

Due date: November 16, 2009

4. The model of first-order autoregressive, AR(1), is specified as $y_t = \rho y_{t-1} + \varepsilon_t$. In this simulation, we simply let $y_1 = 0$ and consider four cases of $\rho = 0.1$, $\rho = 0.5$, $\rho = 0.9$, and $\rho = 0.99$ under $\varepsilon_t \sim N(0,1)$ for $t = 2, 3, \dots, T$. In each case, you should generate corresponding AR(1) sequences with numbers of $T = 50$, $T = 100$, $T = 300$, and $T = 1000$ for 1000 times and compute their sample averages each time, and then plot the histograms of averages as those in Subsection 4.2. Please separately display and discuss your results under different cases.

5. In this simulation, random sequences $\{x_t\}$ with sizes of $T = 10$, $T = 50$, $T = 100$, and $T = 500$ are drawn from a distribution for 1000 times and their normalized sample averages are computed each time as follows:

$$\frac{\sqrt{T}(\bar{x} - \mu)}{\sigma},$$

where \bar{x} , μ , and σ are the sample average, mean, and standard deviation, respectively. Please plot the histograms of averages under different sizes of T and explain the results. Moreover, there are 3 distributions we should consider in this exercise as follows.

- (1) Student t (2) distribution with zero mean.
- (2) Student t (4) distribution with zero mean.
- (3) Lognormal distribution.