

# Contents

<b>1 Linear and Matrix Algebra</b>	<b>1</b>
1.1 Basic Notations . . . . .	1
1.2 Matrix Operations . . . . .	2
1.3 Matrix Determinant and Trace . . . . .	5
1.4 Matrix Inverse . . . . .	7
1.5 Matrix Rank . . . . .	8
1.6 Eigenvalue and Eigenvector . . . . .	9
1.7 Symmetric Matrix . . . . .	11
1.8 Orthogonal Projection . . . . .	13
References . . . . .	15
<b>2 Statistical Concepts</b>	<b>17</b>
2.1 Distribution Functions . . . . .	17
2.2 Moments . . . . .	19
2.3 Special Distributions . . . . .	23
2.4 Likelihood . . . . .	27
2.5 Estimation . . . . .	30
2.5.1 Point Estimation . . . . .	30
2.5.2 Criteria for Point Estimators . . . . .	30
2.5.3 Interval Estimation . . . . .	32
2.6 Hypothesis Testing . . . . .	33
2.6.1 Basic Concepts . . . . .	33
2.6.2 Construction of Tests . . . . .	35
References . . . . .	37
<b>3 Classical Least Squares Theory</b>	<b>39</b>
3.1 The Method of Ordinary Least Squares . . . . .	40

3.1.1	Simple Linear Regression . . . . .	40
3.1.2	Multiple Linear Regression . . . . .	42
3.1.3	Geometric Interpretations . . . . .	45
3.1.4	Measures of Goodness of Fit . . . . .	49
3.2	Statistical Properties of the OLS Estimators . . . . .	51
3.2.1	Classical Conditions . . . . .	51
3.2.2	Without the Normality Condition . . . . .	52
3.2.3	With the Normality Condition . . . . .	56
3.3	Hypotheses Testing . . . . .	58
3.3.1	Tests for Linear Hypotheses . . . . .	58
3.3.2	Power of the Tests . . . . .	62
3.3.3	An Alternative Approach . . . . .	64
3.4	Confidence Regions . . . . .	65
3.5	Multicollinearity . . . . .	66
3.5.1	Near Multicollinearity . . . . .	66
3.5.2	Digress: Dummy Variables . . . . .	68
3.6	Limitations of the Classical Conditions . . . . .	69
	Exercises . . . . .	70
	References . . . . .	74
<b>4</b>	<b>Generalized Least Squares Theory</b>	<b>77</b>
4.1	The Method of Generalized Least Squares . . . . .	78
4.1.1	When $\mathbf{y}$ Does Not Have a Scalar Covariance Matrix . . . . .	78
4.1.2	The GLS Estimator . . . . .	79
4.1.3	Properties of the GLS Estimator . . . . .	81
4.1.4	FGLS Estimator . . . . .	82
4.2	Heteroskedasticity . . . . .	82
4.2.1	Tests for Heteroskedasticity . . . . .	83
4.2.2	GLS Estimation . . . . .	85
4.3	Serial Correlation . . . . .	86
4.3.1	A Simple Model of Serial Correlation . . . . .	87
4.3.2	An Alternative View . . . . .	89
4.3.3	Tests for AR(1) Disturbances . . . . .	90
4.3.4	FGLS Estimation . . . . .	92
4.4	Application: Linear Probability Model . . . . .	95
4.5	Seemingly Unrelated Regressions . . . . .	96

