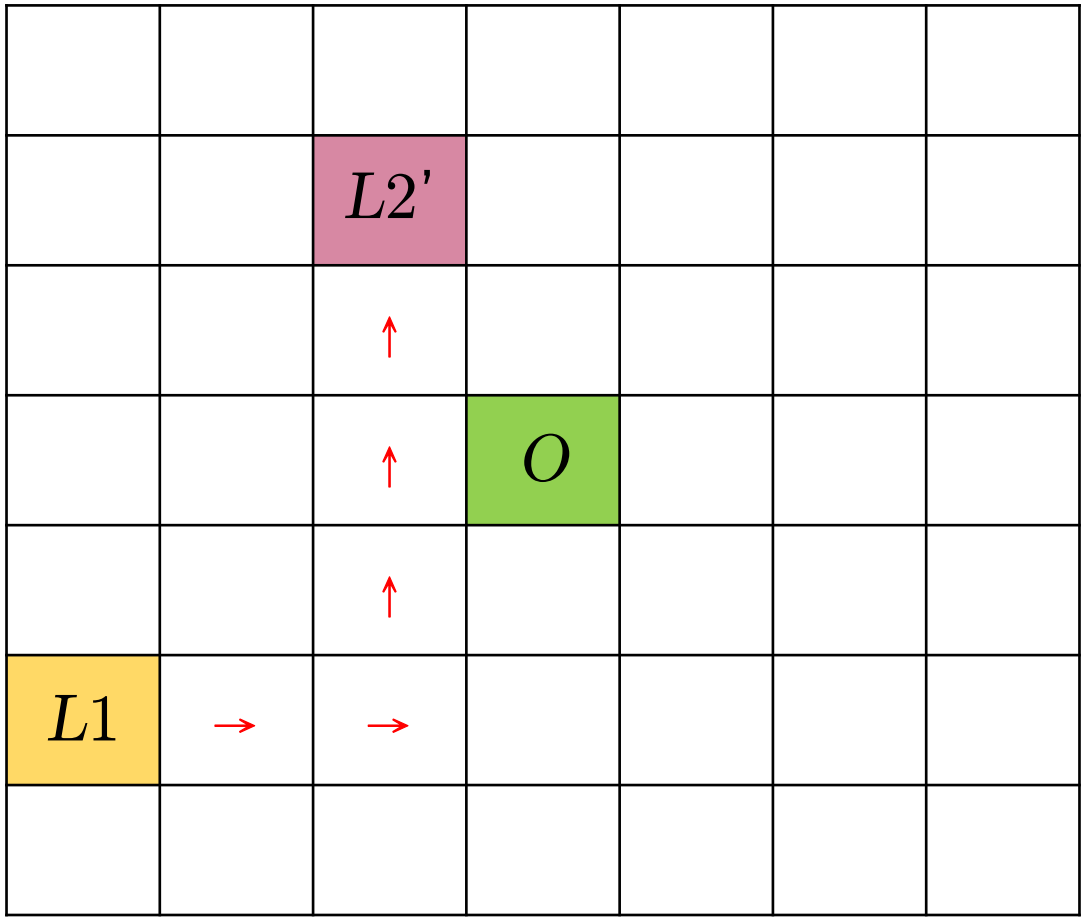
$L0$
$L1$

Game 15

The Level-k Model

Your goal
Left 4, Below 2

Other's goal
Right 2, Above 4



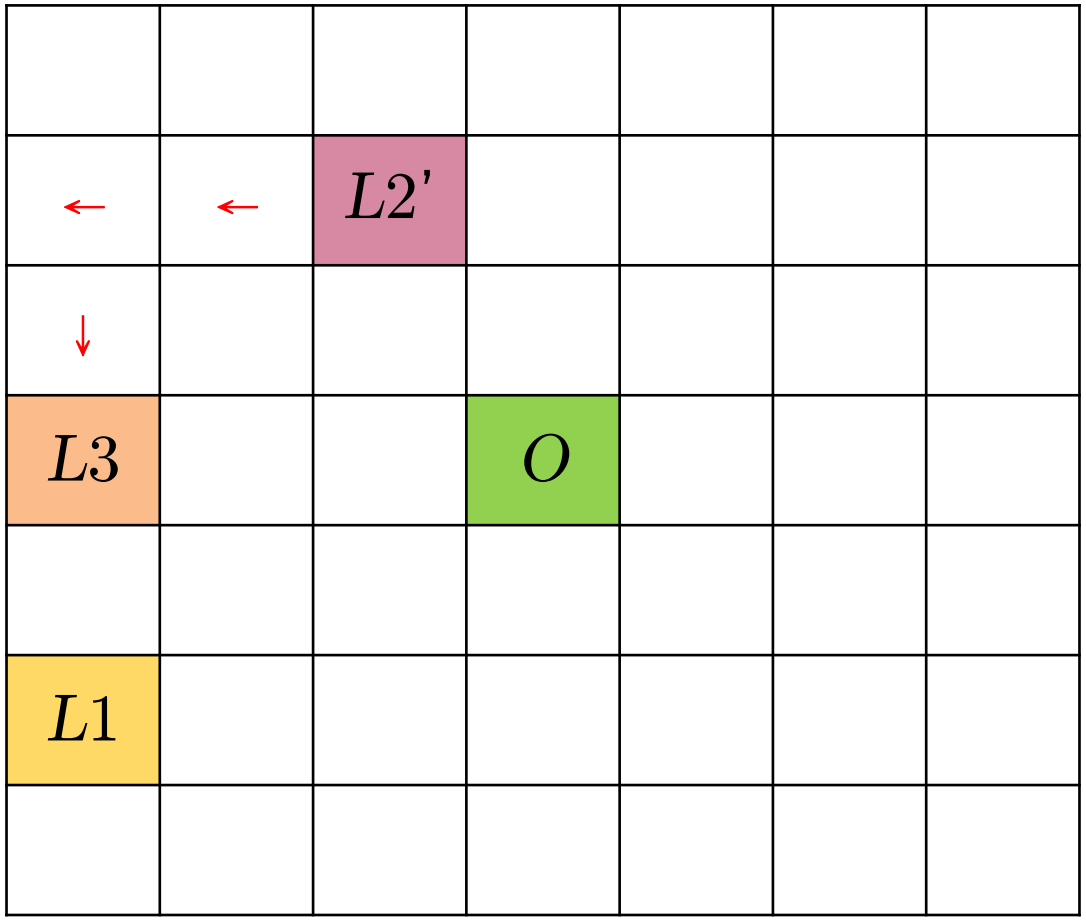
$L0$
$L1$
$L2'$

Game 15

The Level-k Model

Your goal
Left 4, Below 2

Other's goal
Right 2, Above 4



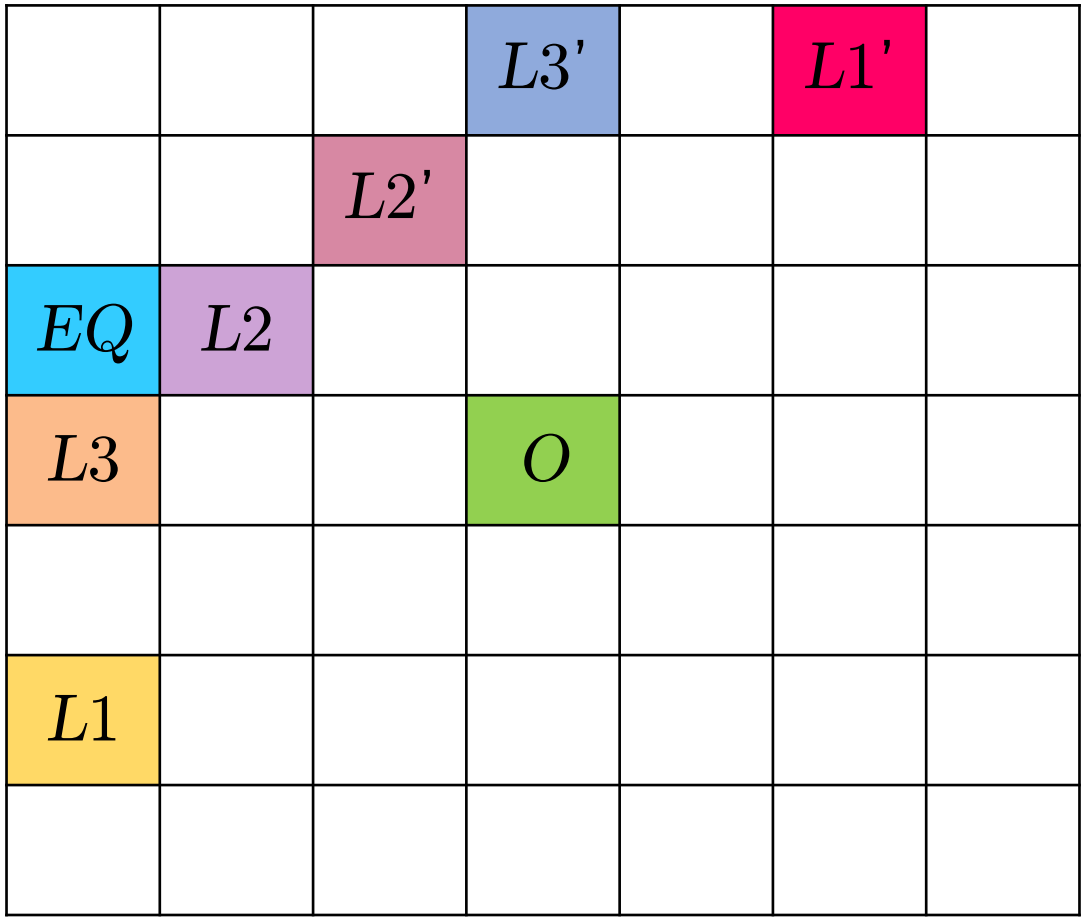
$L0$
$L1$
$L2'$
$L3$

Game 15

The Level-k Model

Your goal
Left 4, Below 2

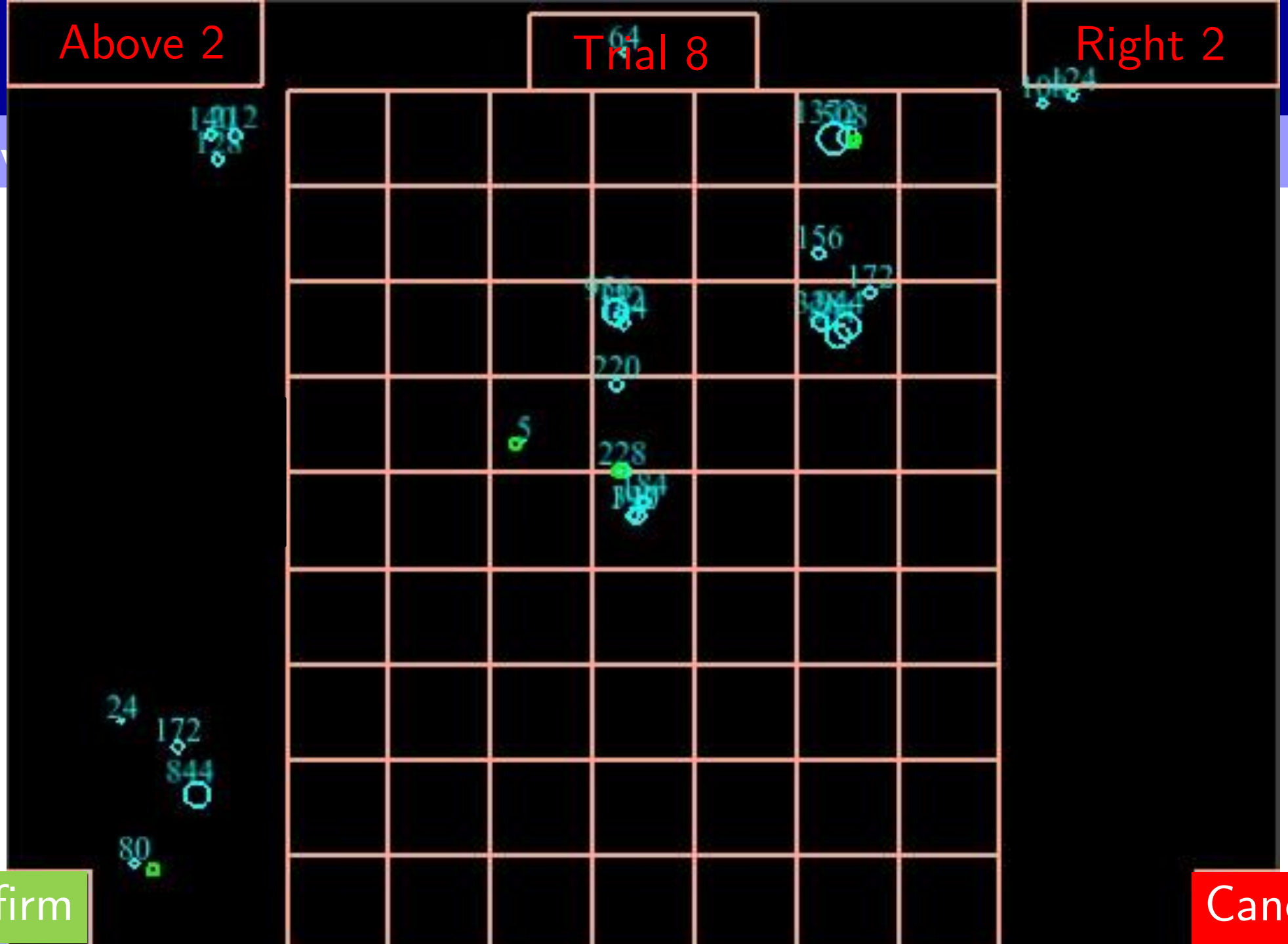
Other's goal
Right 2, Above 4



$L0$ $L1'$
$L1$ $L2$
$L2'$ $L3'$
$L3$ $EQ\ (L4)$

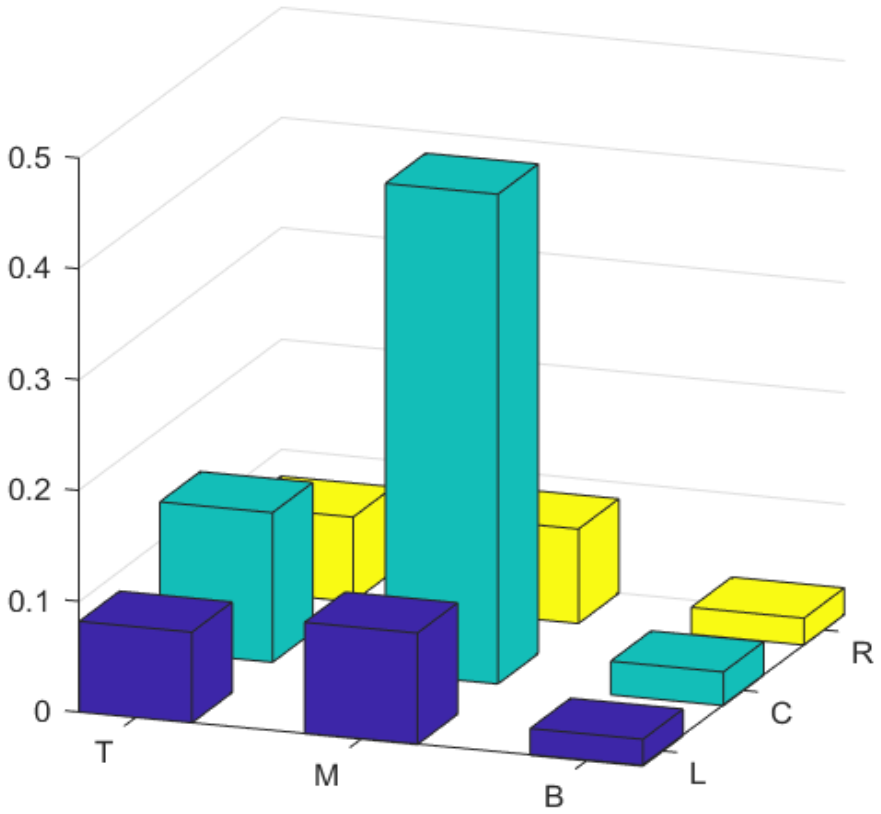
Game 15

The lev



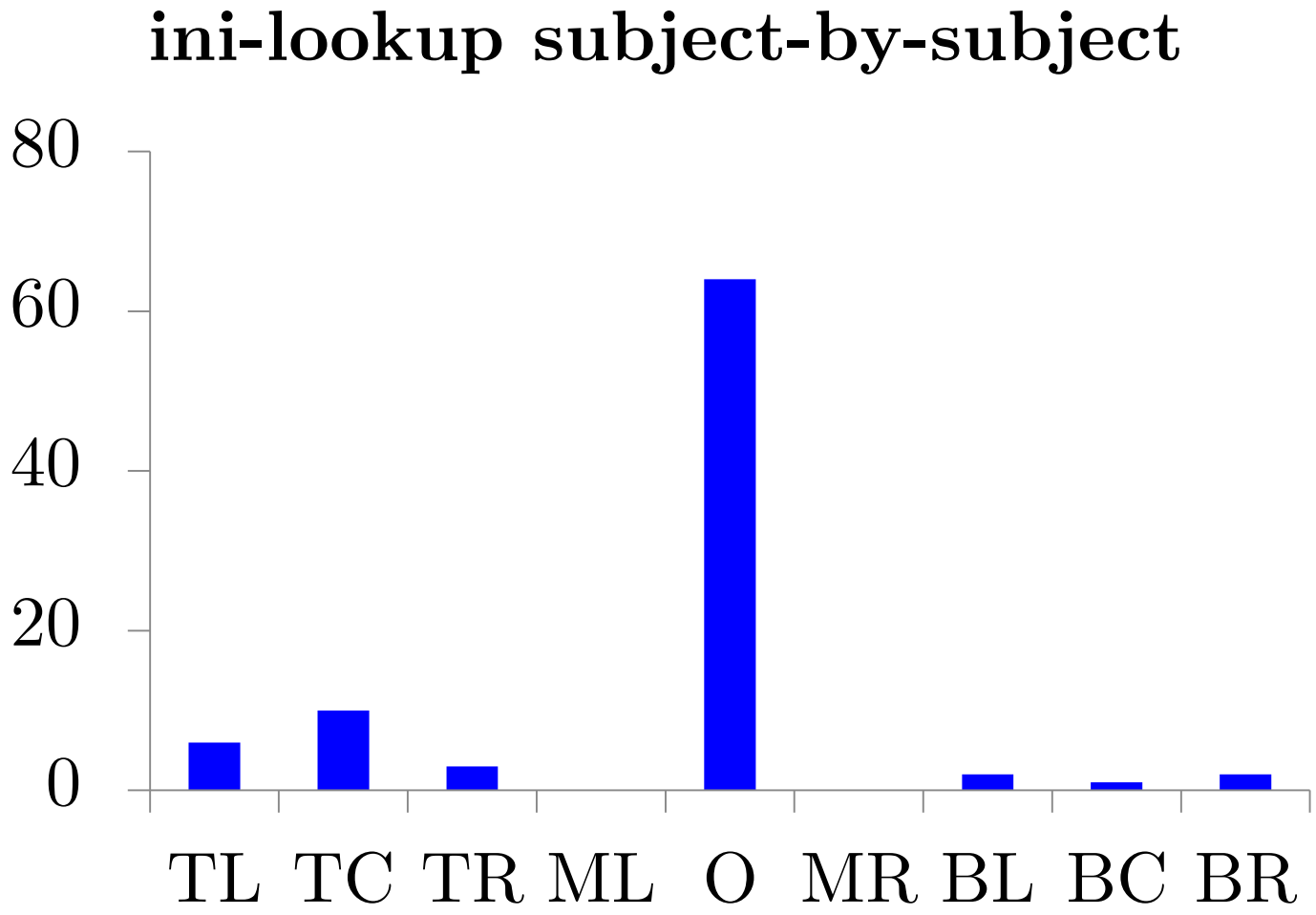
Initial Lookups

- Evenly split each map into 3×3 grids
- Percentage of fixations on each grid in the first 1% of the time
- Most lookups start from the center (L0)

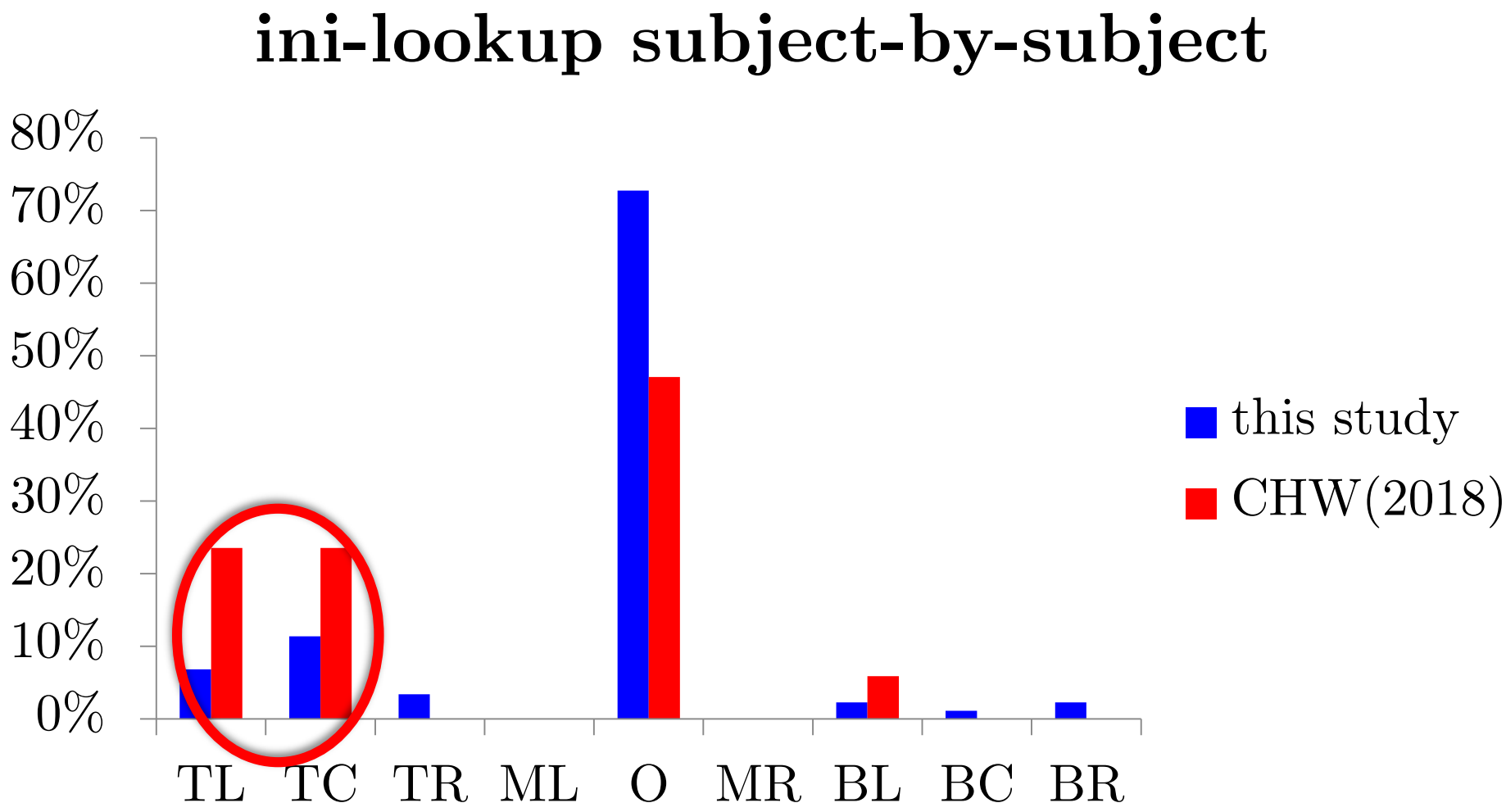


Initial lookup

- Subject's ini-lookup:
  - initial lookups fall mostly in that type.
- Only 6 and 10 people were classified as TL and TC.



No Top-left Focal Point



前言

行爲經濟學的有趣之處

