Topic: LLN & CLT

Example: (pp. 19-20 in lecture note)

1. (LLN) Generate random samples with sample sizes T=50, 100, 300, and 1000 from the following distributions and compute the sample average for each sample. Repeat this procedure 1000 times and plot the resulting histograms. Explain if your results obey the law of large numbers.
   (1) Chi-squared $\chi^2(1)$ distribution
   (2) Student t(5) with zero mean
   (3) Student t(1)

2. (CLT) Generate random samples with sample sizes T=50, 100, 300, and 1000 from the following distributions and compute the normalized sample average for each sample:
   $$\frac{\sqrt{T}(\bar{x} - \mu)}{\sigma}$$
   where x, μ, and σ are the sample average, mean, and standard deviation, respectively. Repeat this procedure 1000 times and plot the resulting histograms. Explain if your results obey the central limit theorem.
   (1) Chi-squared $\chi^2(1)$ distribution
   (2) Student t(5) with zero mean
   (3) Student t(2); for this case, replace σ with its sample counterpart.

Homework

Due: 2011/11/28

1. (LLN) Generate a random sample from an AR(1) model:

$$x_t = \rho * x_{t-1} + \varepsilon_{t-1}, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^{\ 2}), \qquad , \mathrm{t} = 1, ..., \mathrm{T}$$

and compute its sample average based on the following designs.

(1) Given $\sigma_\varepsilon = 1$, change the AR(1) coefficient $\rho = 0.2, 0.5, 0.8, 0.99$

(2) Given $\rho = 0.2$, change σ_ε to $\sigma_\varepsilon = 1, 2, 3, 4$

For each case, consider the sample sizes T=50, 100, 300, and 1000, and the number of replications is 1000. Plot the resulting histograms for each case. Explain your results **in detail.**

2. (CLT) Generate a random sample from an AR(1) model:

$$x_t = \rho * x_{t-1} + \varepsilon_{t-1}, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^{\ 2}), \qquad , \mathrm{t} = 1, ..., \mathrm{T}$$

and compute its normalized sample average,

$$\frac{\sqrt{\mathrm{T}}(\overline{\mathrm{x}} - \mu_x)}{\sigma_x},$$

where $\overline{\mathrm{x}}$, μ_x, and σ_x are the sample average, mean, and standard deviation,

respectively. Please simulate its normalized sample average based on the following designs.

(1) Given $\sigma_\varepsilon = 1$, change the AR(1) coefficient $\rho = 0.2, 0.5, 0.8, 0.99$

(2) Given $\rho = 0.2$, change σ_ε to $\sigma_\varepsilon = 1, 2, 3, 4$

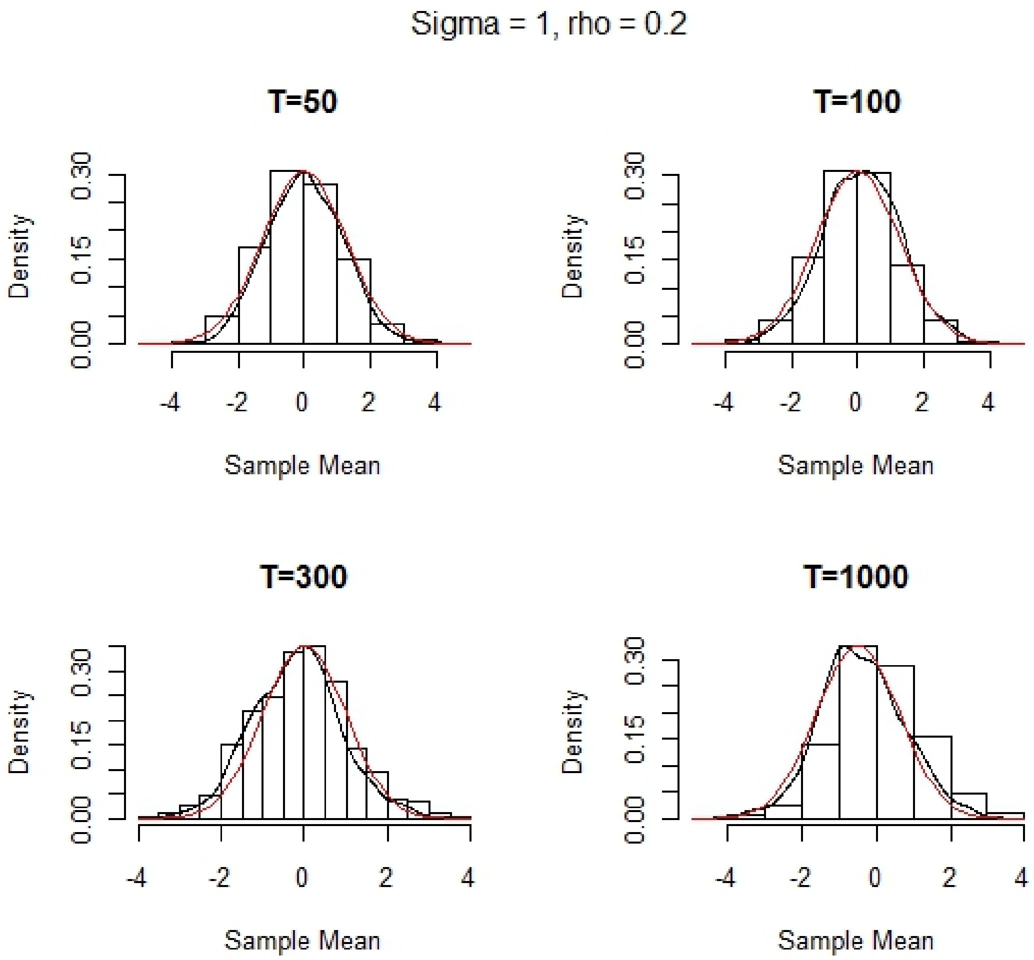
For each case, consider the sample sizes T=50, 100, 300, and 1000, and the number of replications is 1000. Plot the resulting histograms for each case. Explain your results **in detail**.

Hint :

(1) Do not restrict x range between -1 and 1 and try different **breaks number**. You may observe the difference in the figure.

EX : hist(fun_LLN(50,1000), **breaks = 20**, freq=FALSE,main='T=50',xlab='Sample Mean')

(2) In Problem 2 above, you may try to smooth the density (black line) and plot the standard normal distribution (red line) in the same figure for comparison.



normal_density = dnorm(seq(-4, 4, 0.1), mean = 0, sd = 1) #generate normal dist.

par(mfrow=c(2,2), oma=c(2, 0, 3, 0))

xtmp <- fun_CLT(50,1000)

hist(xtmp,freq=FALSE,main='T=50',xlab='Sample Mean')

par(new=T)

plot(density(xtmp), axe = NULL) # smooth your hist result

par(new=T)

plot(normal_density, col = "red", type = "l", axe = NULL) #plot standard normal distribution