- ✧ What is the question?
  - How multiple mutations into tan environment affect the evolutionary stability of preferences? (we define multi-mutation stability criteria in multi and single-population cases.)

✧ Why should we care about this?

幾乎全部的 literature（與indirect evolutionary approach）建立在 symmetric two-player games played by a single population of players.[1] Besides, 此外，這些不同穩定的概念都需要robustness against one single type of mutation.在這篇paper中，我們考量一個 population could be 同時 invaded by new entrant simultaneously within the framework of the indirect evolutionary approach.

✧ What is the answer?

(Multi-population case)When the number of populations is greater than or equal to three, coalitions of mutants may have a chance to gain an evolutionary advantage, directly or indirectly, by deviating. 無法保證一直存在穩定偏好，給定一個 material payoff function，仍可能有無限種類的結果。

(Single-population case)當 configuration 穩定, as in the case of multiple populations, the material payoff corresponding to the aggregate outcome is the same as fitness value received by each incumbent.

✧ How did you get there?

(Multi-population case) n 個不同的 population in this section，每一個 round，每個 population 的一個 agent 隨機抽取來玩這個 complete information 的 game。假設每個 population 內的偏好種類有限且每個人的偏好種類獨立。且 incumbent 確定能得到相同的 average fitness 若 configuration is stable。

(Single-population case)抽幾個 agent 從同一個有限的 population！

例子）全班隨機抽出四個人玩牌，給定一個四人局的牌，假定四人都是理性的，偏好有限且偏好種類獨立，遊戲的結果取決於玩家拿到哪一副牌，不是誰去玩這副牌。而且在indirect evolutionary approach下，可以有新加入者。

Notation

- $\Delta(S)$ denote the set of probability distributions over S
- An element $\sigma \in {}_{i\in N} \Delta(A_i)$: a mixed strategy profile;
- any element ϕ in $\Delta({}_{i\in N} A_i)$: a correlated strategy for matched players i
- $\pi_i(a)$: the fitness that the player i obtains if an action profile a is played.
- For each $i \in N$, let π_i: ${}_{i\in N} A_i \rightarrow R$ be the material payoff function of player i.
- π_i: ${}_{i\in N} A_i \rightarrow R$ be the material payoff function of player i.
- Every mixed strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$ can be interpreted as a correlated strategy ϕ_σ in the following way: $\phi_\sigma(a_1, \ldots, a_n) = {}_{i\in N} \sigma_i(a_i)$ for every $(a_1, \ldots, a_n) \in {}_{i\in N} A_i$.