Predicting Defaults with Regime Switching Intensity: Model and Empirical Evidence

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Motivation

- Default events are strongly related to observable firm specific and macroeconomics fundamentals (Shumway 2001, Duffie et al., 2007).
- Recent research indicates conditional on observable covariates, intensity model are not sufficient to capture the large degree of default clustering (Das et al., 2009) Possible reasons are:
  - Missing observable risk factors: Lando and Nielsen (2009).
  - Complex inter-firm linkages or unobserved fraudulent accounting practice is hard to model.
  - Mis-specification in intensity process: Duan (2010), Azizpour et al. (2010).
- Common frailty factor (latent process) to firms/industries provides more accurate estimation on default probabilities and portfolio loss distribution (Duffie et al., 2009; Koopman et al., 2011).

Main Results

- In addition to common unobserved risk factors, firm's risk exposure to observable covariates are possibly time-variant/regime dependent due to pro-cyclical lending policies of banks toward firms.
- In this work, we propose a regime-switching (RS) intensity model
  - differentiates high-/ low- default risk periods
  - RS in intercept can be proxy for common frailty factor
  - RS in factor coefficients explains time-varying risk exposure to observable risk factors
- Our empirical results of U.S. listed firms during 1990-2009 show
  - regime-switching effect in intensity function is statistically significant.
  - regime-dependent risk exposure can not be omitted.
  - in-sample and out-of-sample default prediction abilities of RS model outperform doubly-stochastic intensity model.

Intensity Models

- Let τ_i be default time of firm i whose default intensity is defined as:

$$\lambda_{i,t} = \lim_{\triangle t \to 0} \frac{P(t < \tau_i \leq t + \triangle t | \tau_i > t, \mathcal{F}_t)}{\triangle t} = \Lambda(\boldsymbol{\mu}' \mathbf{W}_{i,t}),$$

  $\mathbf{W}_{i,t}$ is risk factors/covariates with parameter $\boldsymbol{\mu}$. Probability of default within a small period $\triangle t$ is $1 - e^{-\lambda_{i,t} \triangle t}$.

- Duffie et al. (2007):

$$\lambda_{i,t} = \exp\left(\mu_0 + \mu_1 R_{i,t} + \mu_2 DTD_{i,t} + \mu_3 R_{mt} + \mu_4 R_{ft}\right).$$

  where R_i and DTD_i are *firm-specific* variables, stock return and distance to default; R_m and R_f are *macroeconomics* variables, S&P 500 index return and 3 month Treasury Bill rate.

Intensity Models (Cont'd)

- Duffie et al. (2009) include an additional latent variable as $\mathbf{W}_{i,t}$, $\mathbf{W}_{i,t} = (\mathbf{X}_{i,t}, y_t)$.

$$\lambda_{i,t} = \exp(\gamma y_t + \boldsymbol{\mu}'\mathbf{X}_{i,t})$$

  where y_t is an frailty variable following Ornstein-Uhlenbeck process with parameter κ and standard deviation γ.

$$dy_t = -\kappa y_t dt + dB_t, \quad y_0 = 0.$$

- Due to the unobserved y_t, a computing intensive Monte Carlo Expectation Maximization algorithm is used to estimate unknown parameters.

Regime-switching Intensity Model

- $\mathbf{W}_{i,t} = (\mathbf{X}_{i,t}, s_t)$. $\mathbf{X}_{i,t}$ is an observable risk factors (firm/industry/macro) and s_t is unobservable regime indicator affecting default process.
- s_t is one dimension, K states first-order Markov process.
- $\mathbf{X}_{i,t}$ and s_t are mutually independent processes.
- Condition on $s_t = j$, assume the intensity function is of the form:

$$\Lambda(\mathbf{X}_{i,t}, s_t = j; \boldsymbol{\mu}_j) = \exp\left(\mu_{0j} + \mu_{1j} X_{i,1t} + \cdots + \mu_{pj} X_{i,pt}\right),$$

where $\mathbf{X}_{i,t}$ is observable covariates of firm i at t and $\boldsymbol{\mu}_j := (\mu_{0j}, \mu_{1j}, \cdots, \mu_{pj})'$ is unknown parameter vector specific to regime j.

