NATIONAL TAIWAN UNIVERSITY

Department of Finance: Econometric Theory I — Midterm

Department of Economics: Econometric Theory III — Midterm

Fall 2013 Professor C.-M. Kuan

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1. (10 points) To estimate the variance-covariance matrix of the OLS estimator in a linear regression model, explain clearly in what case the Eicker-White estimator is to be preferred and also in what case the Newey-West estimator is to be preferred.

2. (12 points) Suppose you estimate the model: $y_t = \alpha y_{t-1} + e_t$. What is the probability limit of the OLS estimator $\hat{\alpha}_T$ in the cases below? Let $\{u_t\}$ be a white noise with mean 0 and variance σ_u^2.

   (a) y_t are generated as $y_t = u_t$, $t = 1, \ldots, T$.

   (b) y_t are generated as $y_t = u_t - \pi_1 u_{t-1}$, $t = 1, \ldots, T$, where $|\pi_1| < 1$.

3. (14 points) Suppose $\boldsymbol{y}$ is generated according to the structural change model below with heteroskedastic errors in different periods:

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \end{bmatrix},$$

   where $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ are, respectively, $T_1 \times 1$ and $T_2 \times 1$, $\boldsymbol{X}_1$ ($T_1 \times k$) and $\boldsymbol{X}_2$ ($T_2 \times k$) are non-stochastic matrices, $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ are both $k \times 1$ parameters vectors, and $\boldsymbol{e}_1$ ($T_1 \times 1$) and $\boldsymbol{e}_2$ ($T_2 \times 1$) are error vectors such that

$$\mathbb{E}\left(\begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \end{bmatrix}\right) = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}, \qquad \operatorname{var}\left(\begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \end{bmatrix}\right) = \begin{bmatrix} \sigma_1^2 \boldsymbol{I}_{T_1} & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_2^2 \boldsymbol{I}_{T_2} \end{bmatrix}.$$

   What is the GLS estimator of $(\boldsymbol{\beta}_1' \ \boldsymbol{\beta}_2')'$? How would you implement this GLS estimator? Please give an intuition for this result.

4. (30 points) Answer the following questions with "True" or "False". If you answer is "True," explain your answer clearly; if your answer is "False", provide an example or a counterexample.

   (a) A sequence of i.i.d. random variables must satisfy a strong law of large numbers and a central limit theorem.

   (b) A biased estimator must be inconsistent.

   (c) A weakly consistent estimator must be asymptotically normally distributed.

   (d) If an estimator is $O_p(T^{-1/2})$, it is bounded but does not necessarily converge.

   (e) If an estimator is $o_p(T^{1/2})$, it may diverge or converge.

5. (34 points) Suppose that y_t are generated as an AR(1) process: $y_t = \alpha_o y_{t-1} + u_t$, where α_o is unknown such that $|\alpha_0| < 1$, and $\{u_t\}$ is a white noise with mean 0 and variance σ_u^2. Now you regress y_t on y_{t-1} and obtain the OLS estimator $\hat{\alpha}_T$.

   (a) Is $\hat{\alpha}_T$ unbiased for α_o?

   (b) What is the asymptotic variance-covariance matrix of $\sqrt{T}(\hat{\alpha}_T - \alpha_o)$?

   (c) How would you test if $\alpha_o = 0.5$? Write down you test statistic and its limiting distribution. All notations must be clearly defined.

   (d) Does your result in (b) remain the same when u_t are i.i.d. with mean zero and variance σ_u^2? In this case, how would you test if $\alpha_o = 0.5$? Write down you test statistic and its limiting distribution.

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1. (12 points) Warm-up question. Let $\widehat{\boldsymbol{\Sigma}}_B$, $\widehat{\boldsymbol{\Sigma}}_P$ and $\widehat{\boldsymbol{\Sigma}}_Q$ be the Newey-West type estimators for the positive definite, covariance matrix $\boldsymbol{\Sigma}_o$, computed with, respectively, the Bartlett, Parzen, and quadratic spectral kernel functions. What are the limits of $\widehat{\boldsymbol{\Sigma}}_Q^{-1}\widehat{\boldsymbol{\Sigma}}_B$ and $\widehat{\boldsymbol{\Sigma}}_Q^{-1}\widehat{\boldsymbol{\Sigma}}_P$?

2. (24 points) Answer the following questions with "TRUE" or "FALSE"; explain your answer clearly or illustrate with an example (a counterexample).

   (a) An unbiased estimator must also be consistent.

   (b) Given two consistent estimators, their ratio must converges to the ratio of their respective limits.

   (c) If $\sqrt{T}(\hat{\theta}_T - \theta_o)$ is asymptotically normally distributed, we can conclude that $\hat{\theta}_T$ is weakly consistent for θ_o.

   (d) Let $\widehat{\boldsymbol{\Sigma}}_{EW}$ and $\widehat{\boldsymbol{\Sigma}}_{NW}$ be, respectively, the Eicker-White and Newey-West estimators for the covariance matrix $\boldsymbol{\Sigma}_o$. Then, $\widehat{\boldsymbol{\Sigma}}_{NW} - \widehat{\boldsymbol{\Sigma}}_{EW}$ may not be a positive semidefinite matrix.

3. (32 points) The table below contains the results of regressing Taiwan's suicide rate on the lagged unemployment rate u_{t-1} and the lagged GDP growth rate g_{t-1}. The numbers in the parentheses are the t ratios based on the standard OLS standard errors, as in p. 56 of the first slide file. The sample size is $T = 30$.

const	u_{t-1}	g_{t-1}	$\bar{R}^2$	Reg F
4.512	2.501	0.015	0.65	27.99**
(2.09*)	(5.73**)	(0.08)		

   (a) Find R^2, the centered coefficient of determination.

   (b) Are the t ratios in the table above reliable? Why or why not?

   (c) You are also given below an Eicker-White covariance matrix estimate (left) and a Newey-West estimate (right):

$$\begin{bmatrix} 5.124 & -1.060 & -0.357 \\ -1.060 & 0.259 & 0.063 \\ -0.357 & 0.063 & 0.031 \end{bmatrix}; \qquad \begin{bmatrix} 6.954 & -1.552 & -0.361 \\ -1.552 & 0.388 & 0.073 \\ -0.361 & 0.073 & 0.027 \end{bmatrix}.$$

   Choose a covariance matrix estimate and compute t ratios accordingly; explain your choice.

(d) For the null hypothesis that the sum of the coefficients for u_{t-1} and g_{t-1} is 2, please compute your test statistic explicitly and discuss the limiting distribution.

4. (14 points) Suppose now you regress Taiwan's suicide rate s_t on a constant term, the unemployment rate u_t and the GDP growth rate g_t. State two possible reasons that your estimates may be inconsistent and explain why they are so.

5. (18 points) Let g denote Taiwan's annual GDP growth rate with observations g_t. Suppose you calculate the sample average of g_t over the years 1980–2010 as $\bar{g} = 5.94\%$ and would like to test the null hypothesis $\mathbb{E}(g_t) = 5\%$. How would you test this hypothesis? Please be specific about your test statistic and its limiting distribution. Explain clearly how your test can be implemented and why it works.