進入正題之前，我想先聊聊兩位與我亦師亦友的認知心理學家：阿莫斯．特維斯基與丹尼爾．康納曼。透過幾樁關於他們的往事，或許能讓各位對於本書要表達的內涵略有掌握。

挖空心思討好特維斯基

即使是像我們這種老記不得上回把鑰匙放在哪裡的人，生命中仍不乏難以忘懷的時刻。有些是公開事件，譬如年齡與我相仿的人，就不會忘記甘迺迪總統被暗殺的那一天（當時我還是大一新鮮人，正在學校體育館的籃球場裡鬥牛）。對於年紀大到能夠閱讀本書的人，難忘的時刻應該包括2001年9月11日（我剛起床，聽著國家公共廣播電台，設法搞懂事件的來龍去脈）。

有些難忘時刻則屬私人回憶，從婚禮到一桿進洞皆爲此類。對我而言，一通來自康納曼的電話就是這樣的時刻，雖然我們常交談[illegible]電話，但是

Misbehaving

1 歷史如何終結？

無政府狀態山雨欲來？

一九八九年，法蘭西斯．福山預測「歷史的終結」，所有國家會趨向美國式的政治和經濟制度，形成他所謂的「經濟與政治自由主義定於一尊」。僅僅五年後，羅伯．柯普蘭（Robert Kaplan）就在論著《無政府狀態山雨欲來》（*Coming Anarchy*）中，描繪出截然不同的未來景象。為了說明無政府狀態這種無法無天、暴力橫行的混亂本質，他深感必須從西非開始說起：

> 西非已經成為〔無政府狀態〕的象徵……疾病、人口過剩、犯罪橫行、資源稀少、難民逃離、國家機器日益朽壞，私人軍隊、保全公司、國際毒品卡特爾[1]各自為政的現象，透過西非角度，最能明顯地展現出來。西非為討論起來極為不愉快、很快就會衝擊我們文明的這個問題，提供了適當的

Why Nation Fails?

Choice Based Classification: Method

- Model the choice by logit error structure:

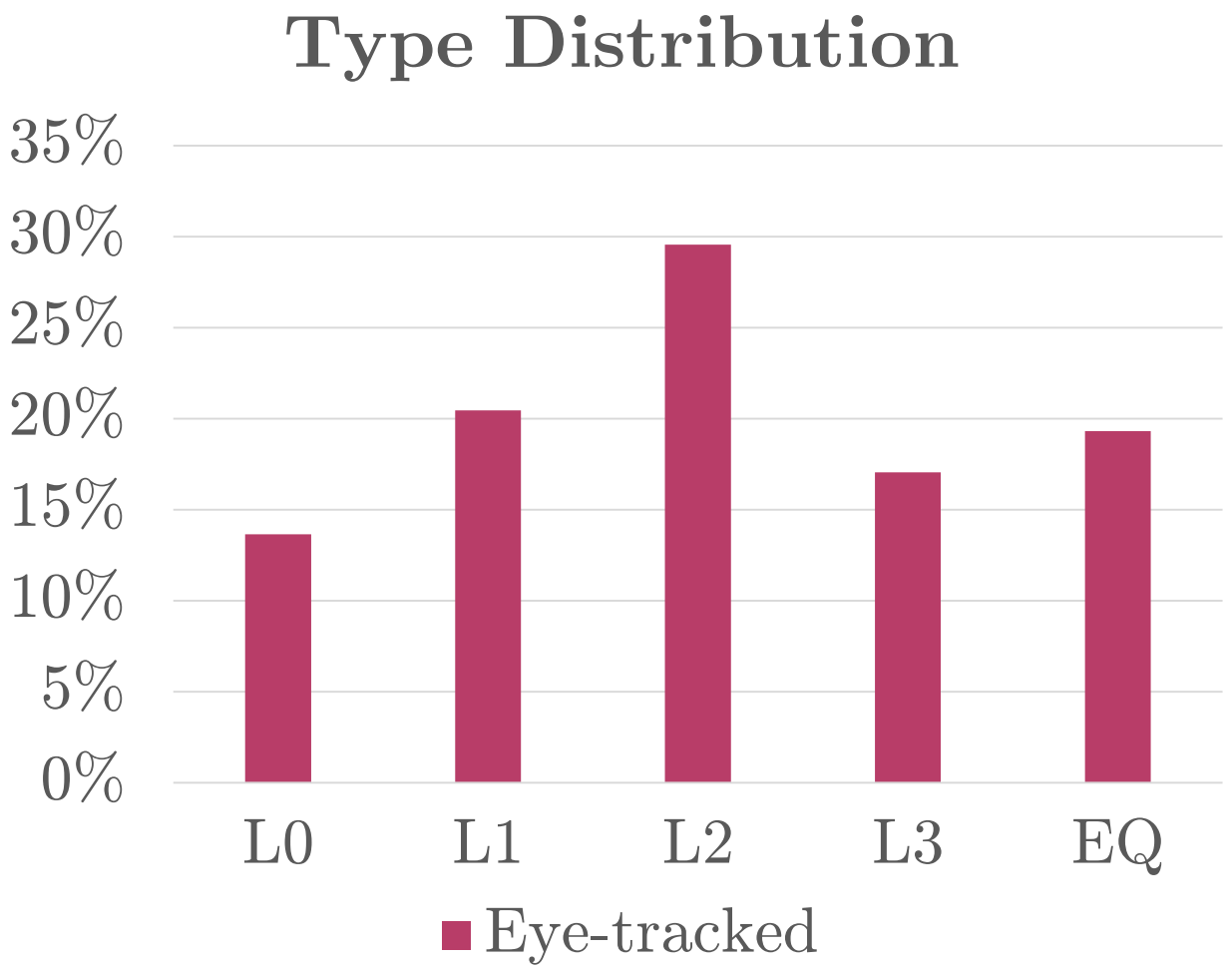
$$d^k(g_n) = \frac{\exp\left(-\lambda_k \times ||g_n - c_n^k||\right)}{\sum_{g \in G_n} \exp\left(-\lambda_k \times ||g - c_n^k||\right)}$$

- Assuming constant λ_k among trials, the MLE objective function is:

