NATIONAL TAIWAN UNIVERSITY

Department of Finance: Econometric Theory I — Final

Department of Economics: Econometric Theory III — Final

Fall 2011 Professor C.-M. Kuan

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1. (6 points) Comment the following statement and explain clearly. "Suppose that y_t are generated according to $y_t = \boldsymbol{x}_t'\boldsymbol{\beta}_o + \varepsilon_t$, with $\boldsymbol{V}_o = \lim_{T\to\infty} \operatorname{var}(T^{-1/2}\sum_{t=1}^{T} \boldsymbol{x}_t\varepsilon_t)$. The diagonal elements of the Eicker-White estimator ought to be smaller than the corresponding diagonal elements of the Newey-West estimator, because the former does not include estimates of the autocovariances of $\boldsymbol{x}_t\varepsilon_t$ but the latter does. As a result, the t-ratio of each coefficient is more likely to reject the null hypothesis if it employs the Eicker-White standard error."

2. (22 points) Suppose you have decided to estimate an AR(1) specification, $y_t = \alpha y_{t-1} + e_t$, using the OLS method.

   (a) What is the probability limit of the OLS estimator $\hat{\alpha}_T$, when y_t are generated according to an MA(1) process: $y_t = u_t - \pi_1 u_{t-1}$, where u_t is a white noise with mean zero and variance σ_u^2?

   (b) How would you estimate the standard error of $\sqrt{T}(\hat{\alpha}_T - \alpha^*)$, where α^* is the probability limit of $\hat{\alpha}_T$? All notations in your estimate must be clearly defined.

3. (20 points) Let y_t be a binary variable taking values zero and one and $\boldsymbol{x}_t$ a vector of regressors. Suppose you approximate $\mathbb{P}(y_t|\boldsymbol{x}_t)$ by

$$F(\boldsymbol{x}_t;\boldsymbol{\theta}) = \frac{\exp(\boldsymbol{x}_t'\boldsymbol{\theta})}{1+\exp(\boldsymbol{x}_t'\boldsymbol{\theta})}.$$

   To estimate $\boldsymbol{\theta}$, one may employ the NLS estimator, the weighted NLS estimator with the conditional variance of y_t as the weight, and the QMLE estimator. Write down the objective functions for these 3 estimators and explain their differences. Which one of these estimators is the most efficient and why?

4. (24 points) Answer the following questions with "TRUE" or "FALSE" and explain why. For (a) and (b), an example or a counter example suffices.

   (a) A biased estimator may be consistent.

   (b) To test a linear hypothesis under the framework of QMLE, the Wald statistic is greater than the LM statistic.

   (c) To estimate an asymptotic covariance matrix, the Newey-West estimator is consistent even when the data are serially independent.

(d) The quadratic spectral kernel function may be negative; as such, the Newey-West estimator based on this kernel may not be positive semi-definite.

5. (10 points) Suppose you believe y_t are generated according to $y_t = \boldsymbol{x}_t'\boldsymbol{\beta}_o + \varepsilon_t$ and would like to test if there is ARCH(1) effect. Describe how the Breusch-Pagan test can be applied in this case. Be specific about the test procedure and its limiting distribution. All notations must be clearly defined.

6. (18 points) Suppose you are given randomly sampled household data and would like to study the binary variable y_t using the probit specification $F(\boldsymbol{x}_t'\boldsymbol{\theta})$. Note that by random sampling we mean the data are independent across households.

   (a) Write down a consistent estimate of the asymptotic covariance matrix of the QMLE $\tilde{\boldsymbol{\theta}}_T$. Be specific about the expected Hessian matrix and information matrix.

   (b) Describe clearly how you can test the null hypothesis $\theta_2 + \theta_3 = 1$, where θ_i is the i-th element of $\boldsymbol{\theta}$. Be specific about your test statistic and its limiting distribution.

7. Bonus (20 points) Suppose $y_t = \sqrt{h_t}\, u_t$, where u_t are i.i.d. with mean zero and variance one, and h_t are such that

$$h_t = \begin{cases} \alpha_0 + \beta y_{t-1}^2, & t = 2, \ldots, s, \\ \alpha_1 + \beta y_{t-1}^2, & t = s+1, \ldots, T, \end{cases}$$

   where $\alpha_0, \alpha_1 > 0$, $\beta \geq 0$, and the change point s is known.

   (a) Write down a quasi-log-likelihood function and explain how to compute the QMLE of unknown parameters.

   (b) Explain clearly how to test the null hypothesis: $\alpha_0 = \alpha_1$. Be specific about your test statistic and the limiting distribution.

   Note: You will NOT receive bonus points unless you have finished answering all other questions.

Happy New Year and See You Next Year!