Regime-switching Intensity Model (Cont'd)

- The simplest case is RS in intercept of intensity function (RS_I):

$$\Lambda(\mathbf{X}_{i,t}, s_t = j; \boldsymbol{\mu}_j) = \exp\left(\mu_{0j} + \boldsymbol{\mu}'\mathbf{X}_{i,t}\right).$$

  If the true parameters $\mu_{01} \geq \mu_{0j}$, $\forall j$, we have regime 1 as the highest intensity level among all other regimes. (cf. Duffie et al., 2009)

- RS in both intercept and risk exposure parameters (RS_{I,X_1}):

$$\Lambda(\mathbf{X}_{i,t}, s_t = j; \boldsymbol{\mu}_j) = \exp\left(\mu_{0j} + \mu_{1j}X_{i,1t} + \boldsymbol{\mu}'\mathbf{X}_{i,t}\right).$$

  where $X_{i,1t}$ is a firm-specific risk factor or macroeconomic variable. This model discusses the regime-specific of risk exposures to observable risk factors by introducing the non-linearity in risk exposure parameter.

Data

- Sample spectrum: 10,950 U.S. listed nonfinancial, nonutility firms, monthly data during 1990-2009.
- Total 1,319 defaulted firms defined as
  - CRSP: delisted code 574
  - Compustat: delist code 02
  - Bloomberg: CACS, default corp action and bankruptcy filing
- Accounting information is of 3 months lag and market information is real time to mimic actual default prediction practice.
- All firmspecific variables are winsorized using a 5/95 percentile interval to prevent outliers.
- DTD is based on rolling window estimates to avoid looking-ahead bias, see Duan (2010) and Wang (2010).

	Name	Definitions / Variables Included
Firmspecific		
	ASSTE*	log of total asset adjusted (TA) deflated to 2005 dollars using GDP deflator
	CASH*	cash and equivalence to TA
	DtD*	distance to default measure
	METL*	market value of asset to total liability
	MKTBE*	market to book ratio
	NITA*	net income to TA
	PROFIT*	operating income before depreciation to TA
	RATING*	debt rating dummy
	RET	$\log(1+R_{i,t})$ - $\log(1+R_{S\&P500,t})$
	RSIZE*	log of market to S&P500 market value
	SALES*	sales to TA
	STD	standdard deviation of RET for one year
	TLTA*	total liability to TA
Macro		
	SR Rate	Treasury constant maturity rate / G3M, G6M, G1
	LR Rate	Treasury constant maturity rate / G3, G5, G7, G10
	Term Spread	G3-G3M, G3-G6M, G3-G1,G5-G3M, G5-G6M, G5-G1
		G7-G3M, G7-G6M, G7-G1,G10-G3M, G10-G6M, G10-G1
	Bond Rate	Moody's seasoned corporate bond yield / Aaa and Baa
	Credit Spread	Baa-Aaa
	VIX	Chicago board options exchange market volatility index
	S&P500	one year trailing S&P500 index return
	CF3*	Chicago Fed national activity index's 3-month moving average
	CPgro*	growth rate of corporate profits after tax
	GDPgro*	growth rate of gross domestic product
	NFCPATAXgro*	growth rate of nonfinancial corporate business profits after tax
	INDPROgro*	growth rate of industrial production index