$$\sum_{n=1}^{36} \ln\left(d^k(g_n)\right)$$

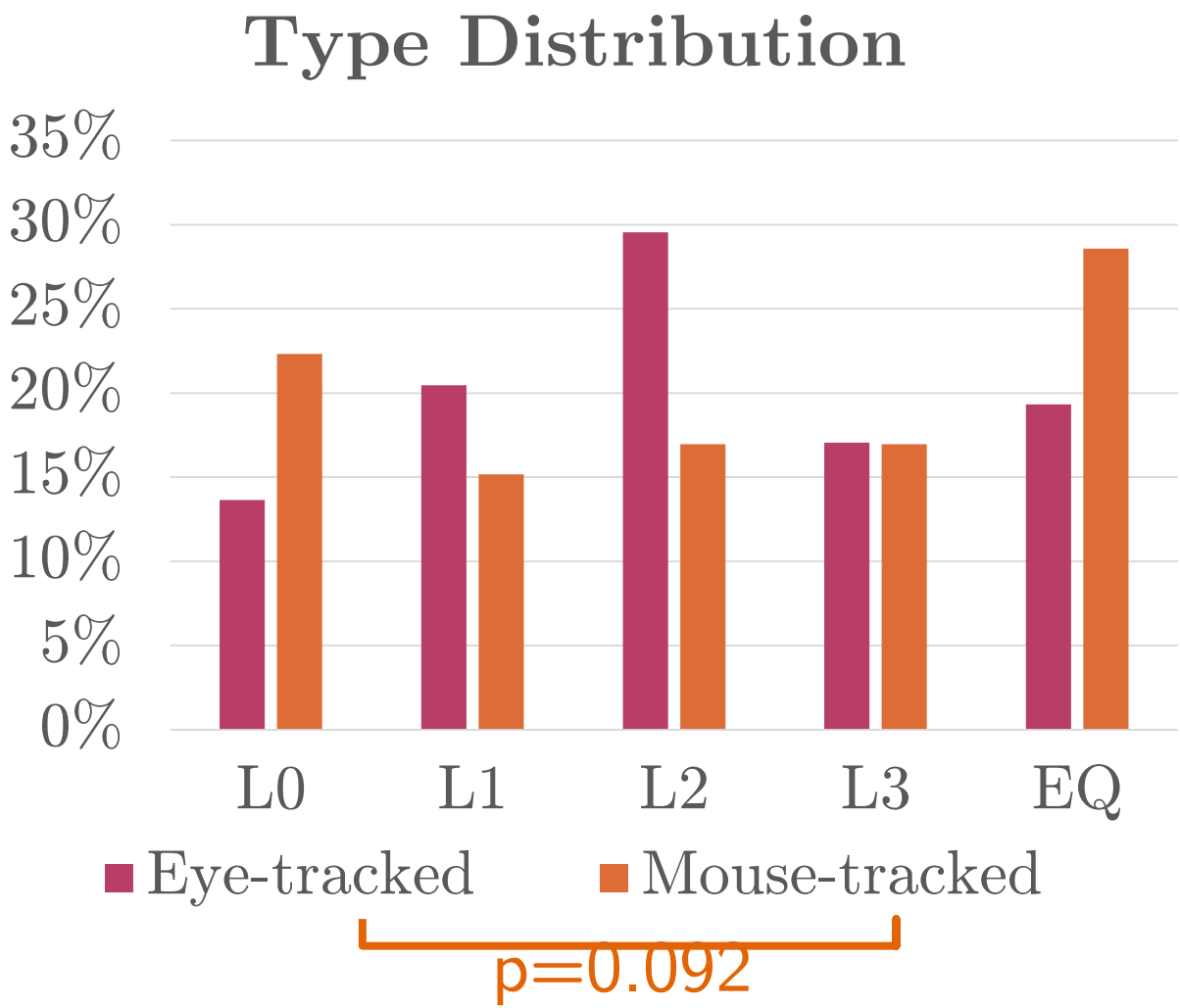
Classification Result

- Eye-tracked, N=88
  - (12, 18, 26, 15, 17)
- Average thinking steps = 2.08



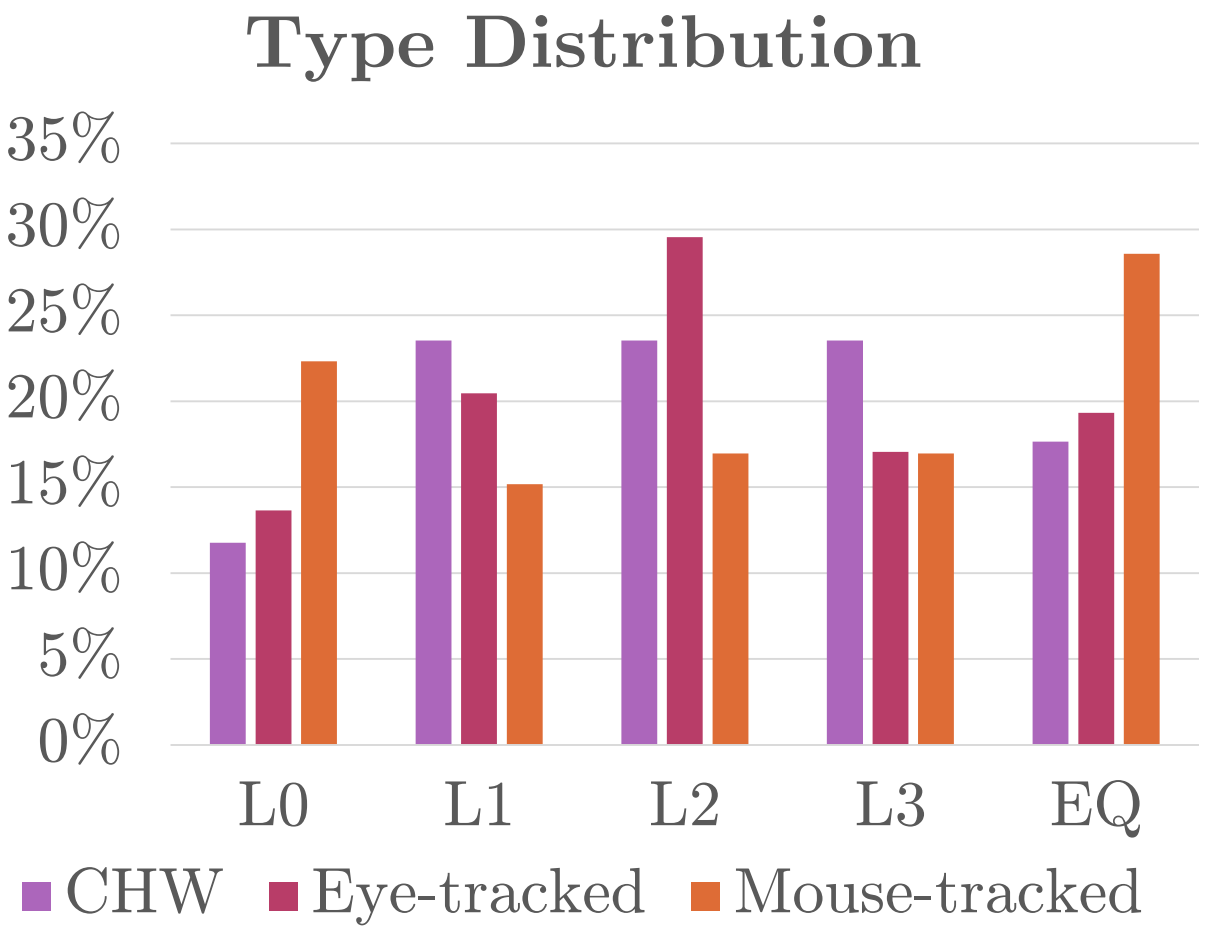
Classification Result

- Mouse-tracked, N=112
  - (25, 17, 19, 19, 32)
- Average thinking steps = 2.14
- Influence of Eye-tracking is not significant



Classification Result

- CHW, N=17
  - (2, 4, 4, 4, 3)
- 0.04 steps less than CHW



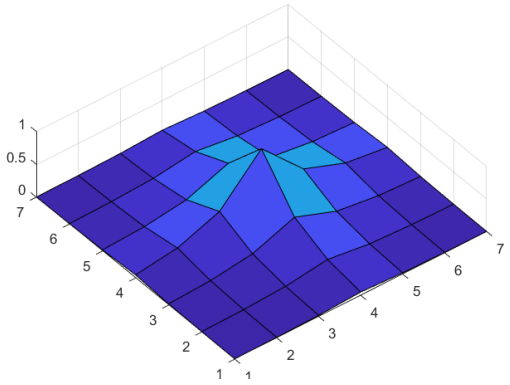
Lookup Based Classification: Method

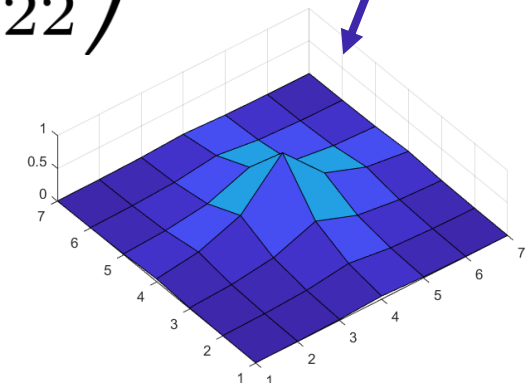
Markov switching matrix

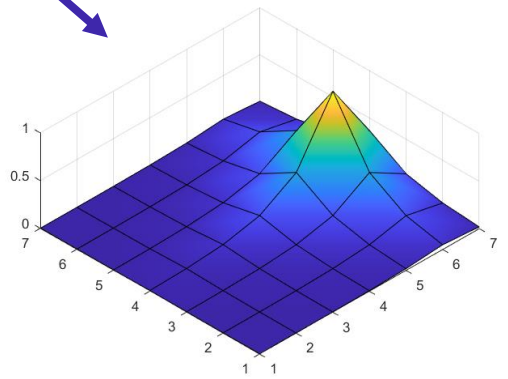
Initial lookup

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \pi_{00} & \pi_{01} & 0 \\ \pi_{10} & \pi_{11} & \pi_{12} \\ \pi_{20} & \pi_{21} & \pi_{22} \end{pmatrix} = \begin{pmatrix} \pi_{00} & \pi_{01} & 0 \end{pmatrix}$$

2nd lookup prior







Lookup Based Classification: Method

t^{th} lookup posterior

$t+1^{th}$ lookup prior

$$\begin{pmatrix} p_0^t & p_1^t & p_2^t \end{pmatrix} \begin{pmatrix} \pi_{00} & \pi_{01} & 0 \\ \pi_{10} & \pi_{11} & \pi_{12} \\ \pi_{20} & \pi_{21} & \pi_{22} \end{pmatrix} = \begin{pmatrix} \sum_{i=0}^{2} p_i^t \pi_{i0} & \sum_{i=0}^{2} p_i^t \pi_{i1} & \dots \end{pmatrix}$$

Classification Result

		Lk_c						
		L0	L1	L2	L3	EQ	RT	Lookup Class.
Lk_l	L0	5	4	6	3	3	18.75	21
	L1	1	8	1	0	1	16.07	11
	L2	2	1	7	3	1	17.83	14
	L3	3	5	9	9	8	19.41	34
	EQ	1	0	3	0	4	20.97	8
	RT	11.53	14.78	20.48	17.74	26.17	33	
	Choice Class.	12	18	26	15	17		N=88

Classification Result

		Lk_c						
		L0	L1	L2	L3	EQ	RT	Lookup Class.
Lk_l	L0	5	4	6	3	3	18.75	21
	L1	1	8	1	0	1	16.07	11
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	L3	3	5	9	9	8	19.41	34
	EQ	1	0	3	0	4	20.97	8
	RT	11.53	14.78	20.48	17.74	26.17	33	
	Choice Class.	12	18	26	15	17		N=88

Lookup x Choice

		Lk_c						
		L0	L1	L2	L3	EQ	RT	Lookup Class.
Lk_l	L0	5	4	6	3	3	18.75	21
	L1	1	8	1	0	1	16.07	11
	L2	2	1	7	3	1	17.83	14
	L3	3	5	9	9	8	19.41	34
	EQ	1	0	3	0	4	20.97	8
	RT	11.53	14.78	20.48	17.74	26.17	33	
	Choice	12	18	26	15	17		N=88

Lookup types and Choice types are not independent (Chi-Square p<0.01)

Are There Other Types?

- *Costa-Gomes and Crawford (2006)* : pseudo-types cluster

- Each subject in a pseudo-type cluster satisfies:
  - *The prediction by any other in the cluster is better than the prediction by the models.*

Pseudo-type: 15 Clusters, 20 Subjects

- This study:
  - 15 clusters
    - Overlap!!
  - 20 subjects
- CGC:
  - 5 clusters
    - Non-overlap
  - 11 subjects

2 clusters:

- 3, 69
- 5, 69
- 22, 33
- 27, 66
- 27, 69
- 35, 69
- 39, 59
- 76, 80

3 clusters:

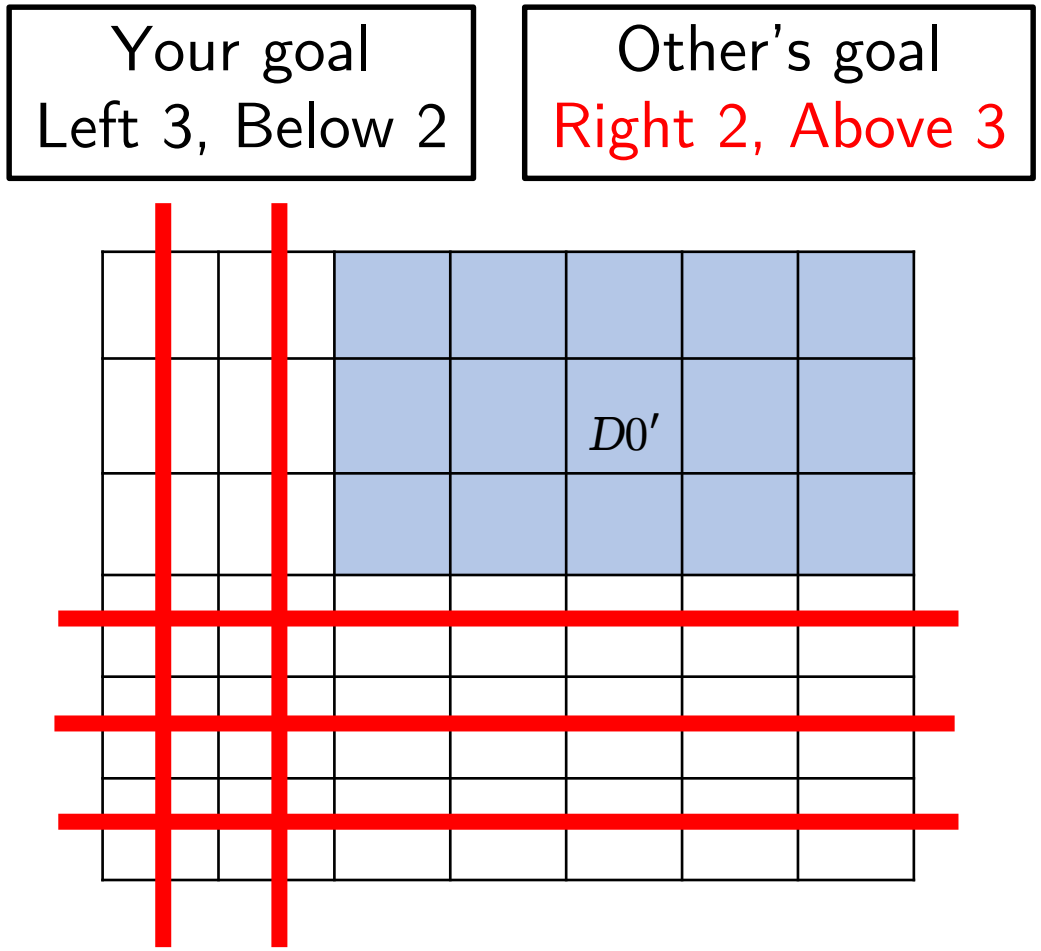
- 27,41,54
- 27,84,87
- 33,46,59

5 clusters:

- 25, 27, 46, 54, 61
- 25, 27, 46, 59, 61
- 27, 46, 54, 61, 84
- 27, 46, 54, 68, 84

Questionnaire of Subject 87

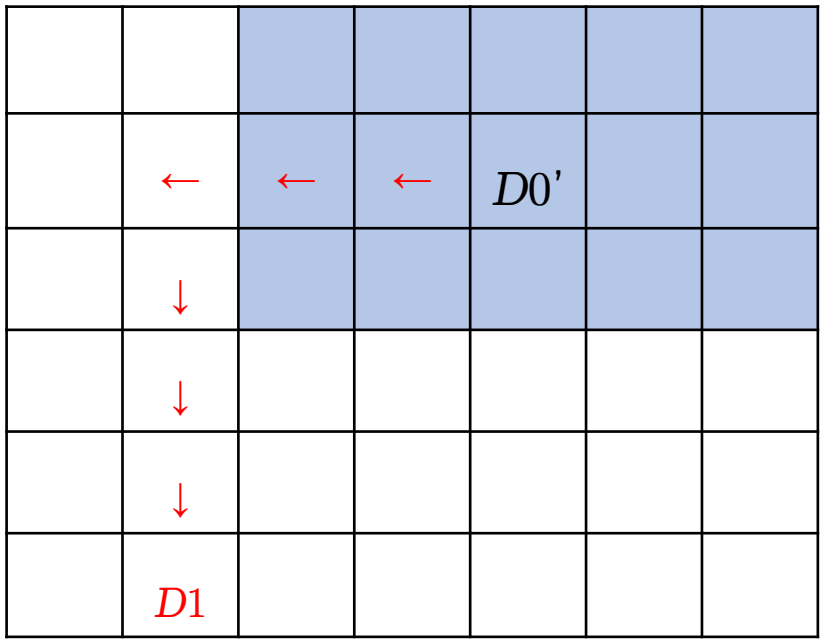
- “If the other's target is right 2 above 3, delete the leftmost two columns and the downmost three rows choose a point near center and move according to own target...”



Questionnaire of Subject 87

- "If the other's target is right 2 above 3, delete the leftmost two columns and the downmost three rows choose a point near center and move according to own target..."

