2010/10/11
Exercise

1. (LLN) Generate a random sample from an AR(1) model:

$$x_t = \rho * x_{t-1} + \varepsilon_{t-1}, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon{}^2), \quad , \ \mathrm{t} = 1, ..., \mathrm{T}$$

and compute its sample average based on the following designs.

(1) Given σ_ε = 1, change the AR(1) coefficient ρ = 0.2, 0.5, 0.8, 0.99
(2) Given ρ = 0.2, change σ_ε to σ_ε = 1, 2, 3, 4

For each case, consider the sample sizes T=50, 100, 300, 1000. Set the number of replications to 1000 and plot the resulting histograms for each case. Do your results obey the law of large numbers? Please explain **in detail** what you see and why.

2. (CLT) Generate a random sample from an AR(1) model:

$$x_t = \rho * x_{t-1} + \varepsilon_{t-1}, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon{}^2), \quad , \ \mathrm{t} = 1, ..., \mathrm{T}$$

and compute its normalized sample average,

$$\frac{\sqrt{\mathrm{T}}(\bar{\mathrm{x}} - \mu_x)}{\sigma_x},$$

where $\bar{\mathrm{x}}$, μ_x, and σ_x are the sample average, mean, and standard deviation, respectively. Please simulate its normalized sample average based on the following designs.

(1) Given σ_ε = 1, change the AR(1) coefficient ρ = 0.2, 0.5, 0.8, 0.99
(2) Given ρ = 0.2, change σ_ε to σ_ε = 1, 2, 3, 4

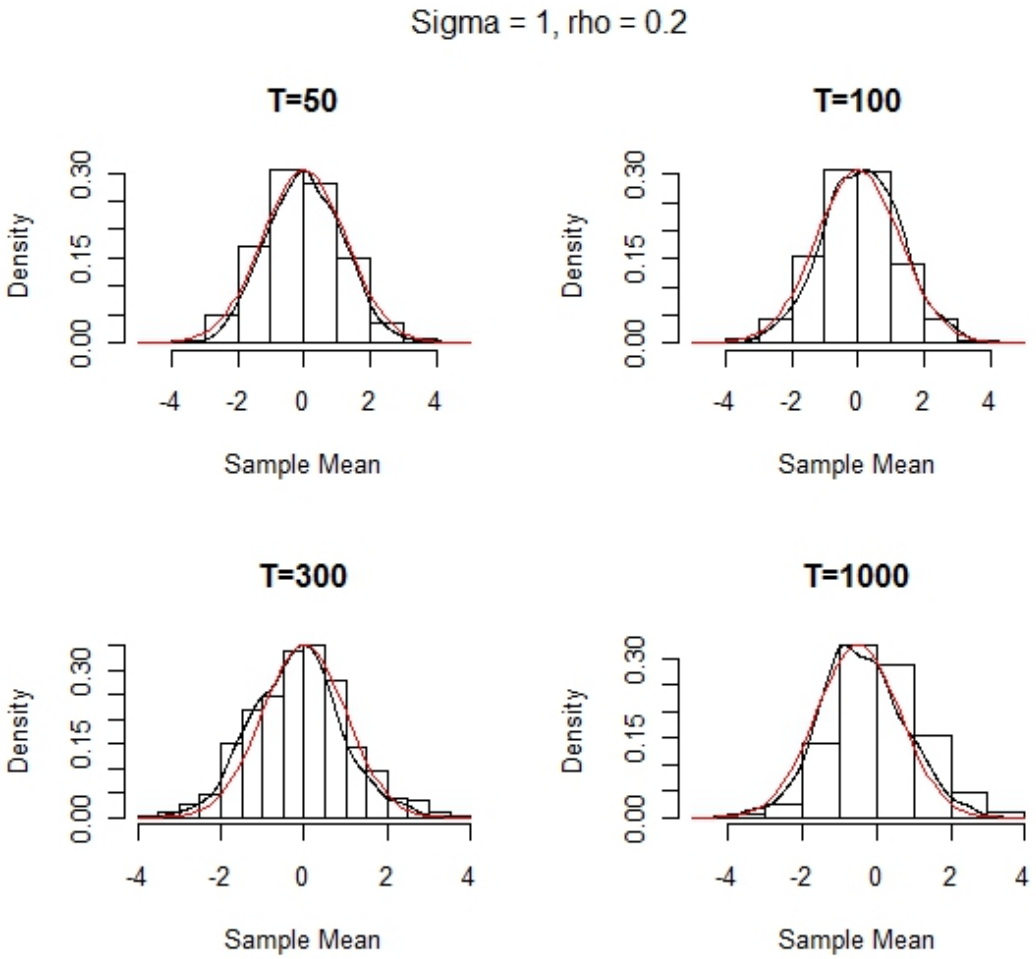
For each case, consider the sample sizes T=50, 100, 300, 1000. Set the number of replications to 1000 and plot the resulting histograms for each case. Do your results obey the Central Limit Theorem? Please explain **in detail** what you see and why.

Hint :
(1)
Do not emphasize x range from -1 to 1, and try to set different **breaks number**, you may observe the difference in the figure.

EX :

hist(fun_LLN(50,1000), **breaks = 20**, freq=FALSE,main='T=50',xlab='Sample Mean')

(2) In problem 2, you may try to smooth the density (black line) and plot the standard normal distribution (red line) at the same figure. Then, observe the result.



normal_density = dnorm(seq(-4, 4, 0.1), mean = 0, sd = 1) #generate normal dist.

par(mfrow=c(2,2), oma=c(2, 0, 3, 0))

xtmp <- fun_CLT(50,1000)

hist(xtmp,freq=FALSE,main='T=50',xlab='Sample Mean')

par(new=T)

plot(density(xtmp), axe = NULL) # smooth your hist result

par(new=T)

plot(normal_density, col = "red", type = "l", axe = NULL) #plot standard normal distribution