2010/11/1 Exercise

1. Wald test : standard linear regression model with heteroskedasticity

Generate random samples from the linear specification:

$$
\begin{aligned}
& y_t = \beta_1 + (\beta_2 + \Delta) * x_{t2} + \beta_3 * x_{t3} + \varepsilon_t \\
& x_{2t} \sim N(0,1), \qquad x_{3t} \sim N(0,1) \\
& \varepsilon_t \sim N(0,1) \quad \text{(homoskedasticity)} \qquad or \qquad . \\
& \varepsilon_t \sim N(0, {x_{t2}}^2) \quad \text{(heteroskedasticity)} \\
& where \quad \beta_1 = 2, \quad \beta_2 = 3, \quad \beta_3 = 5, \quad \Delta = 0.5
\end{aligned}
$$

Regressing y on x, we obtain OLS coefficients $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)'$. Please use Eicker-White covariance matrix estimator to construct a Wald test based on the following designs, and check whether $\hat{\beta}_2$ is "sufficiently close" to 3.

$$
\begin{aligned}
& H_0 : \beta_2 = 3 \\
& H_1 : \beta_2 \neq 3 \\
& W_T \xrightarrow{D} \chi^2(q)
\end{aligned}
$$

(1) When heteroskedasticity is present in your data, please use Eicker-White covariance matrix estimator to construct your test. Given $\Delta = (0,\ 0.1,\ 0.2,\ 0.5,\ 0.8,\ 0.9,\ 1)$, plot power curve of each sample size.

(2) When heteroskedasticity is present in your data, compare two results of Wald Test by using different estimators **under the null.** (1) Use classical OLS variance estimator $\hat{\sigma}_T^2 = \sum_{t=1}^{T} {\hat{e}_t}^2 / (T-k)$ to estimate V_0, then construct Wald statistics . (2) Use Eicker-White (sandwich-type) estimator to estimate $\widehat{D_T}$, then construct Wald statistics

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- When a form of heteroskedasticity is specified in the following form,

$$
\begin{aligned}
& \varepsilon_t \sim N(0, abs(u_t)) \\
& u_t \sim N(0, \sigma_\alpha)
\end{aligned}
$$

(3) (Optional)Given $\Delta = 0.1$, change $\sigma_\alpha = (1{,}5,\ 2,\ 3,\ 5)$. Plot power curves.

(4) (Optional)Given $\Delta = 0.5$, change $\sigma_\alpha = (1{,}5,\ 2,\ 3,\ 5)$. Plot power curves.

(5) (Optional)Given $\Delta = 0$, change $\sigma_\alpha = (1{,}5,\ 2,\ 3,\ 5)$. Plot power curves.

Note 1: For question (3)~(5), You may add one more power curve under homoscedasticity into your plot, and compare together.

Note 2: For each case, consider the sample sizes T= (100, 200, 500, 800, 1000). For each sample size, please simulate the test at least 1000 times and evaluate the proportion of projection. Please explain **in detail** what you see and why.

Hint (It's just a hint, not complete programs):

```
# test whether b2 is equal to zero
b1_no <- 2
b2_no <- 0
b3_no <- 25
delta <- 0.5

R <- matrix(c(0, 1, 0), ncol = 3) # test x2
dimR <- dim(R)
r <- matrix(c(b2_no), ncol = 1)
beta_no <- t(matrix(c(b1_no, b2_no + delta, b3_no), ncol = 3))
```

……………… (neglect details)……………………..

```
########### (Please compare results below)
library(aod)     # aod library supports wald.test()
ols <- lm(y~x2+x3) # regress y on x2 and x3
## using R build-in function ---- vcov
w1 = wald.test(b = coef(ols), Sigma = vcov(ols), L = R, H0 = r)
## using heteroskedasticity-consistent covariance matrix estimator
w2 = wald.test(b = coef(ols), Sigma = Dt/(sample size), L = R, H0 = r)

## using R build-in funciton ---- vcov , and calculate wald statistics
RDR_inv_ols <- solve( R %*% vcov(ols) %*% t(R) )
w3 = t(R %*% b_hat - r) %*% RDR_inv_ols %*% (R %*% b_hat - r)
## note : w1 = w3
# you may compare w1, w2, and w3

## calculate power
pw <- 1- pchisq(W, dimR[1])
########### (End of Comparison)
```