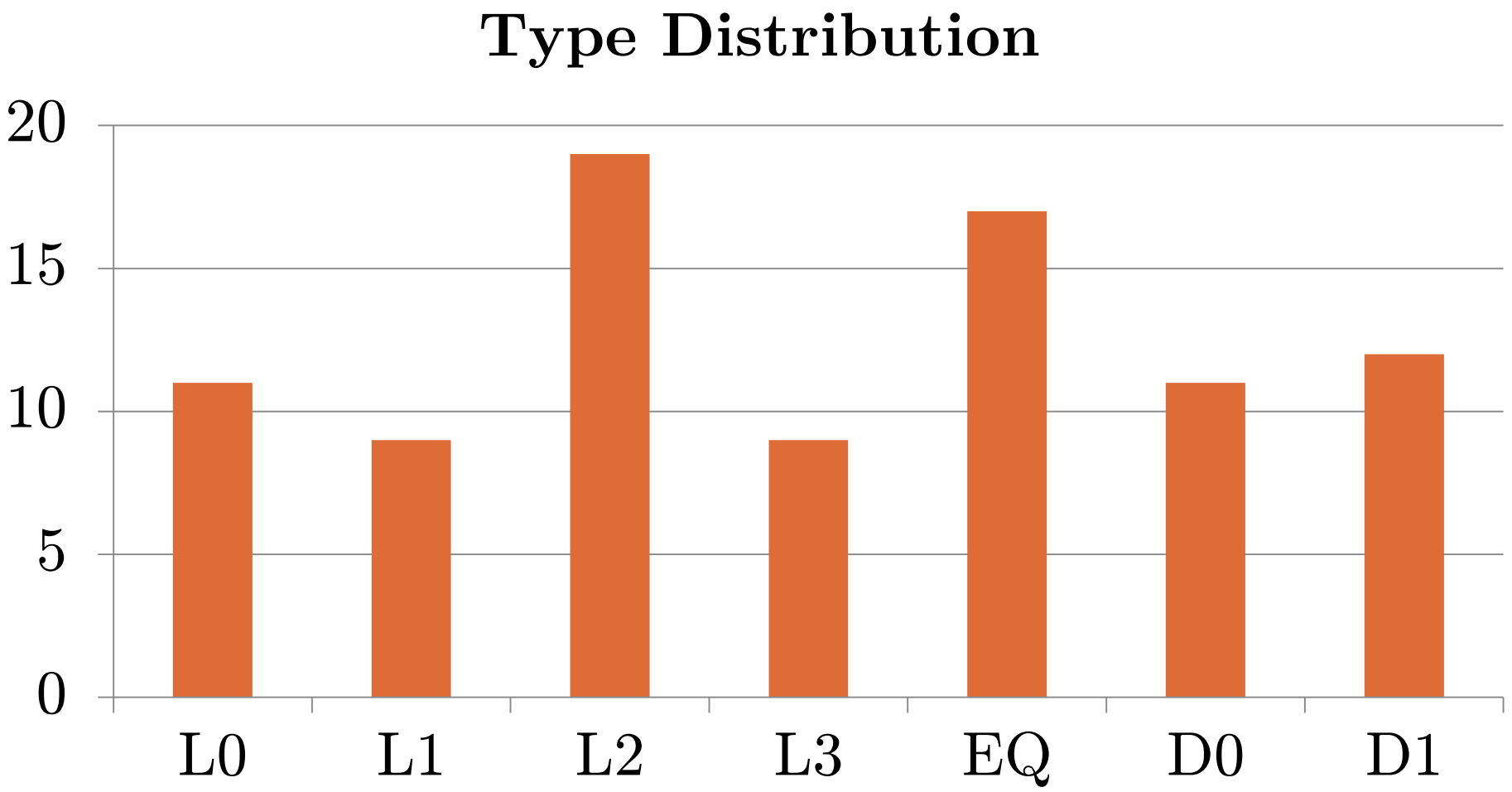
Your goal	Other's goal
Left 3, Below 4	Right 2, Above 3



The Dominance Model

- We use the definition of $D1$ in *Costa-Gomes, Crawford and Broseta (2001)* which is:
  - $D1$: *Does 1 round of deletion of dominance and best responds to a uniform prior over its partner's remaining decisions.*
  - $D0$: *Randomize over remaining decisions after 1 round of deletion.*

Choice-based Classification w/ D0 and D1 Types

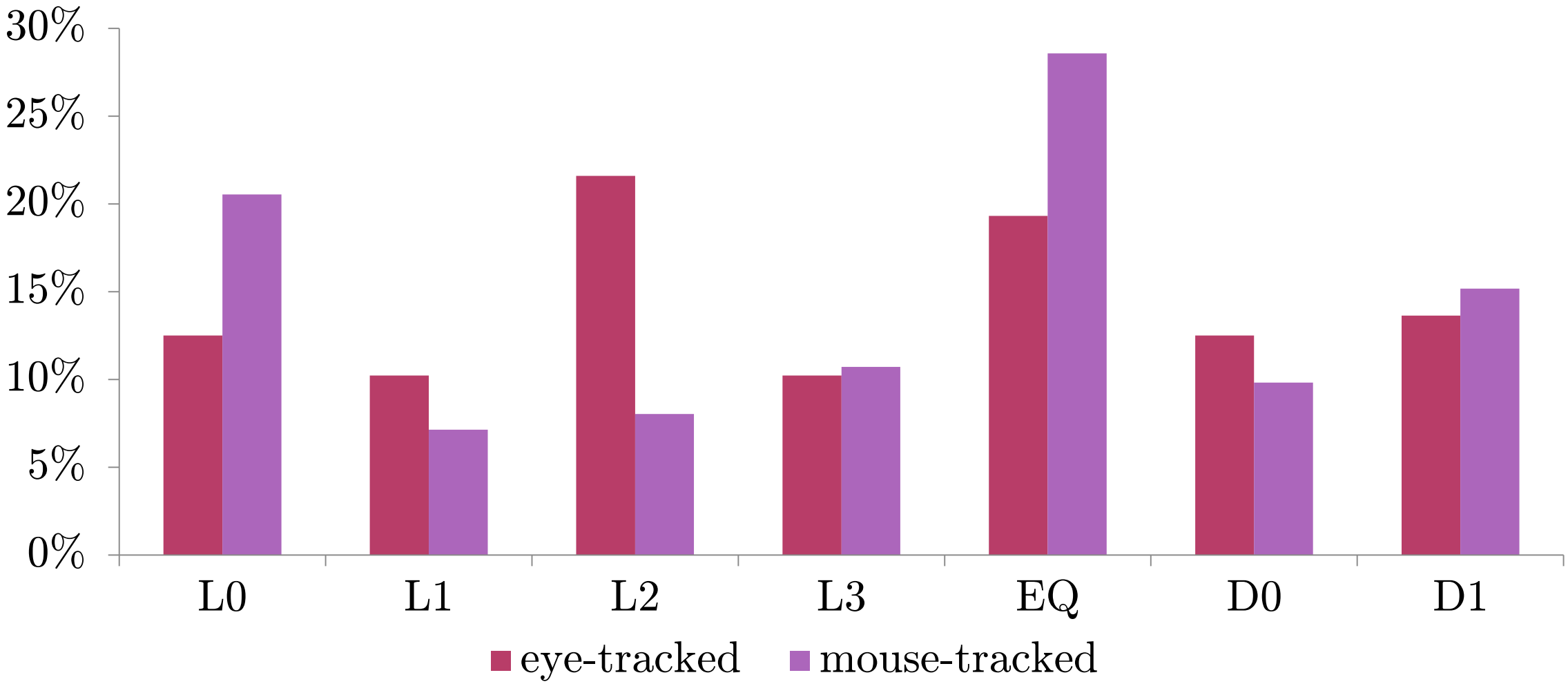


Pseudo-type Clusters

- After adding the 2 additional types...

clusters:

- 39 59
- 54 68
- 76 80
- 84 87

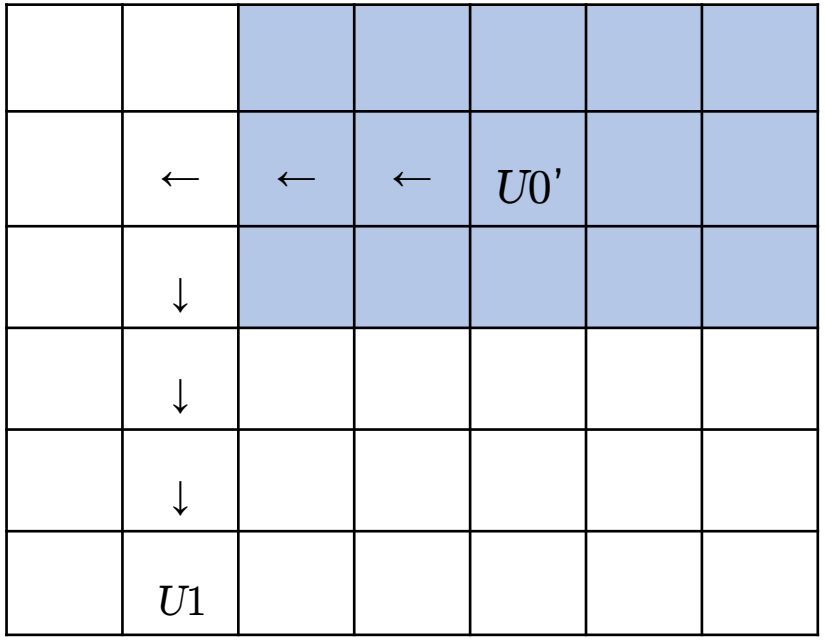

30%
25%
20%
15%
10%
5%
0%
L0
L1
L2
L3
EQ
D0
D1
eye-tracked
mouse-tracked

Omitted Types – $U0 \setminus U1$

- Lookup of $D0$ & $D1$ cannot be model.

- Define U type:
  Start level-k reasoning from the undominated location.

Your goal Left 3, Below 4	Other's goal Right 2, Above 3



Lookup x Choice

Lk_l \ Lk_c	L0	L1	L2	L3	EQ	U0	U1	RT_l	Lk_l
L0	**4**	3	4	0	3	2	3	18.5	19
L1	1	**3**	0	0	1	1	0	14.8	6
L2	2	0	**5**	2	0	1	0	15.1	10
L3	2	1	7	**6**	6	3	6	19.1	31
EQ	1	0	2	0	**4**	0	1	21.0	8
U0	1	2	0	1	3	**4**	0	24.2	11
U1	0	0	1	0	0	0	**2**	10.2	3
RT_c	11.0	12.0	20.2	19.7	26.2	17.7	18.1	**28**	
Lk_c	11	9	19	9	17	11	12		

Omitted Types

- 6 out of 88 subjects are classified as $U0$ and $U1$ consistently.

- The others with $U0$, $U1$ choice might be $D0$, $D1$ subjects.

Summary

- Top-left bias might be related to writing system
- 33 out of 88 follow level-k reasoning
- Identify 23 $D0$ and $D1$ with pseudo-type clusters of choice
- Use $U0$ and $U1$ to model the D-type lookup

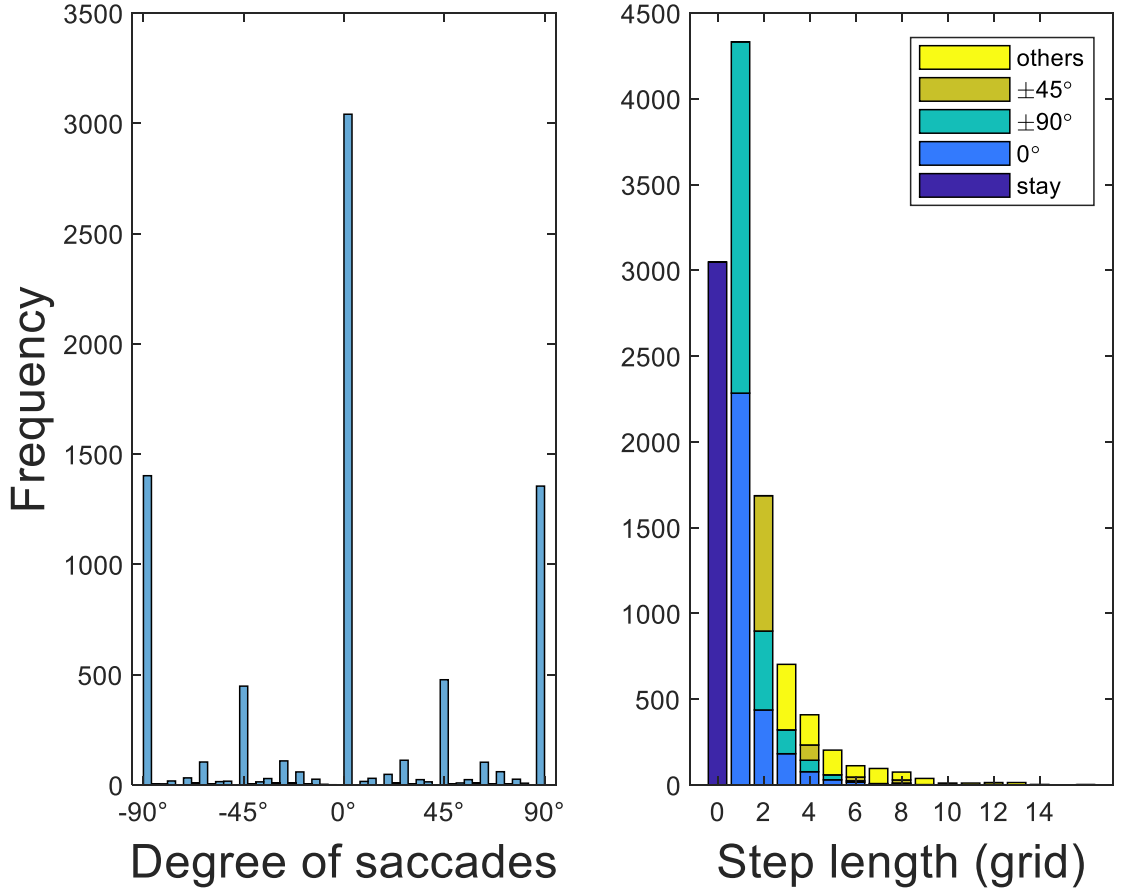
Applying Machine Learning Methods to Eyetracking

- EMHMM in Cognitive Science
  - Chuk, T., Chan, A. B., & Hsiao, J. H. (2014). Understanding eye movements in face recognition using hidden Markov models. *Journal of Vision*, 14(11):8, 1-14.
- Preliminary Results: HMM with logistic errors

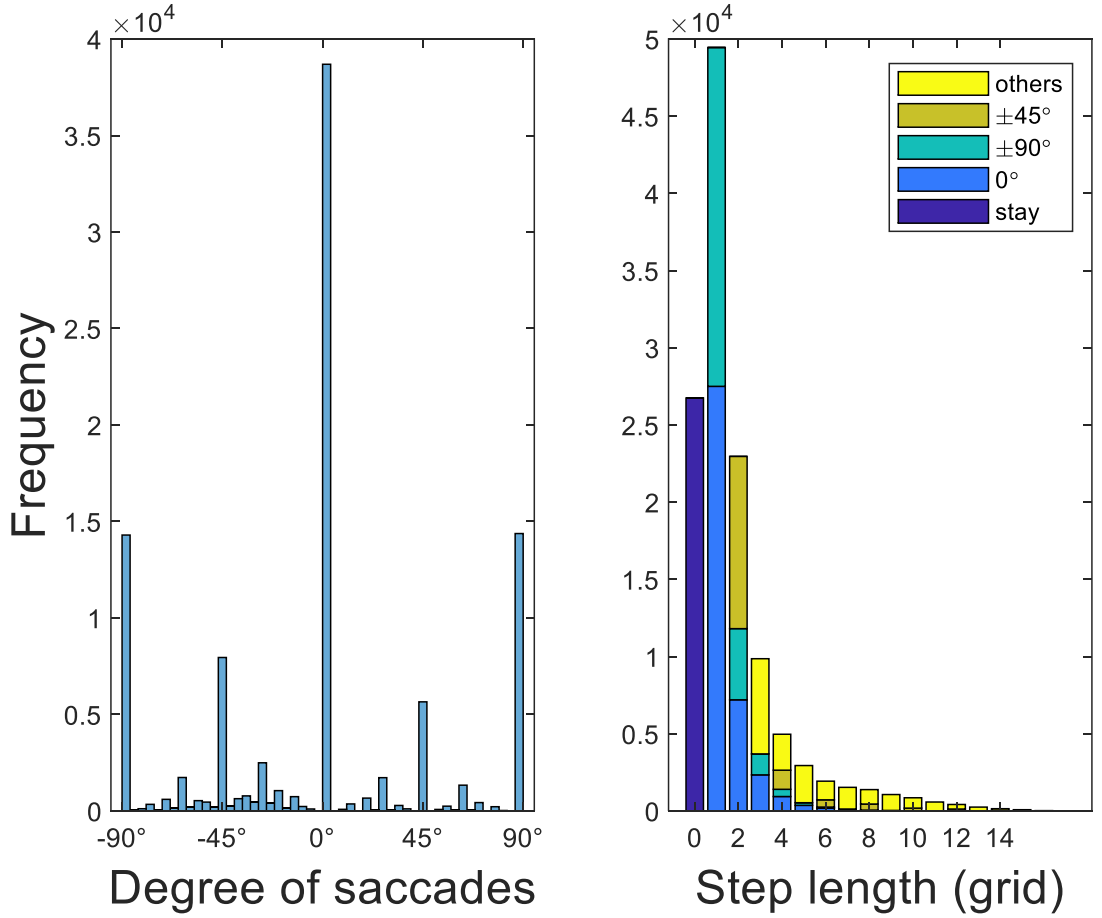
1. EM w/o predefined Lk model captures literal Lk data
2. VBEM with Lk model can predict subjects' choices

Counting grids

CHW2018 (N = 17)

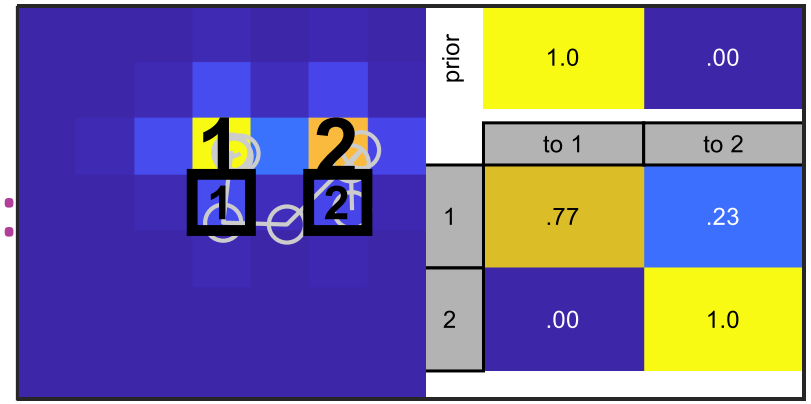


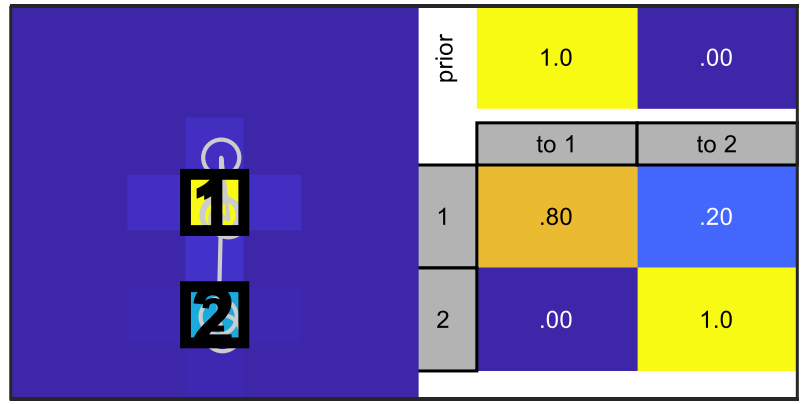
New data (N = 88)

