HW2 (Size Performance)

2011/03/28

Generate sequence from GARCH(1,1) process with non-Gaussian innovations. For each case, consider the sample size T=(100, 300, 500, 700, 900). For each sample size, please simulate two tests (Ljung-Box and Q-Star) at least 1000 times and evaluate rejection frequency.

$$y_t = \sqrt{h_t}u_t\ , u_t \sim t(\nu)$$
$$with\ conditional\ variance:\ h_t = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 h_{t-1},$$
$$where\ \alpha_0 = 0.5,\quad \beta_1 = (0.95 - \alpha_1)$$

1. Given $\alpha_1 = (0.2,\ 0.5,\ 0.8)$, simulate GARCH(1,1) process with t(3) innovations. Plot rejection frequency for two tests. Please specify the color or marks of each line. For example, "dot line : $\alpha_1 = 0.2.$; solid line : $\alpha_1 = 0.5.$"
2. Given $\alpha_1 = (0.2,\ 0.5,\ 0.8)$, simulate GARCH(1,1) process with t(5) innovations. Plot rejection frequency for two tests.
3. How are the changes in ν (degree of freedom) related to rejection frequency? For example, does increasing of degree of freedom cause rejection frequency decrease? Why or why not? Think carefully about the underlying statistical inference.

R : Tips & tricks

- Download library(fGarch), then you can use garchSpec() and garchSim() to generate GARCH sequences.
  Please refer to the following links : http://help.rmetrics.org/fGarch/html/garchSim.html
  http://help.rmetrics.org/fGarch/html/garchSpec.html
- Examples 1 :
  Suppose you want to generate a GARCH(1,1) sequence with coefficient $\alpha_0 = 0.5,\ \alpha_1 = 0.2,$ $\beta_1 = 0.75$ and Gaussian innovations.
  ```
  spec = garchSpec(model = list(alpha = 0.2, beta = 0.75, omega = 0.5));
  y <- garchSim(spec, n = T);    ## T is sample size. For example, T is 100.
  ```
- Examples 2 :
  Suppose you want to generate a GARCH(1,1) sequence with coefficient $\alpha_0 = 0.5,\ \alpha_1 = 0.2,$ $\beta_1 = 0.75$ and t(3) innovations.
  ```
  spec = garchSpec(model = list(alpha = 0.2, beta = 0.75, omega = 0.5, shape = 3), cond.dist =
  "std");
  y <- garchSim(spec, n = T);    ## T is sample size. For example, T is 100.
  ```

HW2 (Bonus)

Similar to HW2, please evaluate size performance of modified Cramer-von Mises test (Deo 2000) and compare to original Cramer-von Mises test (Durlauf 1991). You may try difference GARCH sequence. For each case, consider the sample size T=(100, 300, 500, 700, 900). For each sample size, please simulate at least 1000 times and evaluate rejection frequency.

1. GARCH(1,1) with coefficient $\alpha_1 = (0.2,\ \ 0.5,\ \ 0.8)$ and Gaussian innovations
2. GARCH(1,1) with coefficient $\alpha_1 = (0.2,\ \ 0.5,\ \ 0.8)$ and t(3) innovations
3. GARCH(1,1) with coefficient $\alpha_1 = (0.2,\ \ 0.5,\ \ 0.8)$ and t(5) innovations

Deo (2000) :

$$D_T^c(t) = \frac{\sqrt{2T}}{\pi}\sum_{j=1}^{m(T)} \frac{\hat{\rho}(j)}{\sqrt{\widehat{v_{JJ}}}}\frac{\sin(j\pi t)}{j},$$

where $\sqrt{\widehat{v_{JJ}}} = \frac{1}{\hat{\gamma}(0)}\left(\frac{1}{T-j}\sum_{t=1}^{T-j}(y_t-\bar{y})^2(y_{t+j}-\bar{y})^2\right)^{0.5}.$

When T is large,

$$\text{CVM} = \int_0^1 [D_T^c(t)]^2 dt \xrightarrow{T\to\infty} \int_0^1 [B^0(t)]^2 dt$$

R : Tips & tricks

- Programming flow chart (modified Cramer-von Mises test)