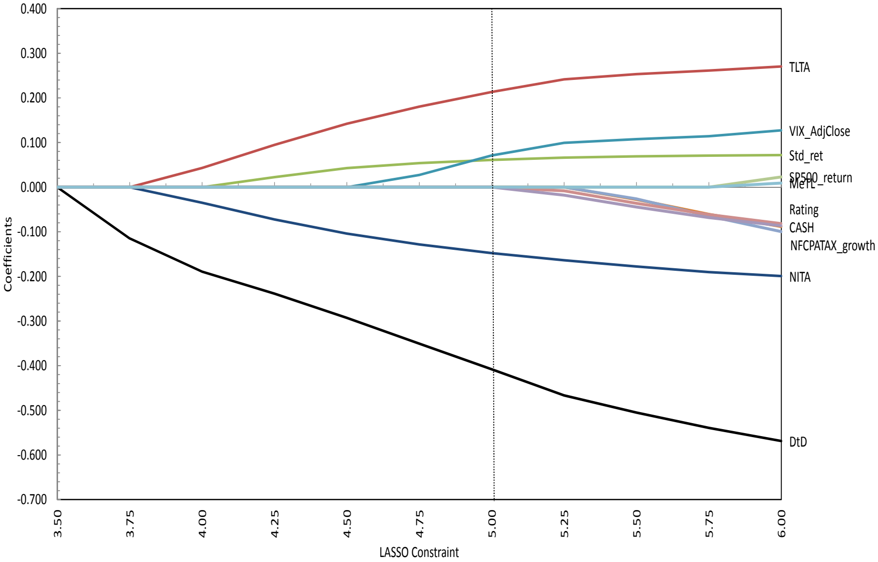
Selecting Covariates via LASSO

- Least Absolute Shrinkage and Selection Operator (LASSO) minimizes the log likelihood subject to the sum of the absolute values of parameters being constrained by a constant.
- LASSO solves the problem

$$\begin{aligned} \max_{\boldsymbol{\mu}} L(\boldsymbol{\mu}|\mathcal{F}_T) &= \sum_{t=1}^{T} \log\left[l_t(\boldsymbol{\mu}|\widetilde{\mathbf{W}}_t, \mathbf{D}_t)\right] \\ \text{subject to} &\sum_{p=1}^{k} |\mu_p| \leq s \end{aligned} \tag{1}$$

where s is a pre-specified shrinkage level.

- We employ the GCV-type statistics to determine s as suggested by Tibshirani (1997) in Cox regression content.


0.400
0.300
0.200
0.100
0.000
-0.100
-0.200
-0.300
-0.400
-0.500
-0.600
-0.700
Coefficients
3.50
3.75
4.00
4.25
4.50
4.75
5.00
5.25
5.50
5.75
6.00
LASSO Constraint
TLTA
VIX_AdjClose
Std_ret
SP500_return
MeTL
Rating
CASH
NFCPATAX_growth
NITA
DtD

Selecting Empirical Regime-switching Model

- The covariates chosen by LASSO approach are: DTD, net income to total asset (NITA), total liability to total asset (TLTA), return annual standard deviation (STD); and a macro variable: VIX index. Denote as M_{LASSO} model.
- We employ Hansen's supreme likelihood ratio test to validate the existence of regime-switching effect in the level or in the factor loadings of M_{LASSO} model. For each time, we only consider one RS effect in one covariate only.
- Hypothesis are

$$H_0 : M_{LASSO} \text{ model}; \quad H_A : RS_{X_i} \text{ model}.$$

where X_i is one of covariates chosen by LASSO method.

Is Regime-switching Effect Statistically Significant?

Table: p-values of supremum LR test

Lag	RS_I	RS_{DtD}	RS_{VIX}	RS_{NITA}	RS_{TLTA}	RS_{STD}
0	0.000	0.000	0.000	0.323	0.061	0.133
1	0.000	0.000	0.000	0.333	0.072	0.113
2	0.000	0.000	0.000	0.299	0.067	0.096
3	0.000	0.000	0.000	0.271	0.086	0.108
4	0.000	0.000	0.000	0.276	0.099	0.151
5	0.000	0.000	0.001	0.262	0.094	0.134
S-LR	6.909	6.634	6.024	1.578	2.337	2.335
LR	160.197	273.461	152.403	63.721	144.265	79.176

Selecting Empirical Regime-switching Model (Cont'd)

- To be comparable to frailty model, we estimate all models with RS effect in intercept and possible RS effects in other factors, such as RS_I and $RS_{I,DtD,VIX,NITA,TLTA}$.
- $RS_{I,DtD,VIX}$ is the best model specification among all RS intensity models estimated in terms of AIC. However, the coefficient of $\mu_{VIX,1}$ is highly insignificant (p-value is 22.50%).
- Finally, we compare Duffie et al. (2007), M_D model, M_{LASSO}, RS_I, and $RS_{I,DtD}$ models in in-sample and out-of-sample default prediction abilities.

MLEs of Regime-switching Intensity Models

- Log likelihoods of M_D, M_{LASSO}, RS_I, and $RS_{I,DTD}$ models.