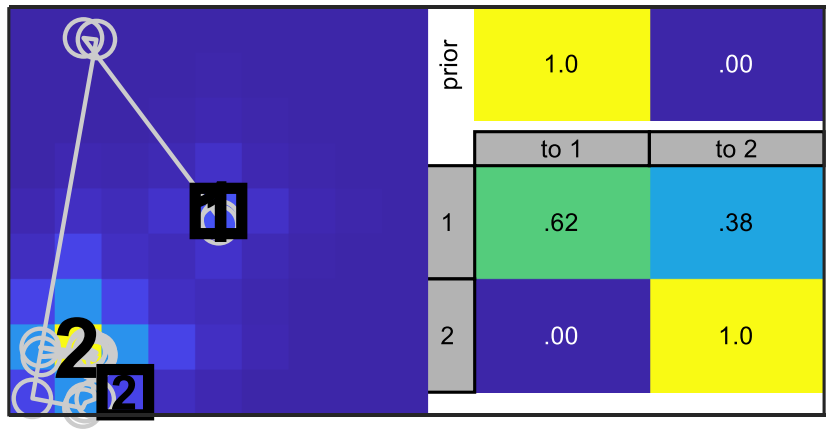


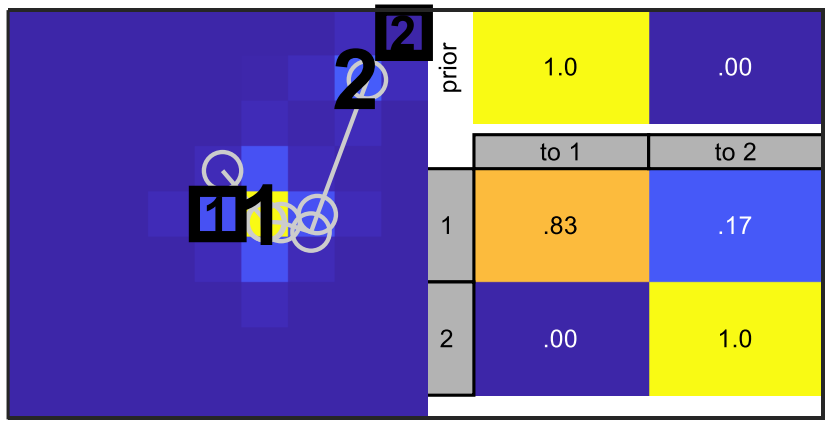
EM w/o predef-Lk: CHW 2018, Subject #10 (Literal L1)

- No predefined states
  - Noise reduction:
  - Drop isolated fixations
- L1 states
  - vs.
- EM-estimated states

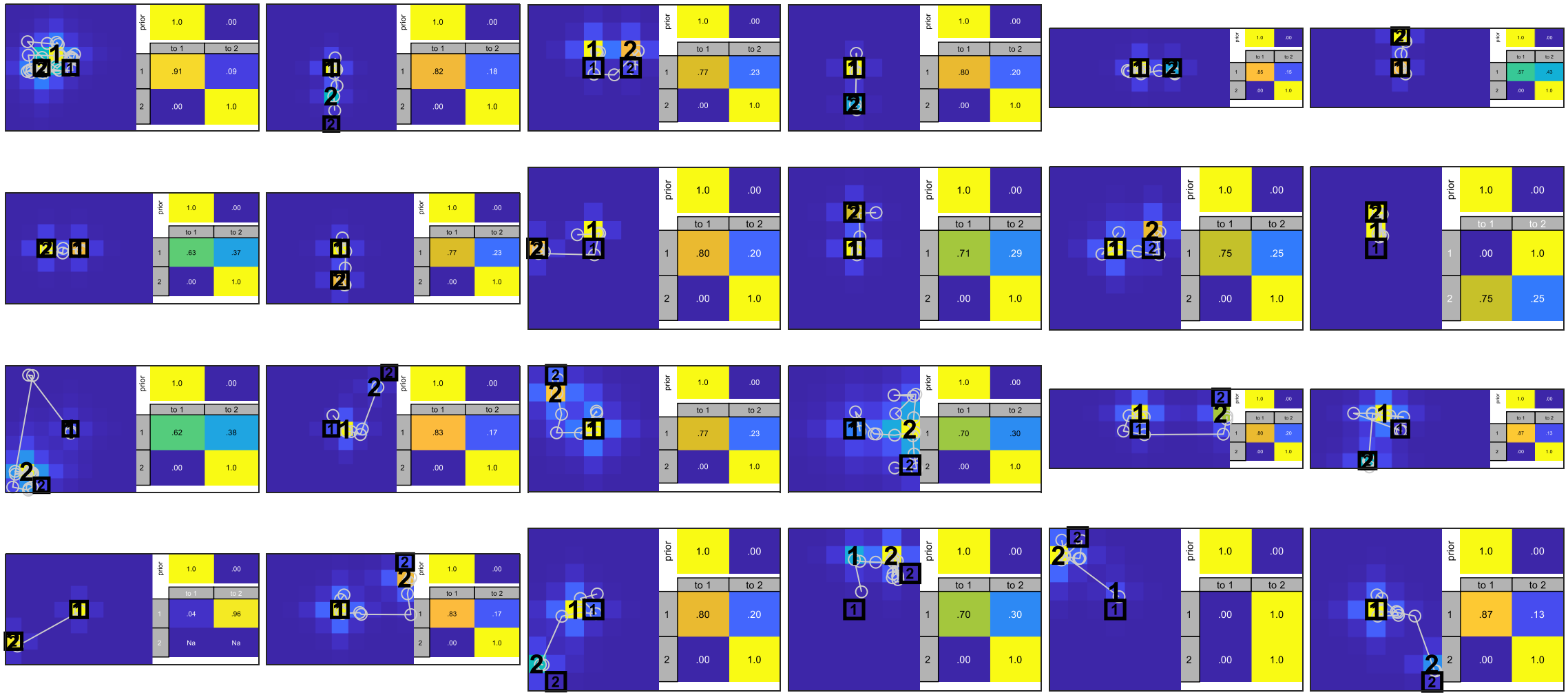








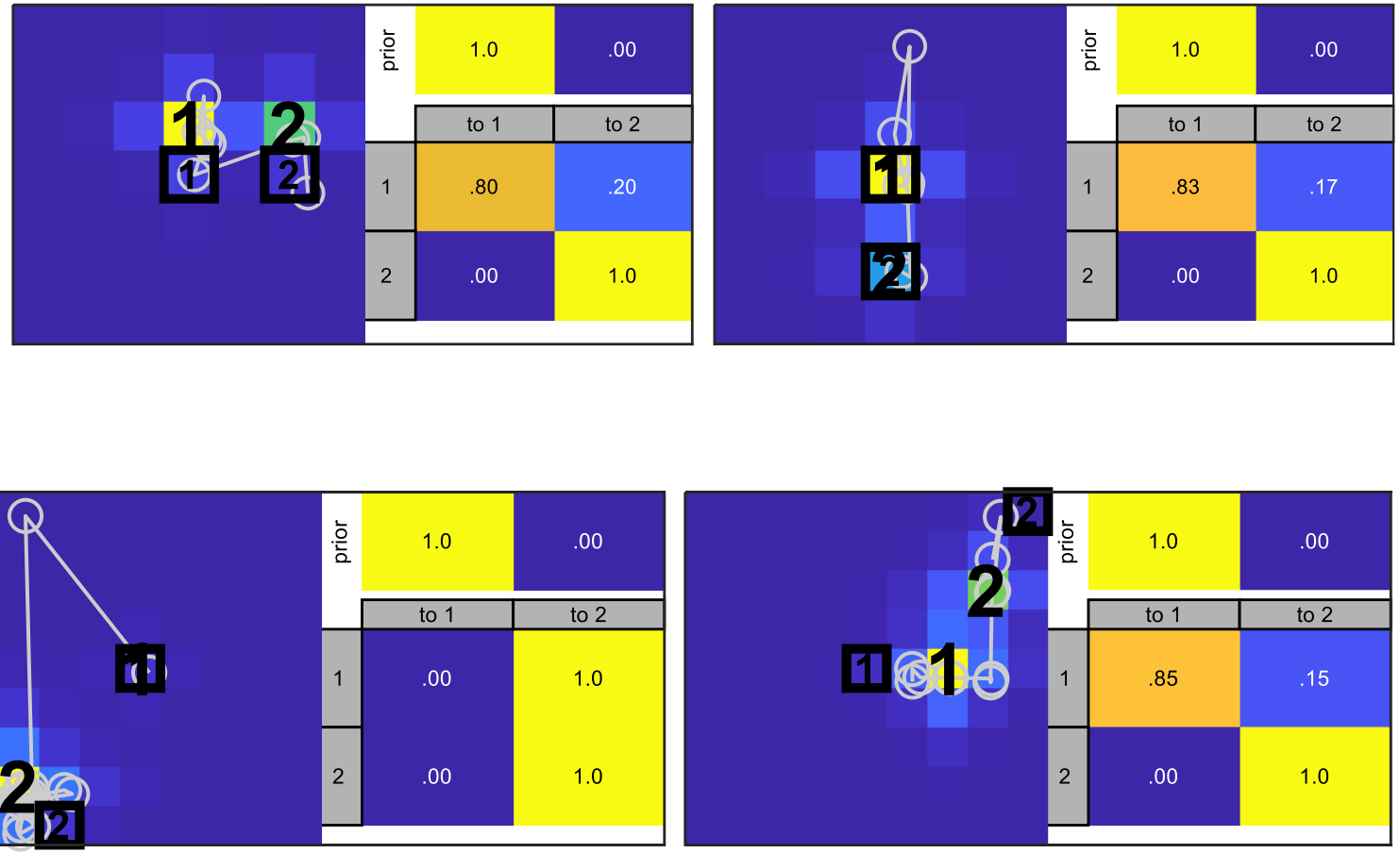
EM w/o predef-Lk: CHW 2018, Subject #10 (Literal L1)



#10 literal L1 ($\lambda = 0.93$) rep = 100, prior fixed. Keep only 90°, 0°

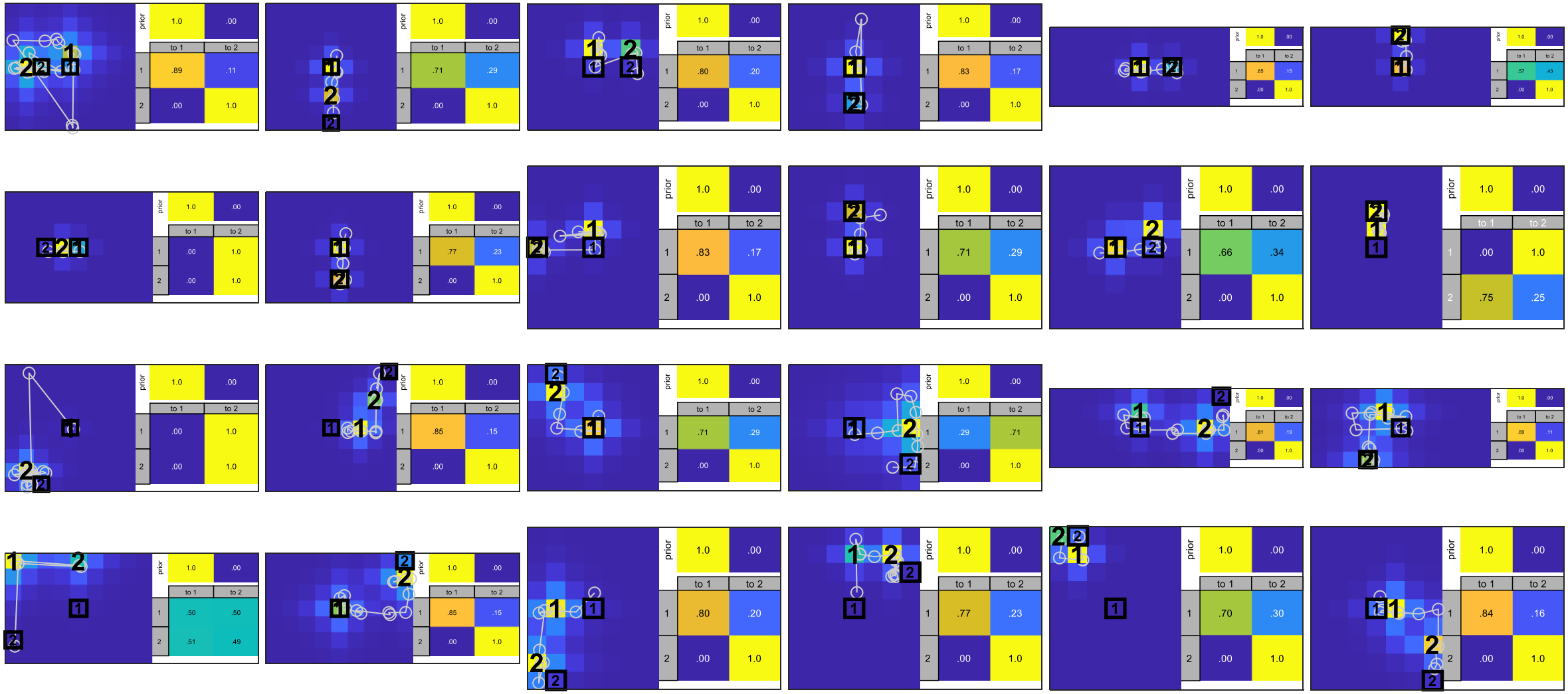
Preliminary Result

- No predefined states
  - Noise reduction:
  - Keep horizontal & vertical saccade
- L1 states
  - vs.
- EM-estimated states



Preliminary Result

#10 literal L1 ($\lambda = 0.93$) rep = 100, prior fixed. Keep only 90°, 0°



VBEM with Lk model

VB \ CHW(Choice)	L0	L1	L2	L3	L4	U0	U1	SUM
L0	2	1	0	0	1	0	0	4
L1	3	6	0	0	0	3	1	13
L2	4	1	9	1	2	2	2	21
L3	1	1	9	7	3	6	6	33
L4	1	0	1	1	11	0	2	16
U0	0	0	0	0	0	0	0	0
U1	0	0	0	0	0	0	1	1
SUM	11	9	19	9	17	11	12	36

VB \ CHW(Lookup)	L0	L1	L2	L3	L4	U0	U1	SUM
L0	3	1	0	0	0	0	0	4
L1	2	3	1	4	0	3	0	13
L2	7	0	5	5	2	1	1	21
L3	4	1	4	18	1	4	1	33
L4	3	1	0	4	5	3	0	16
U0	0	0	0	0	0	0	0	0
U1	0	0	0	0	0	0	1	1
SUM	19	6	10	31	8	11	3	35

Follow-up

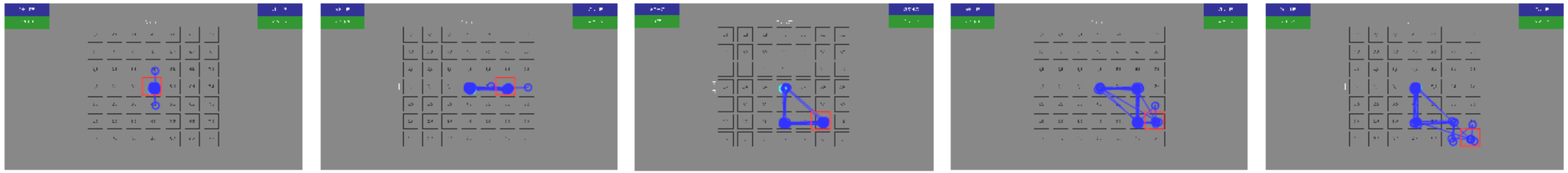
- Preliminary results:
  - EM w/o predefined Lk model captures literal Lk data
  - VBEM with Lk model can predict subjects' choices

- Next step: Build machine learning tool to discover strategies
  - VBEM without predefined models
  - Cluster HMMs (e.g., Lan & Chen, 2021)

