NATIONAL TAIWAN UNIVERSITY

Department of Finance: Econometric Theory I — Final

Department of Economics: Econometric Theory III — Final

Fall 2010 Professor Kuan

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1. (18 points) Given the data $(y_t, \boldsymbol{w}_t')'$, $t = 1, \ldots, T$, let $\boldsymbol{x}_t$ and $\boldsymbol{z}_t$ be two vectors of variables taken from $\mathcal{F}^{t-1} = \{y_{t-1}, \boldsymbol{w}_{t-1}', y_{t-2}, \boldsymbol{w}_{t-2}, \ldots, y_1, \boldsymbol{w}_1'\}$. Your assumption for the QMLE method is:

$$y_t \mid \mathcal{F}^{t-1} \sim \mathcal{N}(\boldsymbol{x}_t'\boldsymbol{\beta},\ h(c + \boldsymbol{z}_t'\boldsymbol{\alpha})).$$

   Given this setup, discuss 4 possible misspecifications under which the information matrix equality would fail.

2. (25 points) Answer the following questions with "TRUE" or "FALSE"; explain your answer clearly or illustrate with an example.

   (a) If $\{y_t\}$ is a stationary ARMA series, $\text{var}(y_t)$ does not change with t.

   (b) If $\{y_t\}$ is an MA series, it must be stationary.

   (c) If $\{y_t\}$ is a white noise, it can not have ARCH effect.

   (d) If $\{y_t\}$ exhibits GARCH effect, $\text{var}(y_t)$ changes with t.

   (e) If $\{y_t\}$ exhibits GJR-GARCH effect, $\text{var}(y_t)$ changes with t.

3. (16 points) Given the specification $y_t|\mathcal{F}^{t-1} \sim \mathcal{N}(\boldsymbol{x}_t'\boldsymbol{\beta}, \sigma^2)$, where $\mathcal{F}^{t-1}$ is the information set up to time $t-1$, suppose that one would like to test if the errors $y_t - \boldsymbol{x}_t'\boldsymbol{\beta}$ follow an AR(1) process.

   (a) Explain why the Breusch-Godfrey test is not applicable when y_t are in fact conditionally heteroskedastic.

   (b) Based on the specification above, how would you test if the errors $y_t - \boldsymbol{x}_t'\boldsymbol{\beta}$ follow an AR(1) process? Write down the test statistic and limiting distribution; you must define your notations clearly.

4. (16 points) Given randomly sampled data $(y_t, \boldsymbol{x}_t')'$, $t = 1, \ldots, T$, where y is a binary variable and $\boldsymbol{x}$ contains k regressors, consider the logit model for y_t with the parameter vector $\boldsymbol{\theta}$. Note that when data are randomly sampled, they are considered *independent* random variables across t.

   (a) To test whether an $s \times 1$ sub-vector of $\boldsymbol{\theta}$ is zero, where $s < k$, what are the Wald statistic and its limiting distribution? Be specific about the asymptotic covariance matrix estimator.

(b) How do you test a single coefficient of $\boldsymbol{\theta}$ is zero? Write down the test statistic and its limiting distribution.

5. (25 points) You are given the data series $\{z_t\}$ and decide to fit an MA(2) model:

$$z_t = \alpha + u_t - \beta u_{t-1} - \gamma u_{t-2},$$

where α, β and γ are unknown parameters and $\{u_t\}$ is a white noise with mean zero and variance σ_u^2.

(a) To estimate this model with the QMLE method, state your assumption on the conditional density.

(b) Write down the resulting quasi-log-likelihood function and explain how the QMLEs of the unknown parameters (α, β, γ and σ_u^2) can be computed. .

(c) What is the null hypothesis for testing if $\{z_t\}$ is in fact a white noise?

(d) Explain how you can test the null hypothesis in (c) and write down your test statistic and its limiting distribution. All the notations in the test statistic must be clearly defined.

6. Bonus (14 points) Discuss why ARMA models are still limited in characterizing stationary time series and explain how they may be extended.
Note: You will NOT receive bonus points unless you have finished answering all other questions.