4.6	Models for Panel Data . . . . .	98
4.6.1	Fixed-Effects Model . . . . .	99
4.6.2	Random-Effects Model . . . . .	104
4.7	Limitations of the FGLS Method . . . . .	107
	Exercises . . . . .	107
	References . . . . .	109
<b>5</b>	<b>Elements of Probability Theory</b>	<b>111</b>
5.1	Probability Space and Random Variables . . . . .	111
5.1.1	Probability Space . . . . .	111
5.1.2	Random Variables . . . . .	113
5.1.3	Moments and Norms . . . . .	114
5.2	Conditional Distributions and Moments . . . . .	117
5.2.1	Conditional Distributions . . . . .	117
5.2.2	Conditional Moments . . . . .	118
5.3	Modes of Convergence . . . . .	122
5.3.1	Almost Sure Convergence . . . . .	122
5.3.2	Convergence in Probability . . . . .	123
5.3.3	Convergence in Distribution . . . . .	126
5.4	Stochastic Order Notations . . . . .	128
5.5	Law of Large Numbers . . . . .	129
5.6	Uniform Law of Large Numbers . . . . .	135
5.7	Central Limit Theorem . . . . .	138
5.8	Functional Central Limit Theorem . . . . .	141
5.8.1	Stochastic Processes . . . . .	141
5.8.2	Weak Convergence . . . . .	144
5.8.3	Functional Central Limit Theorem . . . . .	145
	Exercises . . . . .	148
	References . . . . .	149
<b>6</b>	<b>Asymptotic Least Squares Theory: Part I</b>	<b>151</b>
6.1	When Regressors are Stochastic . . . . .	151
6.2	Asymptotic Properties of the OLS Estimators . . . . .	153
6.2.1	Consistency . . . . .	153
6.2.2	Asymptotic Normality . . . . .	159
6.3	Consistent Estimation of Covariance Matrix . . . . .	164
6.3.1	When Serial Correlations Are Absent . . . . .	165

---

6.3.2	When Serial Correlations Are Present . . . . .	167
6.4	Large-Sample Tests . . . . .	169
6.4.1	Wald Test . . . . .	169
6.4.2	Lagrange Multiplier Test . . . . .	172
6.4.3	Likelihood Ratio Test . . . . .	175
6.4.4	Power of the Tests . . . . .	178
6.5	Asymptotic Properties of the GLS and FGLS Estimators . . . . .	179
6.6	Application: Time Trend Models . . . . .	180
	Exercises . . . . .	182
	References . . . . .	185
<b>7</b>	<b>Asymptotic Least Squares Theory: Part II</b>	<b>187</b>
7.1	Autoregression of an $I(1)$ Variable . . . . .	188
7.1.1	Asymptotic Properties of the OLS Estimator . . . . .	188
7.1.2	Tests of Unit Root . . . . .	191
7.1.3	Test of Stationarity against $I(1)$ . . . . .	194
7.2	Regressions of $I(1)$ Variables . . . . .	196
7.2.1	Spurious Regressions . . . . .	196
7.2.2	Cointegration . . . . .	198
	Appendix . . . . .	203
	Exercises . . . . .	206
	References . . . . .	206
<b>8</b>	<b>Nonlinear Least Squares Theory</b>	<b>209</b>
8.1	Nonlinear Specifications . . . . .	209
8.2	The Method of Nonlinear Least Squares . . . . .	213
8.2.1	Nonlinear Least Squares Estimator . . . . .	213
8.2.2	Nonlinear Optimization Algorithms . . . . .	215
8.3	Asymptotic Properties of the NLS Estimators . . . . .	220
8.3.1	Consistency . . . . .	220
8.3.2	Asymptotic Normality . . . . .	223
8.4	Hypothesis Testing . . . . .	226
	Exercises . . . . .	227
	References . . . . .	228

<b>9 The Quasi-Maximum Likelihood Method: Theory</b>	<b>229</b>
9.1 Kullback-Leibler Information Criterion . . . . .	230
9.2 Asymptotic Properties of the QMLE . . . . .	234
9.2.1 Consistency . . . . .	235
9.2.2 Asymptotic Normality . . . . .	235
9.3 Information Matrix Equality . . . . .	237
9.4 Hypothesis Testing . . . . .	243
9.4.1 Wald Test . . . . .	243
9.4.2 Lagrange Multiplier Test . . . . .	244
9.4.3 Likelihood Ratio Test . . . . .	250
Exercises . . . . .	252
References . . . . .	253
<b>10 The Method of Quasi-Maximum Likelihood: Applications</b>	<b>255</b>
10.1 Discrete Choice Models . . . . .	255
10.1.1 Probit and Logit Models . . . . .	255
10.1.2 Multinomial Logit Models . . . . .	259
10.1.3 Conditional Logit Model . . . . .	261
10.2 Application 2: Models of Count Data . . . . .	263
10.3 Application 3: Models of Limited Dependent Variables . . . . .	264
10.3.1 Censored and Truncated Regression Models . . . . .	264
10.3.2 Models of Sample Selection . . . . .	268