	M_D	M_{LASSO}	RS_I	$RS_{I,DtD}$
loglik	-7827.87	-7313.33	-7233.23	-7154.68
AIC	15665.74	14614.66	14484.46	14329.37
BIC	15682.89	14594.08	14515.32	14363.67

- We also estimate Duffie et al. (2009) frailty model using LASSO covariates. The log likelihood of frailty model is -7214.61.
- Our results imply that the regime-specific intercept and regime-specific risk exposure to observable factors in well-specified intensity all need to be considered in default modelling.

MLEs of $RS_{I,DTD}$ Model

- Signs of MLEs of $RS_{I,DTD}$ model are consistent to previous literatures.
- All parameters are significant at 1% level, except VIX is at 5% level.
- NITA and TLTA have large magnitude in default intensity.

	p_{11}	p_{22}	μ_{01}	μ_{02}		
MLE	0.676	0.649	-5.448	-6.985		
std	(0.053)***	(0.054)***	(0.133)***	(0.186)***		
	DtD_1	DtD_2	NITA	TLTA	STD	VIX
MLE	-0.625	-3.901	-8.081	3.002	0.609	0.005
std	(0.024)***	(0.269)***	(0.378)***	(0.126)***	(0.077)***	(0.003)**

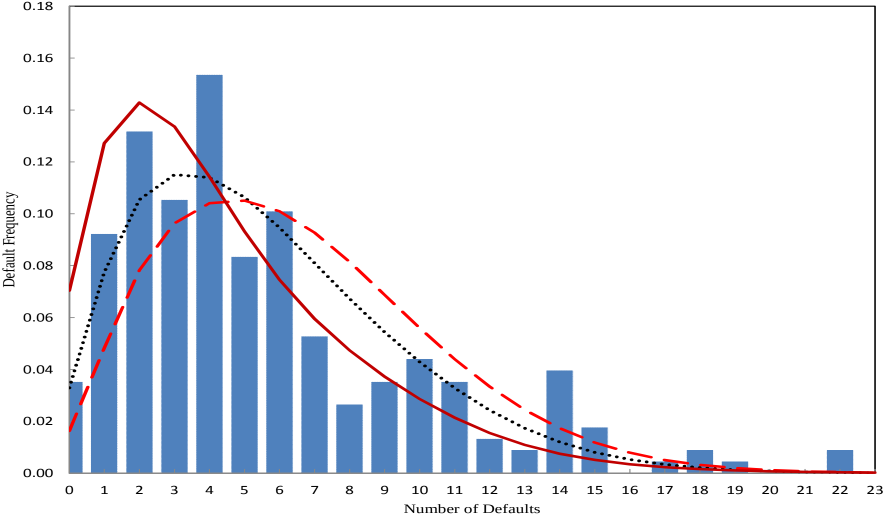
Predicted Default Frequency

- Conditional on regime j and assume that over the period $[t, t + \triangle t]$, the values of covariate are constant, then the predicted probability of $k = 1, 2, \ldots$ defaulters in a N_t companies portfolio at time t will be

$$P\left(\sum_{i=1}^{N_t} D_{i,t} = 0 | s_t = j\right) = \prod_{i=1}^{N_t} e^{-\Lambda(\mathbf{X}_{i,t}, s_t = j; \hat{\boldsymbol{\mu}}_j)\triangle t}$$

$$P\left(\sum_{i=1}^{N_t} D_{i,t} = 1 | s_t = j\right) = \sum_{i=1}^{N_t} [(1 - e^{-\Lambda(\mathbf{X}_{it}, s_t = j; \hat{\boldsymbol{\mu}}_j)\triangle t}) \prod_{l=1, l \neq i}^{N_t} e^{-\Lambda(\mathbf{X}_{lt}, s_t = j; \hat{\boldsymbol{\mu}}_j)\triangle t}]$$

Duan (2010) provides an algorithm to calculate formula above.


