Peakload Pricing

Name
Major
Student ID

Suppose a monopoly utility firm produces electricity for n periods, so its output is $\vec{q} = (q_1, \cdots, q_n)$. The firm builds a power plant with capacity q_0, so the output in each period is limited by the capacity constraints, $q_j \leq q_0, j = 1, \cdots, n$. We further assume firm has a fixed cost F and a constant marginal cost of generating electricity in each period, $\vec{c} = (c_1, \cdots, c_n)$, and a constant marginal cost of adding production capacity c_0. Hence, the firms total cost is

$$C(q_0, \vec{q}) = F + c_0 q_0 + c_1 q_1 + \cdots + c_n q_n = F + c_0 q_0 + \vec{c} \cdot \vec{q}.$$

Let the demand price function in period j be $p_j = p_j(\vec{q})$, which depends on consumption in other periods, and $\vec{p} = (p_1(\vec{q}), \cdots, p_n(\vec{q}))$. So, total revenue is $R(\vec{q}) = q_1 p_1(\vec{p}) + \cdots + q_n p_n(\vec{p}) = \vec{p} \cdot \vec{q}$.

1. State the Kuhn-Tucker version of the Lagrange method (Theorem 18.7).
   (a) Write down the optimization problem.
   (b) State the Kuhn-Tucker version Lagrangian.
   (c) State the corresponding non-degenerate constraint qualification (NDCQ).
   (d) State the first order conditions (FOC).
2. Write down the maximization problem for this profit-maximizing firm.
3. State the Kuhn-Tucker version Lagrangian. Is the corresponding NDCQ satisfied?
4. Write down the first order conditions for this problem.
5. Suppose there are only two periods and the cost parameters are $c_0 = 40$, $c_1 = c_2 = 20$ and demand functions are $p_1(\vec{q}) = 200 - 2q_1 - q_2$ and $p_2(\vec{q}) = 150 - q_1 - 2q_2$. Write down the marginal revenue functions $\frac{\partial R(\vec{q})}{\partial q_1}$ and $\frac{\partial R(\vec{q})}{\partial q_2}$.
6. Is there a set of (q_1^*, q_2^*, q_0^*) satisfying the FOC such that $q_1^* = q_2^* = q_0^*$? Why or why not?
7. Is there a set of (q_1^*, q_2^*, q_0^*) satisfying the FOC such that $q_1^* = q_0^* > q_2^*$? Why or why not?
8. What is the solution to this maximization problem?

Shadow Price

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1. If x thousand dollars is spent on labor and y thousand dollars is spent on equipment, your lovely CAL4 factory produces $Q(x, y) = 50x^{\frac{1}{2}}y^2$ units of output.

   (a) How should \$80,000 be allocated between labor and equipment to yield the largest possible output?

   (b) Use Theorem 19.1 to estimate the change in maximum output if this allocation decreased by \$1,000.

   (c) Compute the exact change in the previous question.

2. Write the statement of the theorem which corresponds to Theorems 19.2 and 19.3, but for both equlity and inequality constraints.