Lagrange Multiplier in Economics

Name
Major
Student ID

Consider the production function $q = f(K, L)$. Given that the unit cost of capital is P_K and the unit cost of labor is P_L, the total cost of production is $g(K, L) = P_K \cdot K + P_L \cdot L$. A firm may want to maximize production under fixed budget or minimize cost under fixed production.

1. For the Cobb-Douglas production function $q = f(K, L) = AK^{1-\alpha}L^{\alpha}$, $A > 0$, $0 < \alpha < 1$, find (K^*, L^*) that maximizes the production q subject to the budget constraint $g(K, L) = I$ where I is a constant.

2. In general, show that (K^*, L^*) satisfies $\dfrac{\frac{\partial f}{\partial K}(K^*, L^*)}{P_K} = \dfrac{\frac{\partial f}{\partial L}(K^*, L^*)}{P_L}$. What is the economic interpretation of this equation?

3. Now suppose that the production is fixed at $\overline{q} = f(K, L) = AK^{1-\alpha}L^{\alpha}$ where $\overline{q}$ is a constant. What values of K and L would minimize the cost function $g(K, L) = P_K \cdot K + P_L \cdot L$?

4. Consider production function $q = f(K, L) = K^{\frac{1}{3}} + L^{\frac{2}{3}}$. Find (K^*, L^*) that maximizes q subject to the budget constraint $K + 2L = I$. If I increases, will $\frac{K^*}{L^*}$ increase (prefer to employ more capital K) or decrease (prefer to employ more labor L)?