0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Default Frequency
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Number of Defaults

ROC Analysis

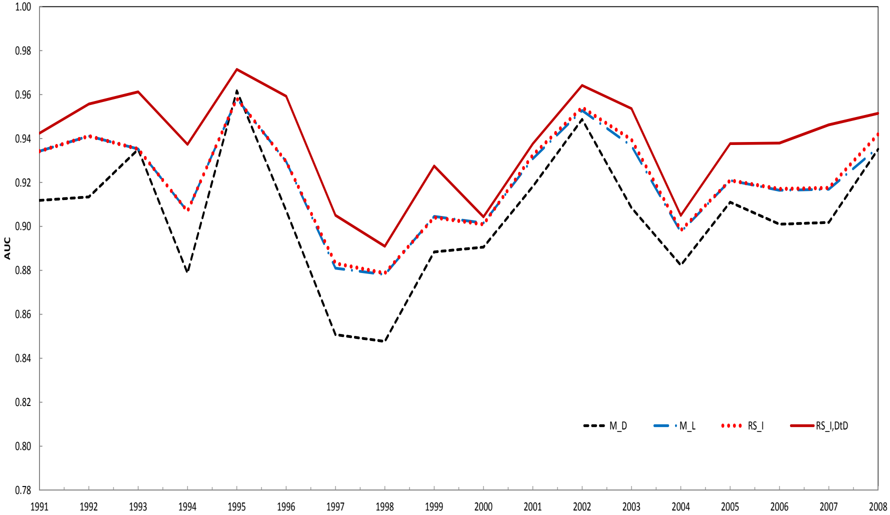
- ROC diagram summarizes the trade-off between false positive rate and true positive rate. Given a predicted PD as a threshold value, a confusion matrix is defined as:

		Actual Value		
		Default	Survive	Total
Prediction	Default	True Positive (TP)	False Positive (FP)	$\hat{D}$
	Survive	True Negative (TN)	False Negative (FN)	$\hat{S}$
	Total	D	S	

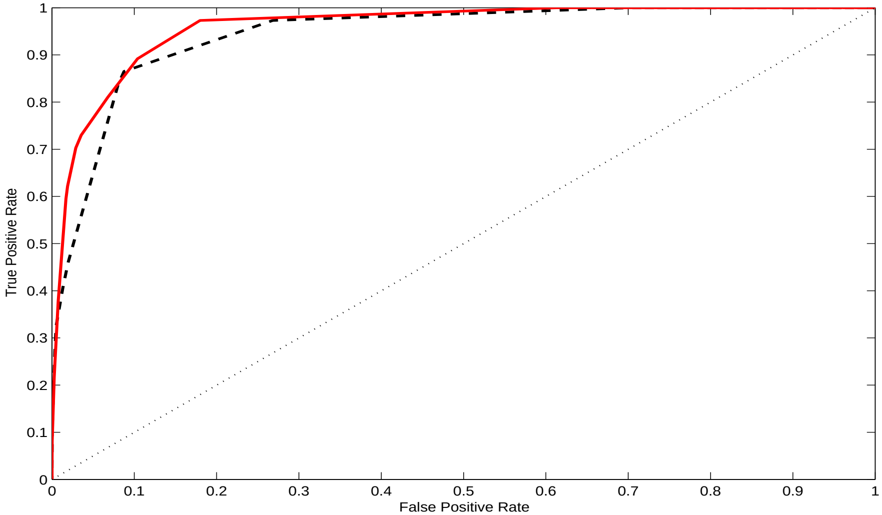
where D and S ($\hat{D}$ and $\hat{S}$) are actual default number and survive number (predicted default number and predicted survive number).

- True positive rate (TPR) is $\frac{TP}{D}$ and false positive rate (FPR) is $\frac{FP}{S}$.
- Flipping coin would give the 45° line to show its no-discrimination nature. Therefore, the area under ROC curve (AUC) is a measure for comparing different models.

In-sample Area under ROC



Out-of-sample ROC Diagram



Conclusion

- In this work, we propose the regime-switching intensity model and provide the estimation algorithm when the unobservable regime indicator follows the Markovian process.
- Our test indicates that the regime switching effect in the intercept of intensity function, risk exposure of distance to default measure of U.S. listed companies during 1990-2009 is significant.
- Regime-switching intensity model characterizes the right tail part of loss distribution plot (average default frequency plot) well.
- Our results imply that the regime-specific intercept and regime-specific risk exposure to observable factors in well-specified intensity all need to be considered in default modeling.