Simulate CHW Level-k Lookups ($\lambda = 5$, 35 fixations)

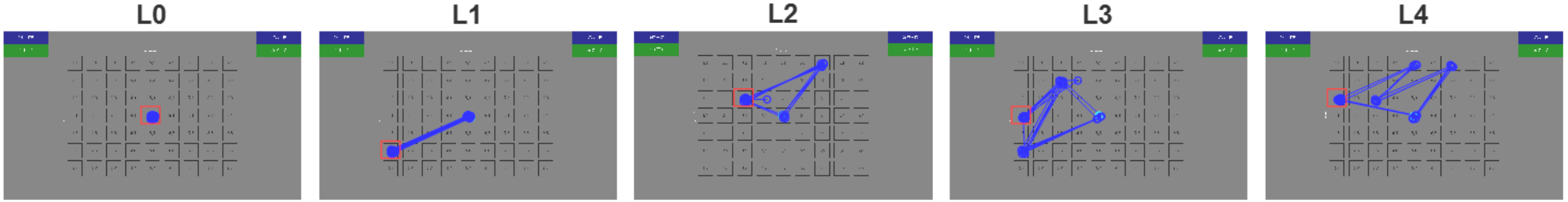
- Own Target: (2, 0) Other Target: (0,-2)

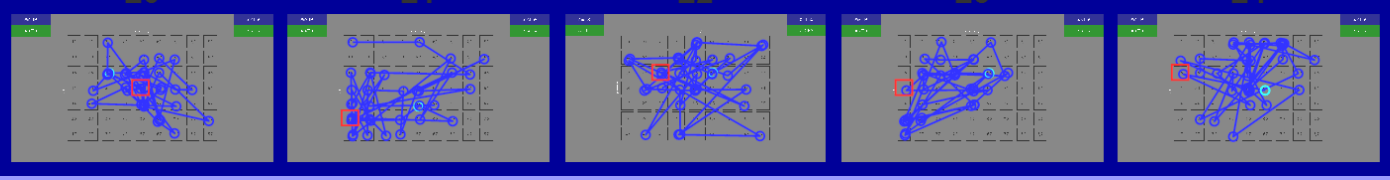
L0 L1 L2 L3 L4



- Own Target: (-4,-2) Other Target: (2, 4)

L0 L1 L2 L3 L4





CHW: Well Recovered for $\lambda = 0.6$ (N = 100)

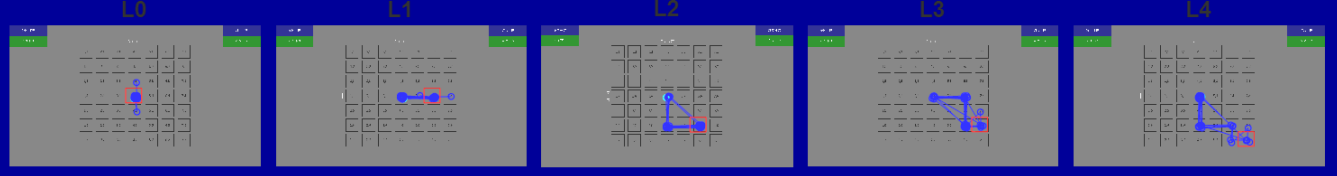
(41/88 of our subjects have estimated $\lambda > 0.6$)

36 Games (k)

	L0	L1	L2	L3	L4
L0	**86%**	11%	2%	1%	-
L1	-	**100%**	-	-	-
L2	-	-	**84%**	-	16%
L3	-	-	-	**100%**	-
L4	-	-	-	-	**100%**

36 Games (λ)

	L0	L1	L2	L3	L4
L0	**0.60** (0.02)	0.60 (0.01)	0.60 (0.01)	0.58 (0.00)	-
L1	-	**0.60** (0.02)	-	-	-
L2	-	-	**0.60** (0.03)	-	0.60 (0.03)
L3	-	-	-	**0.60** (0.03)	-
L4	-	-	-	-	**0.60** (0.02)

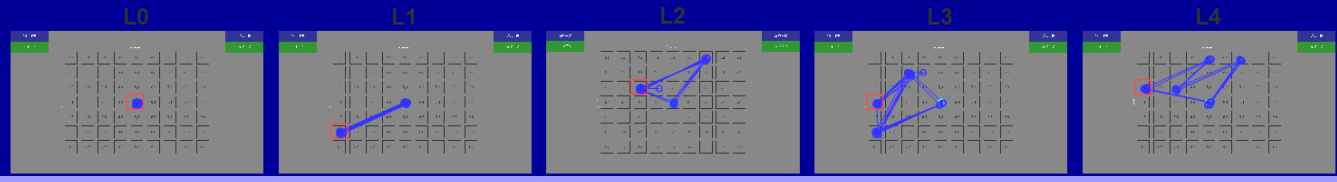


CHW: Well Recovered for $\lambda = 0.6$ ($N = 100$)

Still recover well using only 12 Easy games!

	12 Easy Games (k)				
	L0	**L1**	**L2**	**L3**	**L4**
L0	**72%**	20%	8%	-	-
L1	-	**100%**	-	-	-
L2	-	-	**99%**	-	1%
L3	-	-	-	**100%**	-
L4	-	-	-	-	**100%**

	12 Easy Games (λ)				
	L0	**L1**	**L2**	**L3**	**L4**
L0	**0.60** (0.04)	0.61 (0.04)	0.60 (0.02)	-	-
L1	-	**0.59** (0.04)	-	-	-
L2	-	-	**0.60** (0.04)	-	0.57 (0.00)
L3	-	-	-	**0.60** (0.04)	-
L4	-	-	-	-	**0.61** (0.04)

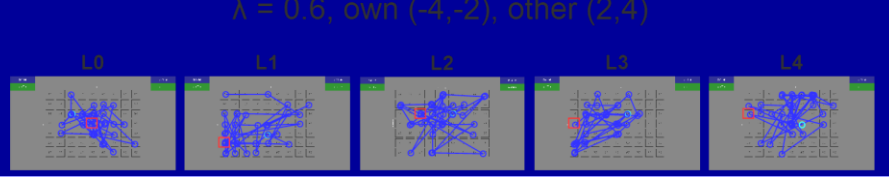


CHW: Well Recovered for λ = 0.6 (N = 100)

Or 12 Hard games!

	12 Hard Games (k)				
	L0	**L1**	**L2**	**L3**	**L4**
L0	**74%**	22%	3%	1%	-
L1	-	**100%**	-	-	-
L2	-	-	**97%**	-	3%
L3	-	-	-	**100%**	-
L4	-	-	1%	-	**99%**

	12 Hard Games (λ)				
	L0	**L1**	**L2**	**L3**	**L4**
L0	**0.60** (0.04)	0.61 (0.04)	0.63 (0.01)	0.49 (0.00)	-
L1	-	**0.60** (0.04)	-	-	-
L2	-	-	**0.61** (0.04)	-	0.59 (0.04)
L3	-	-	-	**0.60** (0.04)	-
L4	-	-	0.58 (0.00)	-	**0.60** (0.04)



CHW: Not Recovered for λ = 0.4 (N = 100)

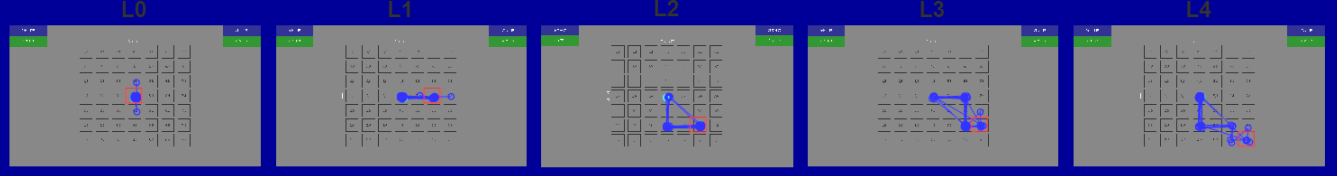
(30/88 of our subjects have estimated λ < 0.5; 16/88 have λ < 0.4!)

36 Games (k)

	L0	L1	L2	L3	L4
L0	**83%**	11%	6%	-	-
L1	-	**66%**	-	34%	-
L2	-	-	**65%**	-	35%
L3	-	-	-	**100%**	-
L4	-	-	-	-	**100%**

36 Games (λ)

	L0	L1	L2	L3	L4
L0	**0.40** (0.02)	0.39 (0.01)	0.40 (0.02)	-	-
L1	-	**0.37** (0.05)	-	0.39 (0.05)	-
L2	-	-	**0.37** (0.03)	-	0.40 (0.02)
L3	-	-	-	**0.35** (0.04)	-
L4	-	-	-	-	**0.35** (0.05)



CHW: Not Recovered for λ = 0.4 (N = 100)

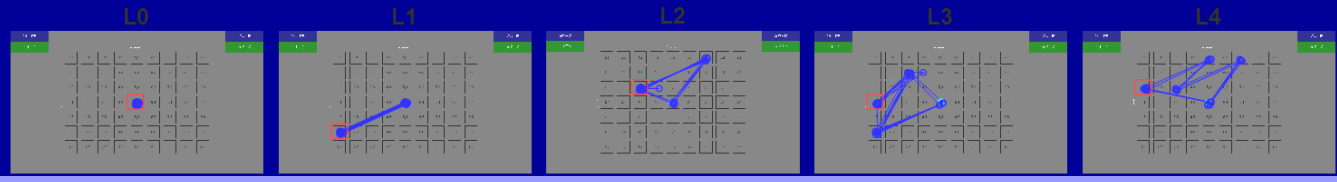
Neither Easy games!

12 Easy Games (k)

	L0	L1	L2	L3	L4
L0	**86%**	13%	1%	-	-
L1	-	**79%**	-	21%	-
L2	-	-	**93%**	3%	4%
L3	-	-	11%	**89%**	-
L4	-	-	18%	3%	**79%**

12 Easy Games (λ)

	L0	L1	L2	L3	L4
L0	**0.40** (0.03)	0.38 (0.03)	0.40 (0.00)	-	-
L1	-	**0.39** (0.05)	-	0.40 (0.03)	-
L2	-	-	**0.39** (0.05)	0.37 (0.03)	0.43 (0.02)
L3	-	-	0.32 (0.05)	**0.40** (0.05)	-
L4	-	-	0.35 (0.03)	0.35 (0.02)	**0.41** (0.03)



CHW: Not Recovered for $\lambda = 0.4$ ($N = 100$)

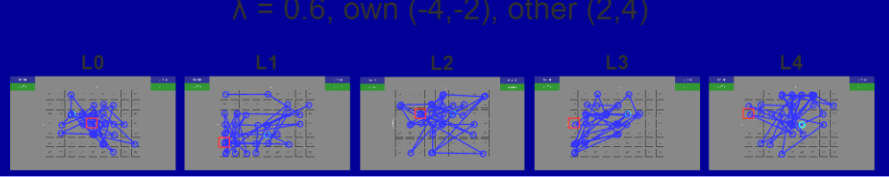
Nor Hard games!

12 Hard Games (k)

	L0	L1	L2	L3	L4
L0	**81%**	17%	2%	-	-
L1	-	**69%**	-	31%	-
L2	-	-	**96%**	-	4%
L3	-	2%	1%	**97%**	-
L4	-	-	51%	-	**49%**

12 Hard Games (λ)

	L0	L1	L2	L3	L4
L0	**0.40** (0.03)	0.40 (0.04)	0.42 (0.02)	-	-
L1	-	**0.38** (0.05)	-	0.41 (0.04)	-
L2	-	-	**0.40** (0.04)	-	0.40 (0.04)
L3	-	0.21 (0.03)	0.28 (0.00)	**0.39** (0.05)	-
L4	-	-	0.37 (0.04)	-	**0.40** (0.05)

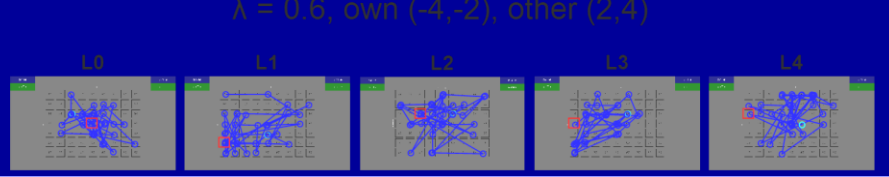


VBEM: Recovered for $\lambda = 0.6$ ($N = 100$)

Even with only 12 Easy or Hard games!

	12 Easy Games (k)				
	L0	**L1**	**L2**	**L3**	**L4**
L0	100%	-	-	-	-
L1	-	100%	-	-	-
L2	-	-	100%	-	-
L3	-	-	-	100%	-
L4	-	-	-	-	100%

	12 Hard Games (k)				
	L0	**L1**	**L2**	**L3**	**L4**
L0	100%	-	-	-	-
L1	-	100%	-	-	-
L2	-	-	97%	-	3%
L3	-	-	-	100%	-
L4	-	-	6%	-	94%



VBEM: Recovered for λ = 0.6 (N = 100)

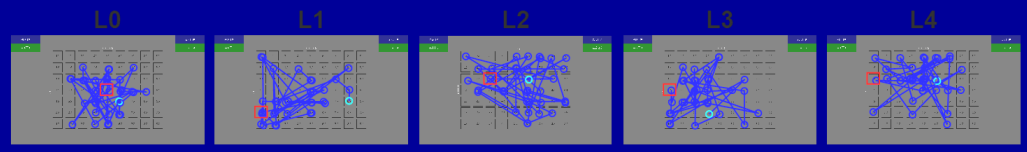
Or, 6 Easy or Hard games!

6 Easy Games (k)

	L0	L1	L2	L3	L4
L0	100%	-	-	-	-
L1	-	100%	-	-	-
L2	-	-	94%	-	6%
L3	-	1%	-	99%	-
L4	-	-	6%	-	94%

6 Hard Games (k)

	L0	L1	L2	L3	L4
L0	100%	-	-	-	-
L1	-	100%	-	-	-
L2	-	-	94%	-	6%
L3	-	-	-	100%	-
L4	-	-	7%	-	93%



VBEM: Still Okay for $\lambda = 0.4$ ($N = 100$)

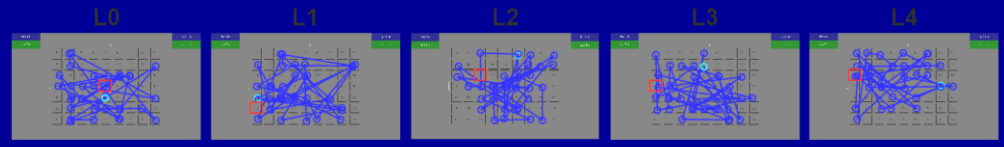
Even with only 6 Easy or Hard games!

6 Easy Games (k)

	L0	L1	L2	L3	L4
L0	100%	-	-	-	-
L1	-	100%	-	-	-
L2	-	-	89%	2%	9%
L3	-	-	-	100%	-
L4	-	-	5%	2%	93%

6 Hard Games (k)

	L0	L1	L2	L3	L4
L0	100%	-	-	-	-
L1	-	100%	-	-	-
L2	-	-	70%	-	30%
L3	-	2%	-	98%	-
L4	-	-	13%	-	87%



VBEM: Or All 36 Games with λ = 0.2 (N = 100)

λ = 0.2 vs. λ = 0.6

	36 Games (k)				
	L0	**L1**	**L2**	**L3**	**L4**
L0	100%	-	-	-	-
L1	-	100%	-	-	-
L2	-	-	81%	-	19%
L3	-	-	-	100%	-
L4	-	-	3%	-	97%

	36 Games (λ)				
	L0	**L1**	**L2**	**L3**	**L4**
L0	100%	-	-	-	-
L1	-	100%	-	-	-
L2	-	-	100%	-	-
L3	-	-	-	100%	-
L4	-	-	-	-	100%

slido

Audience Q&A Session

ⓘ Start presenting to display the audience questions on this slide.

2021/12/10

Thank You

Bootstrap & Vuong's test

Perform **bootstrap** & **Vuong's test** as a robustness check.

Bootstrap:

1. Randomly draw 36 trials with replacement.
2. Perform the former MLE classification method.
3. Repeat 1. and 2. for 1000 times.

Bootstrap & Vuong's test

Perform **bootstrap** & **Vuong's test** as a robustness check.

Vuong's test:

$$V = \frac{\sqrt{N}\left[\frac{1}{N}\sum_{n=1}^{N} m_n\right]}{\sqrt{\frac{1}{N}\sum_{n=1}^{N}(m_n - \overline{m})^2}}$$

m_n : the log likelihood difference of trial n between the largest and second largest level-k type.

id	$\mathrm{Lk_c}$	Lk_c^{alt}	Vuong	Bootstrap Miss Rate
1	L0	L1	**82.90***	**0.000***
2	L2	EQ	**6.50***	**0.000***
3	L1	L0	**2.07***	**0.026***
4	L1	L3	0.62	0.328
5	L1	L2	0.64	0.303
6	EQ	L3	0.57	0.240
7	L3	EQ	1.01	0.198
8	L1	L3	1.41	0.060
9	L0	L2	**3.74***	**0.000***
10	EQ	L3	1.59	0.097
11	EQ	L3	**4.70***	**0.000***
12	EQ	L3	**3.76***	**0.000***
13	L2	EQ	1.27	0.080

id	$\mathrm{Lk_c}$	Lk_c^{alt}	Vuong	Bootstrap Miss Rate
14	L3	L2	0.45	0.522
15	L0	L2	**2.00***	**0.041***
16	L2	EQ	**3.73***	**0.000***
17	L2	EQ	**7.12***	**0.000***
18	L3	EQ	0.97	0.196
19	L2	L3	1.33	0.097
20	L1	L3	0.15	0.405
21	L2	L3	**3.06***	**0.000***
22	L3	L2	0.19	0.459
23	L2	L3	1.55	0.053
24	L1	L3	0.96	0.275
25	L3	L2	0.58	0.444
26	L2	L3	0.90	0.190

id	$\mathrm{Lk_c}$	Lk_c^{alt}	Vuong	Bootstrap Miss Rate
27	L2	L1	0.03	0.497
28	L3	L2	0.65	0.305
29	L2	EQ	0.87	0.233
30	L2	EQ	**3.36***	**0.000***
31	EQ	L3	1.24	0.248
32	EQ	L3	**2.57***	**0.008***
33	L3	L2	0.93	0.162
34	L2	L3	1.17	0.153
35	L1	L3	**3.28***	**0.001***
36	L1	L3	**5.50***	**0.000***
37	L0	L2	0.84	0.218
38	L1	L3	**10.12***	**0.000***
39	L2	L3	**2.22***	0.100

id	$\mathrm{Lk_c}$	Lk_c^{alt}	Vuong	Bootstrap Miss Rate
40	L3	EQ	**3.85***	**0.000***
41	L0	L1	0.72	0.241
42	EQ	L3	1.73	**0.015***
43	L3	L2	1.74	0.054
44	L2	L3	0.45	0.322
45	L0	L2	1.29	0.147
46	L3	L2	0.10	0.493
47	L1	L3	1.17	0.109
48	L3	L2	1.61	0.056
49	L0	L1	0.78	0.228
50	EQ	L3	1.89	**0.030***
51	L0	L1	**2.43***	**0.000***
52	L0	L1	1.04	0.163

id	$\mathrm{Lk_c}$	Lk_c^{alt}	Vuong	Bootstrap Miss Rate
53	L1	L3	0.62	0.300
54	L2	L3	**2.07***	**0.020***
55	L0	L1	**9.12***	**0.000***
56	EQ	L2	0.88	0.400
57	L2	L3	**3.01***	**0.001***
58	L2	L0	0.08	0.527
59	L2	L3	0.74	0.272
60	L1	L3	**2.89***	**0.000***
61	L2	L3	0.82	0.273
62	L1	L3	**5.97***	**0.000***
63	L2	EQ	**2.48***	**0.004***
64	EQ	L3	0.75	0.176
65	L1	L3	**2.42***	**0.010***

id	$\mathrm{Lk_c}$	Lk_c^{alt}	Vuong	Bootstrap Miss Rate
66	L1	L3	0.58	0.278
67	L1	L3	1.26	0.066
68	L2	L3	**2.45***	**0.002***
69	L1	L0	**1.96***	**0.009***
70	L3	EQ	**2.43***	**0.017***
71	EQ	L3	**4.09***	**0.000***
72	L0	L2	**2.18***	**0.026***
73	L2	EQ	**3.66***	**0.036***
74	L0	L2	0.28	0.482
75	L2	EQ	**2.75***	**0.001***
76	EQ	L3	**5.94***	**0.000***
77	EQ	L3	0.36	0.497
78	EQ	L3	1.42	0.100

id	$\mathrm{Lk_c}$	Lk_c^{alt}	Vuong	Bootstrap Miss Rate
79	EQ	L3	**2.66***	**0.007***
80	EQ	L3	**7.36***	**0.000***
81	L3	L2	**2.16***	**0.017***
82	EQ	L3	**2.24***	**0.003***
83	L1	L3	**7.00***	**0.000***
84	L2	L3	**3.47***	**0.000***
85	L2	EQ	**2.75***	**0.000***
86	L3	L2	1.62	**0.047***
87	L2	L3	**2.58***	**0.009***
88	L3	L1	0.07	0.617

Pseudo-type Clusters

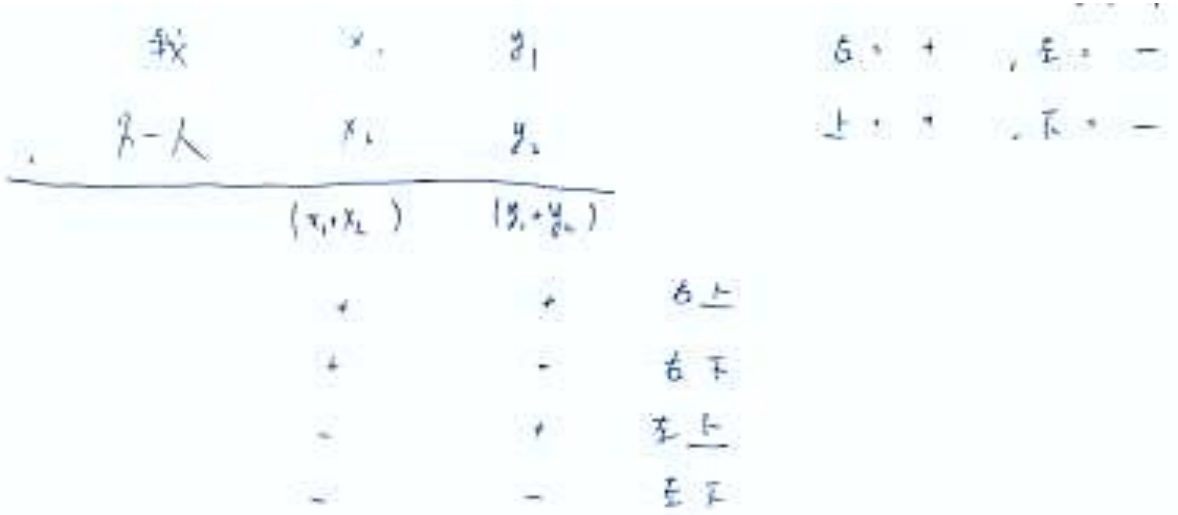
- After adding the 2 additional types...
  - Sum both targets and jump to that corner: #76 & #80

clusters:

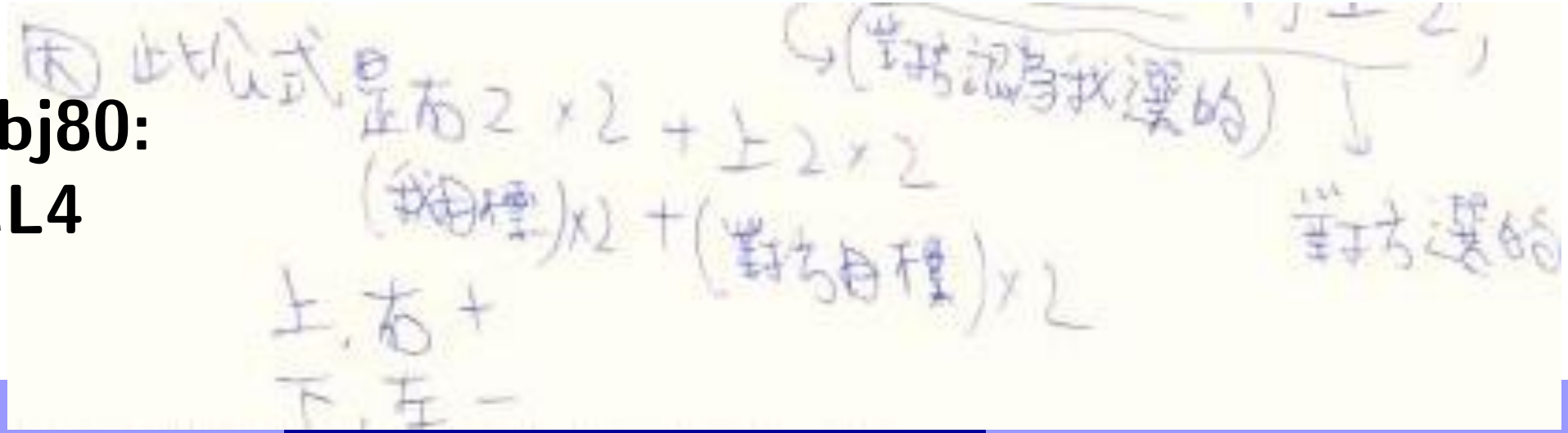
- 39 59
- 54 68
- 76 80 → Sum target corner/ SCL4
- 84 87

id	Lk_c^*	Lk_c^a	Vuong	signi	lambda
76	Corner	SCL4	1.37	0	1.66
80	SCL4	Corner	1.54	0	3.64

Subj76:
Sum target corner



Subj80:
SCL4



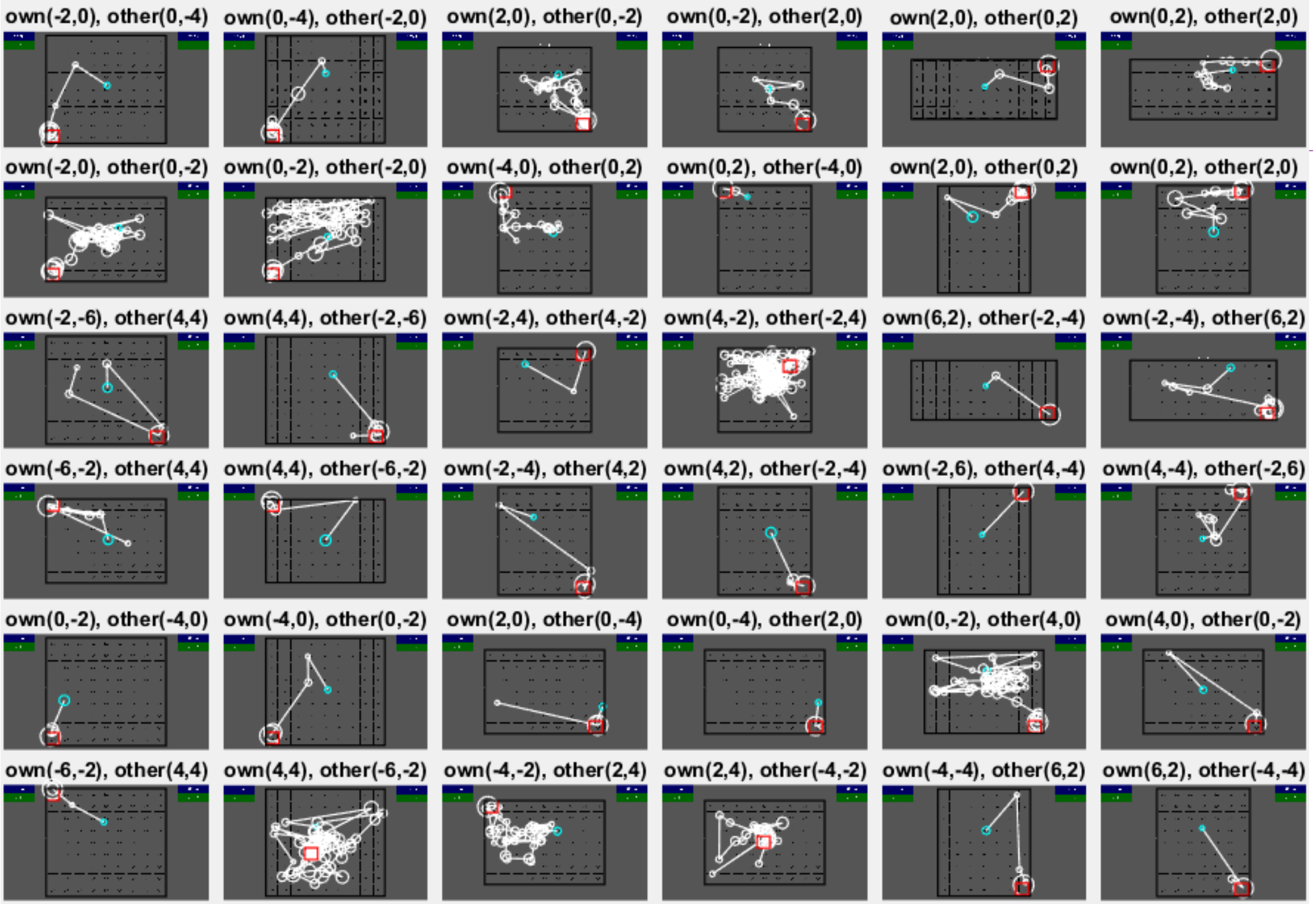
id	Lk_c^*	Lk_c^a	Vuong	signi	lambda
76	Corner	SCL4	1.37	0	1.66
80	SCL4	Corner	1.54	0	3.64

Subj76:
Sum target corner

	Me	x_1	y_1	
+	Another	x_2	y_2	
		+	+	(top-right)
		+	-	(bottom-right)
		-	+	(top-left)
		-	-	(bottom-left)

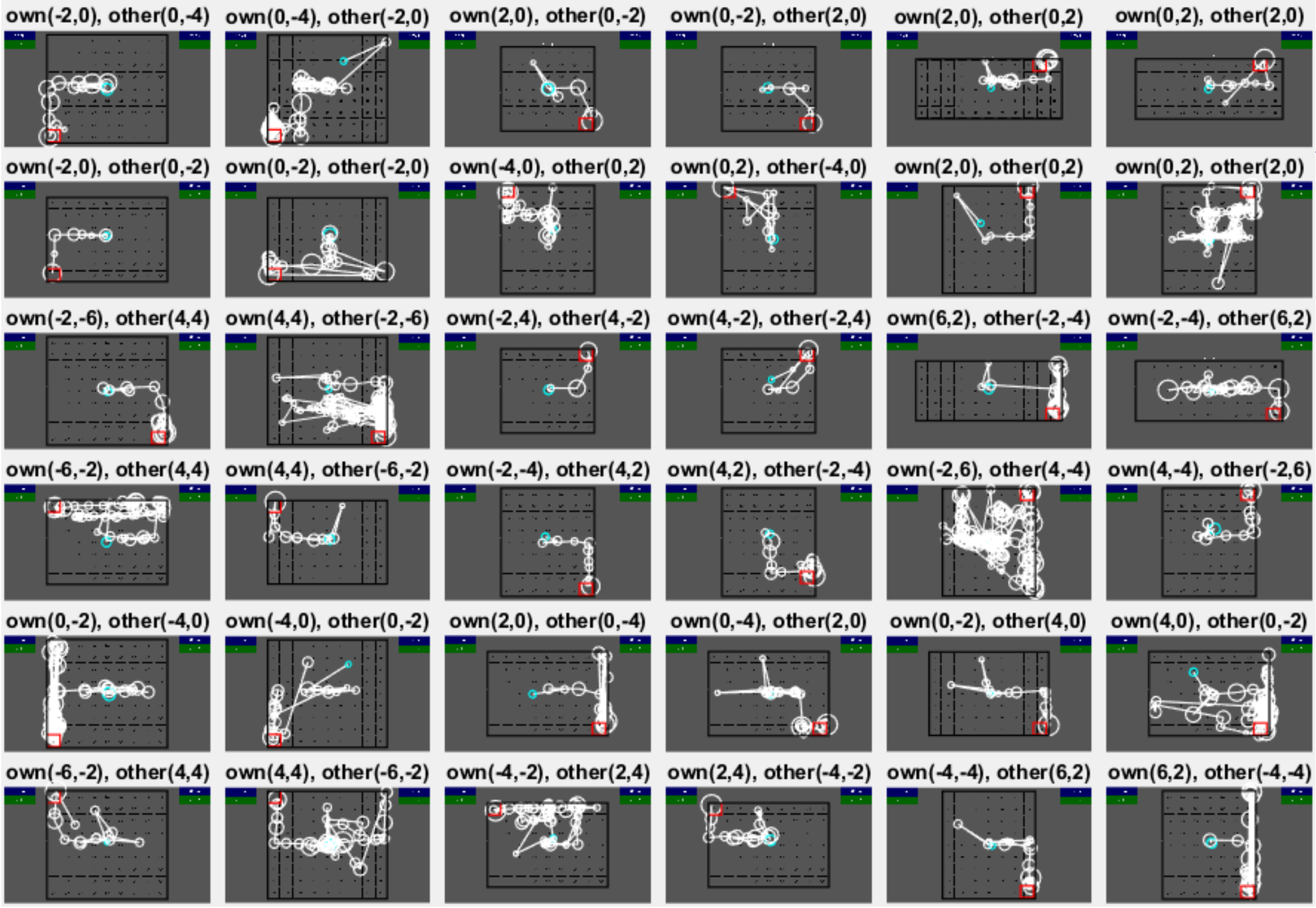
Subj80:
SCL4

The formula is:
(own target)×2+ (opponent's target)×2



Subj76

Choice

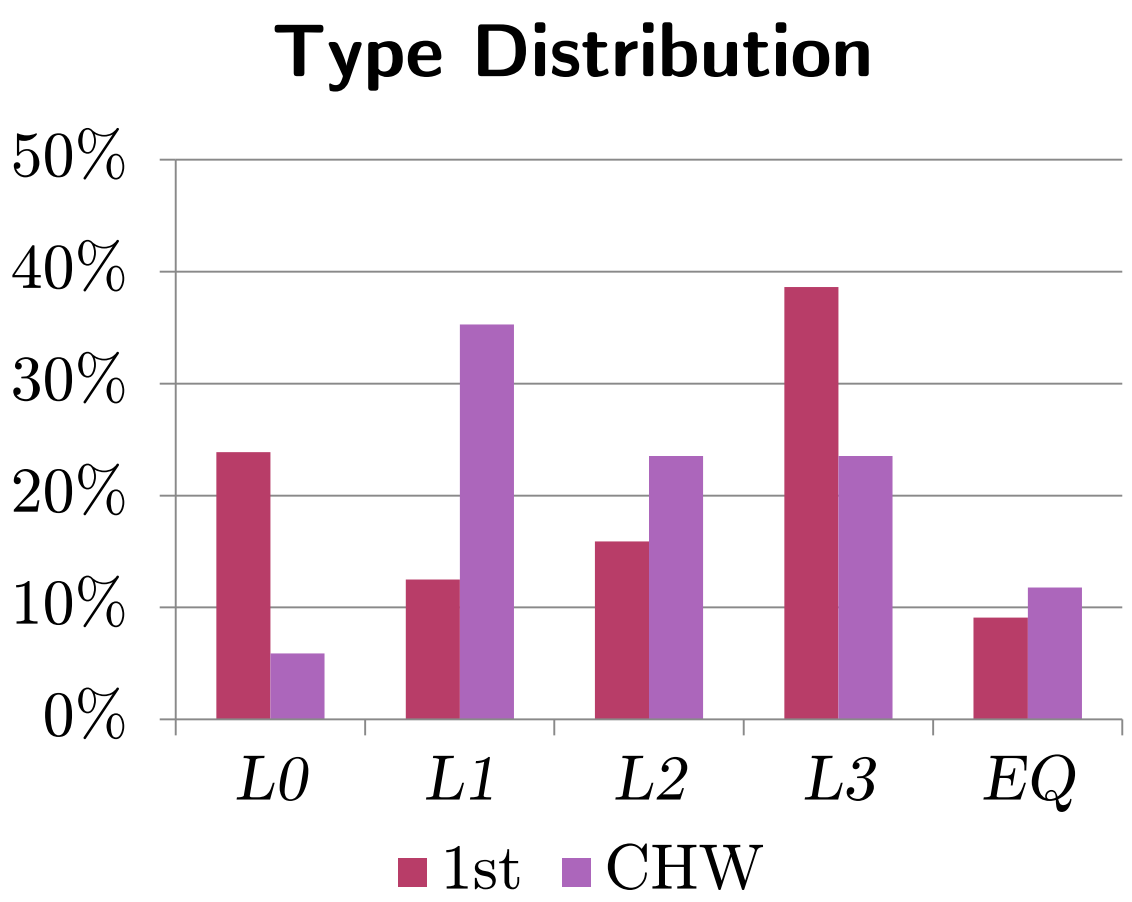


Subj80

Choice

Classification Result: Lookup

- Average thinking steps = 1.97
- 0.03 step less than CHW



id	Lk_l	Lk_l^{alt}	Vuong	Lk_l^*	Lk_c
1	L1	L0	0.67	L0	L0*
2	EQ	L2	2.21*	EQ	L2*
3	L3	L1	0.02	L1	L1*
4	EQ	L2	0.00	L2	L1
5	L3	L0	3.65*	L3	L1
6	L3	L0	13.40*	L3	EQ
7	L3	L1	6.42*	L3	L3
8	L3	L1	0.00	L1	L1
9	L2	L0	0.00	L0	L0*
10	L3	L2	5.67*	L3	EQ
11	EQ	L2	0.00	L2	EQ*
12	EQ	L0	8.79*	EQ	EQ*
13	L3	L2	2.56*	L3	L2

id	Lk_l	Lk_l^{alt}	Vuong	Lk_l^*	Lk_c
14	L0	L0	-	L0	L3
15	L3	L2	2.81	L3	L0*
16	EQ	L2	1.83	L2	L2*
17	L3	L2	0.07	L2	L2*
18	L0	L0	-	L0	L3
19	EQ	L2	0.00	L2	L2
20	L3	L2	5.00*	L3	L1
21	EQ	L2	0.96	L2	L2*
22	L3	L1	5.00*	L3	L3
23	L3	L2	5.23	L3	L2
24	L2	L0	0.00	L0	L1
25	L3	L1	2.68*	L3	L3
26	EQ	L3	4.64*	EQ	L2

id	Lk_l	Lk_l^{alt}	Vuong	Lk_l^*	Lk_c
27	L2	L1	0.86	L1	L2
28	L3	L1	6.54*	L3	L3
29	L2	L0	0.00	L0	L2
30	L3	L1	3.93*	L3	L2*
31	L0	L0	-	L0	EQ
32	EQ	L3	1.27	L3	EQ*
33	L3	L1	6.48*	L3	L3
34	L3	L2	0.00	L2	L2
35	L2	L1	-0.00	L1	L1*
36	L1	L0	5.75*	L1	L1*
37	EQ	L2	0.98	L2	L0
38	L1	L0	6.53*	L1	L1*
39	L3	L1	2.84*	L3	L2+

id	Lk_l	Lk_l^{alt}	Vuong	Lk_l^*	Lk_c
40	L3	L1	8.86*	L3	L3*
41	L3	L1	2.05*	L3	L0
42	EQ	L2	5.33*	EQ	EQ+
43	L3	L2	3.72*	L3	L3
44	L3	L1	4.37*	L3	L2
45	EQ	L2	2.29*	EQ	L0
46	L2	L0	5.91*	L2	L3
47	L1	L0	0.00	L0	L1
48	EQ	L2	1.93	L2	L3
49	L0	L0	-	L0	L0
50	L3	L2	4.36*	L3	EQ+
51	L1	L0	0.00	L0	L0*
52	L3	L1	0.00	L1	L0

id	Lk_l	Lk_l^{alt}	Vuong	Lk_l^*	Lk_c
53	L3	L1	2.12*	L3	L1
54	L3	L1	4.10*	L3	L2*
55	EQ	L2	0.00	L2	L0*
56	L1	L0	0.00	L0	EQ
57	L3	L2	3.64*	L3	L2*
58	L0	L0	-	L0	L2
59	L0	L0	-	L0	L2
60	L1	L0	0.00	L0	L1*
61	L1	L0	0.00	L0	L2
62	L3	L1	1.59	L1	L1*
63	L3	L1	3.52*	L3	L2*
64	EQ	L3	0.44	L3	EQ
65	L1	L0	0.00	L0	L1*

id	Lk_l	Lk_l^{alt}	Vuong	Lk_l^*	Lk_c
66	L3	L1	4.45*	L3	L1
67	L3	L1	0.00	L1	L1
68	L2	L1	2.45*	L2	L2*
69	L1	L0	5.24*	L1	L1*
70	L3	L2	0.88	L2	L3*
71	L1	L0	0.00	L0	EQ*
72	L3	L2	2.96*	L3	L0*
73	EQ	L3	0.00	L3	L2*
74	L0	L0	-	L0	L0
75	EQ	L3	7.48*	EQ	L2*
76	EQ	L2	3.30*	EQ	EQ*
77	L3	L2	6.40*	L3	EQ
78	EQ	L3	0.39	L3	EQ

id	Lk_l	Lk_l^{alt}	Vuong	Lk_l^*	Lk_c
79	EQ	L3	1.65	L3	EQ*
80	EQ	L3	4.23*	EQ	EQ*
81	L3	L2	5.77*	L3	L3*
82	EQ	L1	0.00	L1	EQ*
83	L3	L1	2.03*	L3	L1*
84	L2	L0	0.00	L0	L2*
85	EQ	L2	0.92	L2	L2*
86	L3	L2	9.14*	L3	L3+
87	L1	L0	1.90	L0	L2*
88	L1	L0	0.00	L0	L3

Initial-lookups– U0 & U1

- 6 out of 11 U0 (choice) subjects starts from U0
- 2 out of 12 U1 (choice) subjects starts from U0'

- U0 exists
- U1 is rare

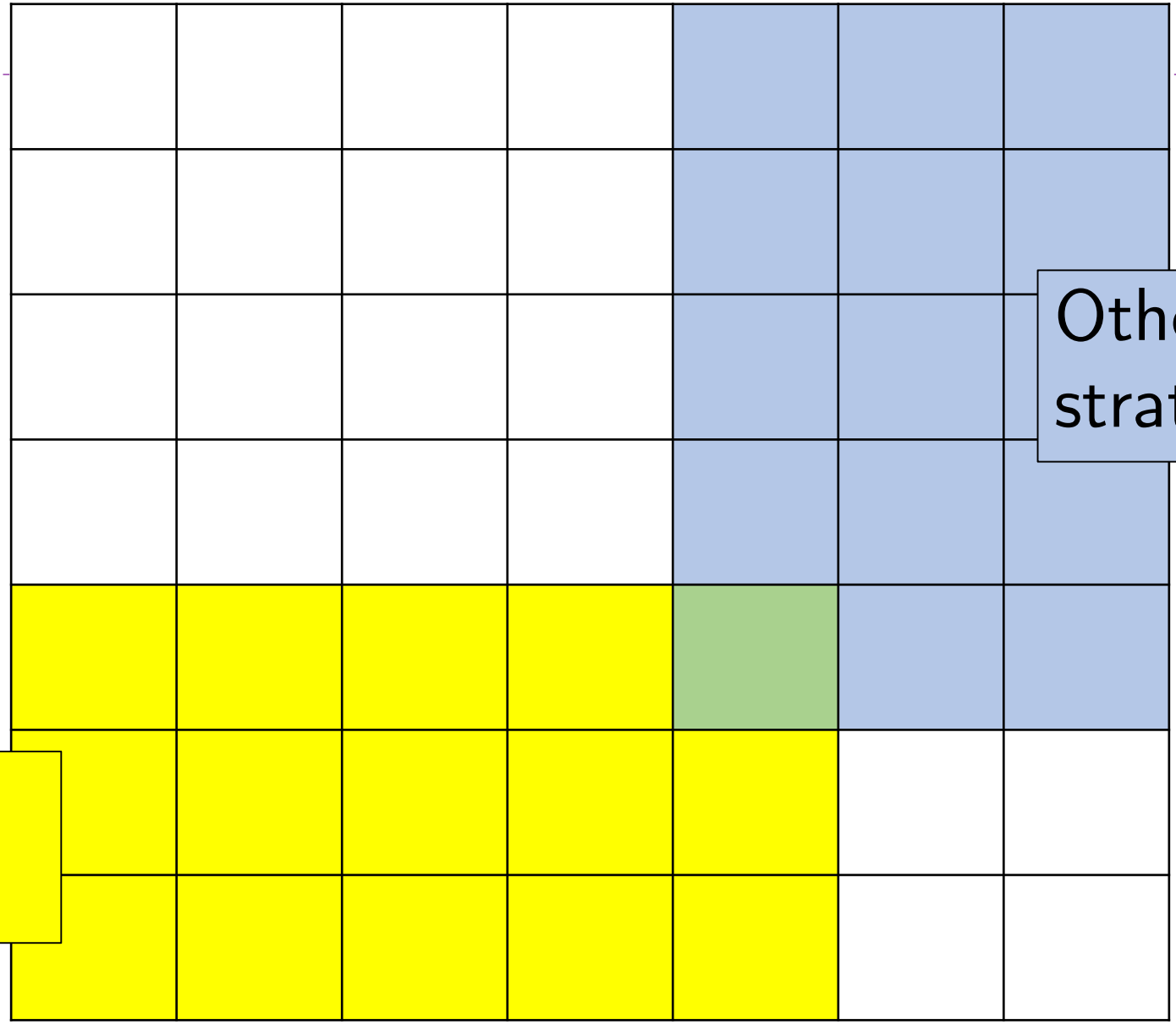
Initial-lookups– U0 & U1

id	ini_type	own_first	other_first	Lk_l	Lk_c
3	U0	8	28	U0	U0
8	U0	**31**	5	U0	U0
35	U0’	1	35	U0	U0
69	U0	5	31	U0	U0

id	ini_type	own_first	other_first	Lk_l	Lk_c
46	U0	24	12	U1	U1
68	U0	4	32	U1	U1

Your goal
Left 2, Below 4

Other's goal
Right 4, Above 2

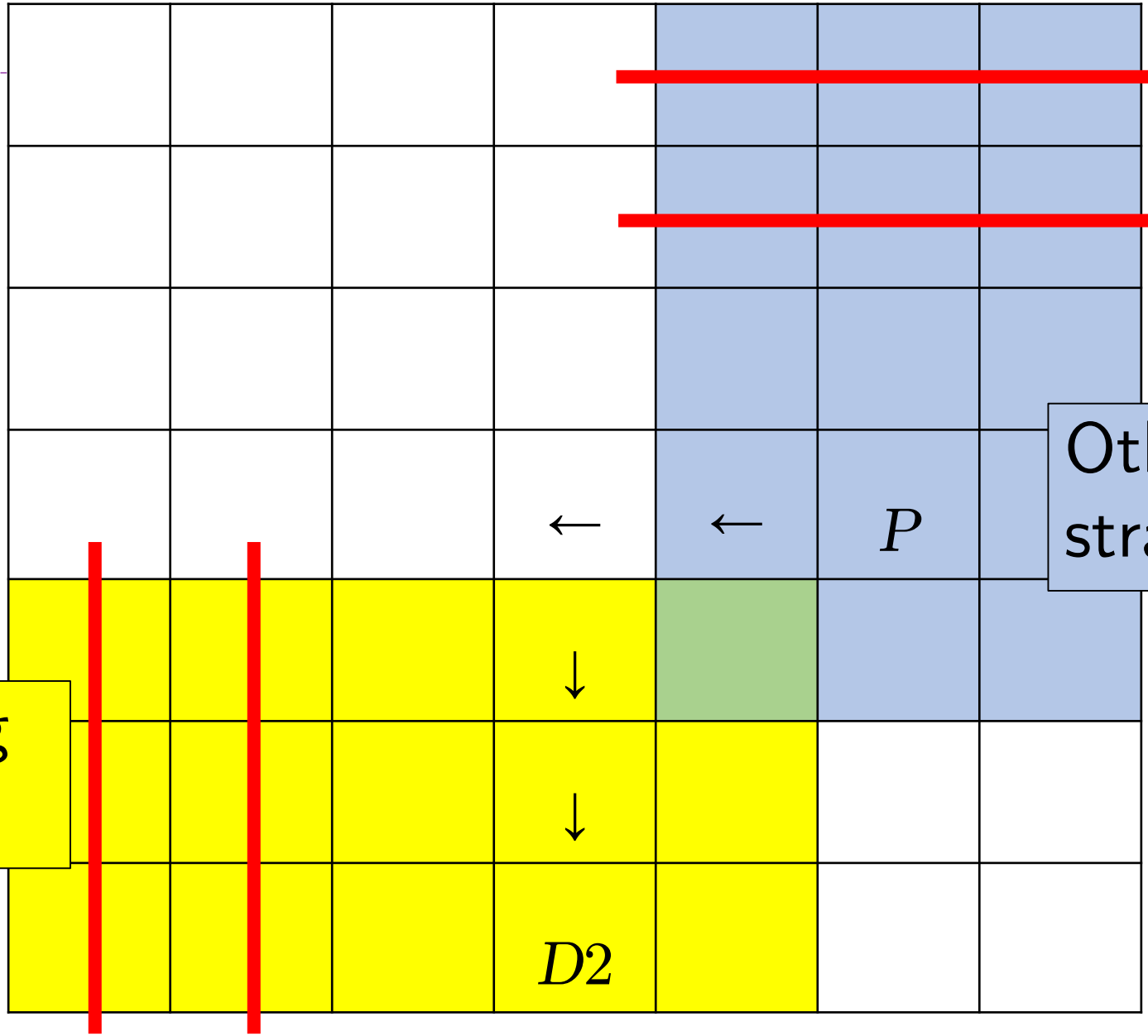


Other's remaining strategies

Own remaining strategies

Your goal
Left 2, Below 4

Other's goal
Right 4, Above 2



Other's remaining strategies

Own remaining strategies

Your goal
Left 2, Below 4

Other's goal
Right 4, Above 2